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LOGIC

BERNARD BOSANQUET

LOGIC

OR THE MORPHOLOGY OF KNOWLEDGE

BY

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PREFACE TO THE FIRST EDITION

THE conception of Logical Science, which has been my guide in the present work, is that of an unprejudiced study of the forms of knowledge in their development, their inter-connection, and their comparative value as embodiments of truth. If an attempt founded on such a conception appears to err by over-ambitiousness, I can only plead that an honest effort in the right direction does not depend for its value solely on its intrinsic success It is at any rate a heavy wager laid by the author on his judgment of the true aim and future of his science, and may attain results as a suggestion which it misses as an achievement

In the present centrifugal state of logical research, no undertaking, perhaps, can be entirely valueless which aims at reintroducing some sort of unity into the enquiry Such an aim is necessarily involved in the idea of a single connected science of logical processes and products It has therefore been my object to maintain the central identity of judgment and reasoning throughout, and in no case to permit the variety of applications due to diversity of matter to interrupt the connection and subordination demanded by unity of principle Although in periods of transition logical science has been most effectually advanced by detached discussion, in separate chapters, 'books,' or volumes, of Inductive or experimental method, of equational reasoning, and of the Logic of Chance, yet it seems plain that a time must come when the various cases and species of inference shall fall into their respective places

as organic members within the intellectual whole. But when this transformation is effected it is necessary that they should lose something of their interesting peculiarity and novelty; and it is natural, from the difficulty of the task, that they should lose more of these qualities than is necessary. For this reason, as well as from incompetence in the field of exact science, I claim no discoveries in scientific method or in mathematical reasoning, but shall be content if my attempts to represent them have not many more errors than are either necessary or natural

There is in England a healthy objection to system-making, and a preference for free criticism, against which I should be sorry to offend. I think however that systematic form is essential to clear exposition and to really effective criticism, and I have not supposed that my work will be considered as a system in any other sense than that thus implied. In particular, I do not hope or even desire that the specific names which I have invented for some kinds of Judgment and argument should come into general use I have endeavoured in every case to retain the accepted usage of the generic names, which are all that are recognised in ordinary logical discussion, and have only used strange titles for subdivisions which have no accepted place in existing logic, and which merely serve to insist upon certain views of logical evolution.

It is almost superfluous to acknowledge here what I owe to other writers, as the Index bears copious testimony to the amount of my borrowings. In bringing together 'Inductive' and 'Deductive' Logic I have followed more particularly Sigwart and Jevons, in the arrangement and analysis of Judgment-forms and forms of Inference I have gone to a great extent in the track of Lotze, and also of Hegel, to whom, so far as I know, the idea of this organic treatment of Logic is primarily due, and in fundamental theory of judgment as

in many details I owe even more, probably, than I have acknowledged, to Mr Bradley, whose work, 'Principles of Logic,' appears to me no less valuable now than it did three years ago [1] On one particular point, relating to the simplest process of equation (colour-equation, for example), I am especially indebted to Mr Spencer's Psychology.

Every student will understand that my obligation to former writers is frequently as great when I diverge from them as when I assent to their opinion As I have often been led to express disagreement with Lotze, portions of whose views certainly appear to me strange in so eminent a philosopher, I ought to say that but for his great work on Logic the larger part of what I have written would never have come into my head I may also express my strong conviction that the reform of Logic *in this country* dates from the work of Stuart Mill, whose genius placed him, in spite of all philosophical short-comings, on the right side as against the degenerate representatives of Aristotle. A glance at the Index will suffice to show how constantly I have referred to his treatise

I may venture, finally, to discharge an obligation of older standing, and at the same time to emphasise the guiding idea of my work, by observing that the first germ of unprejudiced interest in the forms of knowledge was implanted in my mind, when wholly innocent of Hegel or Lotze, by some remarks made in a course of Logic lectures which I had the good fortune to hear about twenty years ago, the lecturer being Mr. Alfred Robinson, of New College, Oxford. A comparison, which he threw out, between the study and analysis of judgment-forms and the study and analysis of the forms of flowers or plants has never left me since, and I have never seemed to myself able to exhaust its suggestiveness If I have at all reproduced for others the spectacle of continuity and unity in the intellectual

[1] See the author's 'Knowledge and Reality, a Criticism of Mr Bradley's "Principles of Logic"'. Kegan Paul, Trench & Co 1885.

life, combined with the most varied and precise adaptation of its fundamentally identical function to manifold conditions and purposes, which this comparison never fails to present to my own mind, I shall so far have succeeded in the object of my work.

BERNARD BOSANQUET.

LONDON, *April*, 1888.

PREFACE TO THE SECOND EDITION

In preparing the second edition of this work, I have endeavoured to put myself in the position of a reader of the first edition, aware of recent logical discussions, and interested to know how the book he is reading would respond to them I have therefore offered explanations which I hoped might be of service to such a reader, by enabling him to judge of the attitude which my theory would naturally assume towards a number of problems which have recently been emphasised.

I have made few alterations in the text,[a] but have added a good many footnotes, referred to by letters of the alphabet, while the old ones are as before referred to by numbers I subjoin at the end of the Preface a list of substantive additions other than footnotes.

I am gratified by Mr Keynes's appreciative reference to some of my views, in the fourth edition of his Formal Logic. I note that while favourable on the whole to my treatment of the concept as in close union with the judgment, he remains cold to my interest in the reciprocity of propositions and in the exclusiveness of members under a disjunction I only mention these points, because they seem to me to be involved in the ideal of complete expression by adequate conditioning of connections, which I take as defining the essential cognitive interest, on which, as the central idea of my work, I especially desire to insist.

It is, in my view, from the operation of this ideal, that there

[a] The addition on p. 57, concerning Mr Keynes's view of Extension, has involved some alteration of the text

arises the development of substantial knowledge within forms of sentence framed for all sorts of varied emergencies, and diversely one-sided in their emphasis This is why the dispute as to the meaning of propositional forms, on the field of mere usage, is really unending, but also why it is worth while to take pains in eliciting, from the full spirit and purpose implied in sentences, an import in the way of judgment relative to the ultimate interest which is continuously operative within them. The true meaning of propositions lies always ahead of fully conscious usage, as the real reality lies ahead of actual experience (II 303–4 below) To maintain the reality of a central cognitive interest as such, and the doctrine that it expresses its nature, though roughly, through forms of sentence [a] which cover phases of judgment progressive towards its satisfaction, is the double motive which inspires my logical theory.

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[a] I may venture to cite from Knowledge and Reality, p 57, a few words which express the problem as I stated it to myself *ab initio*, and still envisage it 'Affirmation about the unanalysed present perception may be considered as one extreme, supposal, having its subject indicated by a mere idea, definite but fictitious, as the opposite extreme. The forms of proposition from which the kinds of judgment have respectively drawn their names, are most precisely adapted for the expression of these two contrasted activities For the true region of human knowledge, which for the most lies between the two, neither form of sentence, neither class of proposition, or even of judgment,

The addition 'On a Defective Formulation of the Inductive Principle' is based on a paper published in the Proceedings of the Aristotelian Society of London, that 'On the Limits of a Genetic Theory of Logic' on a paper published in the Psychological Review I have to thank the proprietors of both these journals for their permission to use the papers in question

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OXSHOTT, 1911

is, if pressed home, absolutely and unrestrictedly appropriate,' and the passage proceeds to describe the transition between the two extremes.

I may here refer very briefly to Miss Jones's interesting treatise on 'A New Law of Thought', with Professor Stout's Preface, both of which contain much that would repay a fuller discussion

The substitution of S is P, as the general formula of thinking, for A is A, which, taken as such a formula, is meaningless, will I hope be adopted by logical theory and practice By dealing with a difficulty which so great a logician as Lotze could not overcome, the suggestion shows itself to possess a considerable value

The form S is P is certainly appropriate to the expression of the judgment But the precise analysis by which Miss Jones interprets it raises, I think, a very fundamental difficulty, to which I propose merely to call attention (The same analysis has been repeatedly urged *totidem verbis* by Mr Bradley and others, but with restrictions which point to the difficulty in question See Bradley's Logic, pp 29, 167, and the author's Knowledge and Reality, 273, and Essentials, 140, where the story from Thackeray has the same moral as Professor Stout arrived at in his Preface, though the point is less clearly stated)

That a judgment asserts difference of Intension along with Identity of Extension is an interpretation which holds good *prima facie* over the area of what commonly passes as the Categorical Judgment But (1) beyond this area it is not applicable, at least in any natural sense And (2) we are thus forced to see that within this area it is not perfectly applicable

(1) When what the judgment affirms is explicitly a law of connection or implication between intensions—when, say, you have before you a true hypothetical judgment, the analysis in question does not really apply. You cannot reduce the meaning of such a judgment to the identification of cases Systematic continuity of any kind, causality relation of antecedent to consequent, of condition to conditioned—these seem quite irreducible to an identity of subjects plus difference of attributes But that one intension can 'be'—involve or imply—another is a possibility which, as I understand, Miss Jones absolutely and in principle denies (cp, however, p 46 of her work) I should have thought that this denial brings her view into sharp collision with usage,

e g on p 41, where, if I understand right it is contended that admitting the judgment 'All politicians are statesmen' we are yet entitled to deny that the quality of statesmanship can be inferred from that of being a politician And so elsewhere, pp 45–6, and 73

Here is the root of the difference between us referred to on p 7 of 'A New Law of Thought' Between the letters *A* and *B* I recognise no such continuity as I look for, say, between a cause and its effect. And therefore I call them unfit to symbolise the continuous identity expressed in a judgment which states a law '*A* is *B*' would not, in fact, be self-explaining as a formula for the new law of thought '*S* is *P*' is rightly chosen, because it obviously refers to the structure of the judgment. But what this is, still needs explanation

(2) 'Un giudizio e sempre la *formulazione* d'una legge,' Varisco, 'I massimi Problemi,' p 89 The point here is in the word 'sempre' At bottom, according to the view which I have adopted, ***all*** judgment, and even association, contains in it the principle of universal connection, of a law, however imperfectly formulated From the very beginning, conjunction covers connection The doctrine is well known, and it would be idle to re-argue it here I need only point out that for good or evil, what stands between us is the conception that identity must be reduced to numerical identity The judgment is split up into numerical identity plus intensional difference This answers *prima facie* for apparent categoricals, but I doubt whether it is sound in principle even for them

It occurred to me, indeed, that the author might intend to restrict the explanation to these judgments, which play so large a part in the operations of Formal Logic But I believe that any such restriction would be disowned (See analysis of a geometrical truth on p 46, and note the treatment of the inseparability of the *S* and *P* intensions as merely *compatible* with the meaning)

I may add that I welcome the criticism of Jevons's 'Simple and Obvious Law of Thought', p 36, for which, so far as I know, the author is right in claiming priority And I do not deny that in so far as a universal connection presupposes a world or system within which it falls, a further analysis of identity in difference might bring out features bearing a superficial likeness to those of Miss Jones's conception But I cannot think that any analysis can finally acquiesce in a view which restricts itself to irrelevant conjunctions

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ABBREVIATIONS

Bradley's Appearance = Bradley's Appearance and Reality, ed. 2. Sonnenschein, 1897.

Bradley's Logic = Bradley's Principles of Logic Kegan Paul, Trench & Co, 1883.

Keynes = Keynes's Formal Logic, ed 4 Macmillan, 1906

Prichard = H A Prichard's Kant's Theory of Knowledge Clarendon Press, 1909

Essentials = Bosanquet's Essentials of Logic. Macmillan, 1895.

Sigwart = Logik, von Dr Christoph Sigwart 2nd edition Freiburg, 1889

INTRODUCTION

'If it is held a valuable achievement to have discovered sixty and odd species of parrot, a hundred and thirty-seven species of veronica, and so forth, it should surely be held a far more valuable achievement to discover the forms of reason, is not a figure of the syllogism something infinitely higher than a species of parrot or of veronica?'—HEGEL, *Wissenschaft der Logik*, p 139

'We have seen that the members of the same class, independently of their habits of life, resemble each other in the general plan of their organisation This resemblance is often expressed by the term "unity of type", or by saying that the several parts and organs in the different species of the class are homologous The whole subject is included under the general term of morphology This is one of the most interesting departments of natural history, and may almost be said to be its very soul'—DARWIN, *Origin of Species*, p 382

Title of the Work

1 IN giving to the present work the title of 'Morphology of Knowledge' I intended to indicate as its purpose the unprejudiced study of judgment and inference, throughout the varied forms in which their evolution may be traced and their relationships determined Mere classification, therefore, or mere enumeration of the species of judgment or of inference, would not achieve the aim which I have set before myself, although I am wholly of Hegel's mind when he says that the species of syllogisms are at least as well worth discovering as those of parrots or of veronicas The two quotations which stand at the head of this Introduction may together exhibit my conception of a logical system, if interpreted to mean that I would treat the forms of judgment and of inference as science treats the forms of animals or of plants, not in the spirit of enumerative classification, but in the spirit of morphological analysis In the conception so determined, however, one further correction must be made Morphology, as the science of external shape, involves, I understand an antithesis to physiology as the science of vital function This contrast belongs to the distinction of outer and inner, of persistent bodily form and living productive process, which appears at

first sight to prevail in the world of visible and tangible matter Were we to transfer any such contrast to the scientific treatment of intelligence, we should obviously be forced to identify the morphology of knowledge with the science of language or of grammar, and reserve for the analysis of the vital thinking function some such appellation as that of Mental Physiology I do not say that such an antithesis would be false, but it would fail to illustrate the point of view on which I am now desirous to insist In the systematic activity of thought the contrast between bodily shape and vital process is non-existent. Even the evolution of the animal organism might be considered as the development of a function which is a system of functions, and the science of life if thus regarded would unite, as it appears to me, what is valuable in morphology with the essence and spirit of physiology Of the system of knowledge, at any rate, this is the true account The form of thought is a living function, and the phases and moments of this function are varieties and elements of the form Therefore the 'Morphology of Knowledge' must be construed as not excluding the Physiology of Thought The science of intellectual form includes the science of intellectual life

Knowledge and its Content. Truth

2 Knowledge involves the ideas (i) of Truth and (ii) of Meaning 1 How does the analysis of knowledge as a systematic function, or system of functions, explain that relation in which truth appears to consist, between the human intelligence on the one hand and fact or reality on the other? At first sight, even the genetic[a] analysis of a systematic development which we propose to undertake, though a more genuine explanation of that development than any mere summary of types, is powerless to grasp the relation between the system so developed, and an object-matter that lies outside it To this difficulty there is only one reply If the object-matter of reality lay genuinely outside the system of thought, not only our analysis, but thought itself, would be unable to lay hold of reality For logic, at all events, it is a postulate that 'the truth is the whole'[b] The forms of thought have the relation

[a] For a discussion of the Genetic Theory of Logic, in relation to the standpoint of the present work, see II, pp. 238, 269

[b] See further, on the coherence theory of truth, II, ch IX.

which is their truth in their power to constitute a totality; which power, as referred to the individual mind, is its power to understand a totality The work of intellectually constituting that totality which we call the real world is the work of knowledge The work of analysing the process of this constitution or determination is the work of logic, which might be described as the self-consciousness of knowledge, or the reflection of knowledge upon itself Logic has no criterion of truth nor test of reasoning Truth is individual, and no general principle, no abstract reflection, can be adequate to the content of what is individual It is indeed impossible to study the growth of knowledge without lighting upon confusions of thought that evoke a warning word But even a confusion of thought may have a material significance, and therefore contain a material truth, which escapes the logical critic who perforce ignores its individual content The relation of logic to truth consists in examining the characteristics by which the various phases of the one intellectual function are fitted for their place in the intellectual totality which constitutes knowledge The truth, the fact, the reality, may be considered, in relation to the human intelligence, as the content of a single persistent and all-embracing judgment, by which every individual intelligence affirms the ideas that form its knowledge to be true of the world which is brought home to it as real by sense-perception.

The real world for every individual is thus emphatically *his* world, an extension and determination of his present perception, which perception is to him not indeed reality as such, but his point of contact with reality as such Thus in the enquiry which will have to be undertaken as to the logical subject of the judgment, we shall find that the subject, however it may shift, contract, and expand, is always in the last resort some greater or smaller element of this determinate reality, which the individual has constructed by identifying significant ideas with that world of which he has assurance through his own perceptive experience In analysing common judgment it is ultimately one to say that *I judge*, and that *the real world for me, my real world, extends itself*, or maintains its organised extension Thus is the ultimate connexion by which

the distinction of subject and predication is involved in the act of affirmation or enunciation which is the differentia of judgment.

Meaning

11 To speak of consciousness as a single persistent judgment is at first sight a paradox, in view of the distinction between an idea and the affirmation of an idea It is not easy to deny that there is a world of ideas or of meanings, which simply consists in that identical reference of symbols by which mutual understanding between rational beings is made possible A *mere* suggestion, a *mere* question a *mere* negation, seem all of them to imply that we sometimes *entertain* ideas without affirming them of reality, and therefore without affirming their reference to be a reference to something real or their meaning to be fact We may be puzzled indeed to say what an idea can mean, or to what it can refer, if it does not mean or refer to something real—to some element in the fabric continuously sustained by the judgment which is our consciousness. On the other hand, it would be shirking a difficulty to neglect the consideration that an idea, while denied of reality, may nevertheless, or even must, possess an identical and so intelligible reference—a symbolic value—for the rational beings who deny it A reference, it may be argued, must be a reference to something But it seems as if in this case the something were the fact of reference itself, the rational convention between intelligent beings, or rather the world which has existence, whether for one rational being or for many, merely as contained in and sustained by such intellectual reference

I only adduce these considerations in order to explain that transitional conception of an objective world or world of meanings, distinct from the real world or world of facts, with which it is impossible wholly to dispense in an account of thought starting from the individual subject The paradox is that the real world or world of fact thus seems for us to fall within and be included in the objective world or world of meanings, as if all that is fact were meaning, but not every meaning were fact This results in the contradiction that something is objective, which is not real

We do not meet this paradox adequately by reminding ourselves that e g. a negation or a question is a fact in some one's

mind, and therefore as a psychical occurrence is a real event, and in that sense falls within reality In considering an idea as a psychical occurrence we abstract from its meaning, which apparently consists in some identity or persistent unity that extends beyond the isolated occurrence of the idea in time

The solution of the difficulty appears however to be suggested by the distinction which we have just taken. As the psychical occurrence and objective reference are distinguishable by abstraction only, so objective reference and reference to reality (affirmation) are distinguishable by abstraction only. The world of objective reference and the world of reality are the same world, regarded in the former case as composed of isolated though determined contents, and in the latter case as composed of contents determined by systematic combination in a *single* coherent structure The former point of view is an abstraction that goes near to be a fiction, for isolated contents *qua* isolated are not determined But it is possible and usual to consider ideas, by help of abstraction, in respect of those relations which especially determine them, as detached islands or spheres of knowledge, without actually rêferring them in a thoroughly determinate judgment to a place in the *one* individual system which for each of us is the reality Ideas thus isolated are what we 'entertain' as significant or symbolic ideas which have meaning or objective reference, and yet are not affirmed of Reality Objective reference is the substance of the convention by which rational beings communicate with each other and with themselves, and which, though a consequence of the unity of Reality, can be regarded without being directly identified with that unity To show that every idea which is *entertained*, as for example *suggested* or *supposed*, must be taken to be *ultimately* affirmed of reality, is the task of those portions of logic which deal with question, suggestion, supposition, and denial Thus in the discussion of mere or bare denial we shall be forced to the conclusion that where, in an attempted judgment of denial, no positive basis nor positive consequence of the denial is to be discovered,—where, that is, the idea which is verbally *denied* forms no element *in any affirmation of any kind* about Reality, there no judgment can really take place, nor can the idea in question be enunciated

as an intelligible idea The meaning which an idea seems to retain when named in a spurious judgment or unintelligible proposition of this kind depends on its initiation of other acts of thought than the denial in which *ex hypothesi*, we were to look for its unaffirmed signification Our treatment of supposition will lead us to the same result All significant, i e all logical, ideas are ultimately elements in a single judgment, the judgment which sustains the ideal structure of the real world.

Thus the world of truth and the world of meaning are not really distinct, and the process which logic investigates is the single process and individual self determination of the whole which is the truth or reality

What knowledge are we investigating?

3 It is natural to ask 'Where and what is this self determination? Where does it begin? Where does it end? Is it in the individual mind or in the history of the race, or in an arbitrary combination of the two?'

I cannot attach much importance to this question, which might be asked with no less justice as it appears to me of any other science But no other science would admit that it has a meaning A science deals with its object-matter wherever it finds it The self-determination of the knowing intelligence as the real world takes place wherever there is knowledge and in as far as there is knowledge If the question is whether the process as described is necessarily involved in knowledge or is a mere fact like any isolated fact, there can be no answer except that the question is either superfluous or meaningless Of any particular logical theory, such as that contained in the present work it is superfluous to ask whether it is in all respects necessary and absolute Of course it will contain many erroneous and many accidental elements But to ask whether a true account of the logical process would be necessary and essential or would contain mere fact or accident, is simply to ask whether a true account may not perhaps be false. All truth is necessary in as far as it is rationally known And if a scientific analysis were to lead to no truth at all, it would simply lead to nothing Whether in any particular instance such a nothing may have resulted is a matter for consideration on the merits of the individual case But in general I must protest that there is no more reason in asking

what evolution of thought we are studying,[a] than in asking *what* laws of motion are studied by mechanical science. The laws of motion express the characteristics of moving bodies *qua* moving, and the laws of knowledge express the characteristics of knowing subjects *qua* knowing It is no more necessary to specify in what particular cases you find knowledge, than to specify in what particular cases you find motion. If knowledge has a nature that can be studied[b]—and if not, there is no logic—it can only exist where that nature is realised, and however the progress of the race may have prepared the intellectual inheritance which is devolved upon successive individuals, those individuals can only make it their own by fulfilling the conditions which constitute its nature I do not deny that the type and content of knowledge must change with the advance of evolution, I only affirm that any such change depends on the modes and degrees in which the general conditions of knowledge are fulfilled in successive generations and not in any difference between knowledge in its essential nature, knowledge as developed in the race, and knowledge as a process within the individual intelligence

4. To give a name is for civilised thought the first step in knowledge It at once depends upon, and in a sense creates, a recognisable arrangement of things, qualities, and relations Wherever new ground has to be appropriated, whether actually or in metaphor, the first necessity is to find recognisable points, by which, being named, we can observe and communicate our whereabouts The act Naming

The value of this first step is only to be estimated by experience, now necessarily exceptional, of the attempt to attain knowledge without it We do not experience this simply by going where a language is spoken that we do not understand.

[a] Genetic theorists say, if I understand them rightly, that it is mainly or wholly the evolution of thought as an adaptation to social or external needs It is all very well to illustrate the stimuli of thought in this way, but to restrict them in this way does seem to me ridiculous There is nothing in the universe of experience which cannot act as a stimulus to thought See II, pp 271 275

[b] Of course this does not mean 'studied apart from reality'. The world of knowledge is one of the worlds of reality, organised according to its law, under certain limitations

No doubt, in the long run, lack of communication with our fellows would affect our reasoning power, but we are now looking for an instance in which we ourselves, within our minds, have no names to aid us in distinguishing and recognising things We find such an instance in our attempts to deal with any new region of knowledge of which we have not mastered the rudiments and in which we have neither books nor teachers to guide us This is merely individual ignorance, but the great pioneers of knowledge must be in the same predicament when, going beyond established distinctions and taking note of new phenomena, they lay the foundations of a fresh structure of science. Great discoverers are able to add fresh names to language, ordinary men content themselves with learning the meaning of those in common use. The limitations of popular nomenclature form the limitations of popular observation When we are brought face to face with a scientific classification and the terminology it involves, we are astounded at the blindness in which we had contentedly been living Every yellow ranunculus we call a buttercup, every myosotis a forget-me-not, every large white umbellifer a hemlock, not merely as an epithet, but because we really see no difference. So in the history of architecture or of fine art, popular knowledge is confined for the most part to the application of two or three terms which have gained currency Few people are able to observe without the help of names

It is true that there is something ludicrous in the tendency of common minds to cling to a name; in the insistence of an inexperienced art-critic on superficial characteristics which happen to be nameable, when he ought to be looking into the special significance of a work of art, in M Jourdain's delight at the discovery that he conversed in prose, or in the sudden zeal of Strepsiades for the correct employment of the masculine and feminine terminations Nevertheless, the current censure of verbal knowledge is itself largely founded on ignorance, and actually on the same ignorance which creates the risk and opprobrium of mere verbal knowledge In the annals of philosophy there is nothing more tragi-comic than Mill's condescending excuse for Plato's discussions about related

existence—discussions which laid the permanent foundations of scientific logic, the modern logician showing his superiority to verbal quibbles by an attempt to dissociate existence from attribution based simply and solely on the fact that existence is sometimes expressed by a peculiar kind of attribution The condemnation of a knowledge which rests in mere words too often means that the word and no more has reached the understanding of the critic It is probable that we think too little rather than too much of Naming as a first step in knowledge To give names which endure is with few exceptions the prerogative of genius The number of terms which we inherit from Plato and Aristotle is among the most striking proofs of the immense advance which they won for the human intellect These two great minds mapped out the world of knowledge in its essential features much as we have it before us now, and gave to its main divisions the names which they still retain Or, again, what a gigantic advance was made by the work of Linnæus, though it now serves as the stock example of an 'artificial' classification! It was the indispensable starting-point for the more profound and rational researches of modern times, and thus if not one of the most arduous, at least one of the most valuable, of scientific achievements

I spoke of Naming as the first step in knowledge *for civilised thought* Plato recognised the operation much as we recognise it And yet there must have been a time when it was not easy thus to isolate a single word from the sentence Indeed, even now, the single word *is* not really isolated Except in the instances in a logical text-book, the utterance of a single word always implies a sentence, and usually a judgment We now print the title of a book or the description of a species of flower, contrary to older custom, without a verb, but none the less they are read off into propositions or judgments The artificial nature of the supposed concept which is thought to correspond to a name and to be generically other than a judgment is curiously illustrated by the fact that Linnæus was the first to omit the verb in the descriptions of flowers Such descriptions, though in appearance reduced to logical names, are of course understood as sets of judgments No doubt however

all these habits, including the use of dictionaries, familiarise us at least with the *appearance* of the significant noun in complete isolation from the sentence, and lead us to imagine that in such isolation it is still a fair representative of some individual object or quality (also isolable) in a way in which the complete sentence is not But I must repeat that under no circumstances does a man in his senses make use of an isolated noun, except to indicate an assertion, wish, or command. A dependent sentence, as its name shows, cannot stand alone; and a name is for grammar in this respect like some form of dependent sentence If a man were to say 'the sun', the difference between this and a given proposition like 'the sun is low', is not that in the first case we have a mere name and in the second a proposition, but that in the second case we have a given proposition, while in the first we are set to make propositions at random The same is true of any logical noun, i e descriptive or appositional sentence —and a noun may, we shall see, be equivalent even to a conditional sentence—such that it can stand as subject or predicate in a proposition No such sentence is ever used independently, for all thought, if not optative or imperative,[1] is categorical

Thus it would seem that the isolation of the significant name from its context, which is even now more apparent than real, must in very early stages of language have been a wholly unfamiliar process The history even of proper names shows a tendency to illustrate this, though proper names for human beings would be, one would suppose, among the earliest productions of language. As we go back, we find the 'proper' name less purely distinctive, less 'proper', and more significant or predicative The animal names borne by some savages must be significant, though how or of what may be doubtful The Roman Agnomen and the Athenian Deme name were directly significant, as of course are many modern surnames in their recent origin The addition of a name of father (also husband or master) in the possessive case was not originally

[1] Whether optative and imperative forms can or should be analysed into categorical propositions is a psychological rather than a logical question.

a mere appellation, it was an assertion of ownership[1] It is well known that among the Romans the gentile name (*nomen*) was the name *par excellence*, as indeed for most purposes the surname is now. It is less well known that the 'fore-name', which, if any, was individual, was not at all freely chosen to serve as a distinctive sign, it was characteristic of family, and the choice was exceedingly restricted Among all the Roman patricians only about thirty 'fore-names' were in use, and the 'fore-name' of women was constantly omitted in Cicero's time It would be interesting to find parallels for some of these features in modern usage, a small number of Christian names no doubt serves to name by far the greater number of individuals in one country, and the selection of Christian names is to a certain extent characteristic of families or of family But on the whole it is now admitted that the chief purpose of a proper name is to be a name, i e a constant sign, and even the surname, although significant in many respects, is not really to be relied on as an indication of family. The law lets a man bear what name he pleases and change it as he pleases, so long as he makes his desire sufficiently well known, in other words, the law accepts no purpose in the name beyond that of mere recognition

And in the case of the common or significant name the same thing is more evident Even so highly modernised a language as classical Greek has no unambiguous expression for 'a word', though 'noun' and 'predicate' or 'verb' were familiar terms to Plato and Aristotle The latter has to describe a 'word' by the periphrasis, 'the least portion of discourse which is significant when taken by itself' The Greeks did not separate their words in writing, and in their inscriptions a terminal consonant is affected by the following initial consonant, as it would have been in a compound word I do not think it is fanciful to refer to these facts in illustration of the closer cohesion of sentences in ancient than in modern speech The solvent of the sentence is obviously writing You cannot take

[1] Mommsen, Romische Forschungen, vol ii, p 5 He shows good ground for supposing that the original form was 'Marcus Marci', used equally of wife, slave or son, and that Marcus Marci filius (no corresponding form was adopted in Greek) was a later modification

to pieces the spoken sentence as you can that which lies written before you If you ask an uneducated man about some one point in what he told you, he will say it all over again. But this enquiry hardly belongs to logic, though it helps to rouse us out of the analytic abstractions in which we are now at home I only wished to guard myself against asserting that the conscious selection of an individual object and the appropriation to it of an element of language common to all sentences in which that object is referred to—that this act of Naming comes first in history as it does in modern science It appears to me that the descriptive sentence must have furnished the material for a subsequent appropriation of names, and that the appropriation of names by habitual description must have been quite a different process from methodically searching for new 'points d'appui' and fixing their appellations at one blow.

What the act of Naming implies

Logical Significance

5 The act of Naming implies in the first place 'Logical Significance'. 1 A name then is a sign which rouses the mind to a set of activities having an identical element. In the purely artificial case, when a name is spoken in my hearing without any context expressed or implied, such activities may probably take the shape of interrogation or suggestion; i e as is commonly supposed (but see sect 2, above), a review of matters which we might employ in judgments, but do not yet know how to, being unable to attach any of them to our real world. The meaning of the name consists in its power of suggesting and controlling these activities, these judgments completed *or inchoate* If, to return to the example suggested above, I hear some one say 'The sun', my first idea is that the speaker is thinking aloud, and that I have caught the fragment of a sentence which he has completed in his mind. But with a view to logical theory we may neglect the speaker's intention (though no theory should forget that it has neglected this feature of the case), and simply consider what the word does for the hearer It makes him think of something, and in this case of what is called *par excellence* a *thing* had the word been 'red', it would have suggested a *quality*, had it been 'parallel', a *relation* We have not to do just now with the difference between these three kinds of signification, but only with what signification is as such In thinking of something

without more guidance than a significant name, we find ourselves involuntarily thinking not merely *of* it, but *about* it. And this is inevitable That which the name signifies *is*, for us at all events, an identical character exhibited by different contexts, or different contexts united by a common character. Any one who has been told, by an old-fashioned mesmerist, to ' think of nothing but ' a copper and zinc disk which is put in his hand, and which he is expected to contemplate for some minutes, must have found (supposing that he attempted to do what he was told) that his thoughts traversed in a series of judgments the various ways in which the thing affected his perceptions, or reacted in the comparisons that suggested themselves [1] These judgments, if expressed in language as propositions, would all contain the same name, that of the thing which they described But although connected with different standards of comparison in the different judgments, the thing spoken of in them all is not different things, but the same thing If you persevere and try to elicit the root and basis of its identity, you may indeed fix more or less arbitrarily upon certain ' essential ' attributes, but these attributes represent the thing in different contexts, and are also themselves, as Mill has explained,[2] elements of identity between different contexts You may judge the thing to be round, hard, heavy, flat, cold, and to be on the palm of your hand and you may define circular form, hardness, weight, etc as you please , but you will not express either thing or attribute as other than an element of identity which is exhibited and takes shape in different aspects or relations Mr Bradley [3] has pointed out that

[1] I suspect that the particular mesmerist to whom I refer was influenced by the fallacy which has been combated in the text, and imagined that to think continuously of an individual thing involved an immobility of thought, as though the thing were for thought like an image in space fixed and isolated In attempting to attain such an immobility (which attempt, on this hypothesis, the operator intended to be made) the patient would simply arrest the operation of his intelligence, and would thus approach to that withdrawal of attention from all specific stimuli which is perhaps a condition of the mesmeric sleep How far this principle is connected with that of Braid s and similar experiments, I am not expert enough to say Cp Lotze, Metaphysik, sect 304

[2] See Mill s Logic, 1 201 (sixth edition)

[3] See Bradley s Principles of Logic, p 44

extension in space or duration in time are sufficient to invest that which has them with the character of an identity into which differences enter This—an identical element which enters into and is entered into by differences—is what we might call the logical significance, the significance which must be postulated in all cases, of a name as such

The process of Naming as known to our reflective thought, is to adopt an individual element of language as the instrument of intellectual reference to an individual identity in the knowable world The conscious adoption or appropriation of the linguistic sign is the same thing in principle, whether the sign is employed solely within my own consciousness, or is applied to communication with other intellectual beings The 'convention'[1] or agreement which has been said to give language its meaning, would be the same thing between other persons and me that the employment of significant signs is between me, so to speak, and myself It is as wonderful, and as much a proof of 'convention', i e of the power to agree, that 'goodness' should mean the same to me yesterday and to-day, as that in this sentence it should mean the same to me and to my reader It may be said, and here we anticipate a difficulty which must be treated later in this chapter, 'But goodness does not mean *quite* the same to you and to your reader, or to you at different times' Then I will change the phrase and substitute 'refer to' It will be seen at once that if 'goodness' in my mouth and in yours does not *refer to* the same characteristic, it is not intended to *mean* the same, and its meanings however different cannot be conflicting In that case the two characteristics referred to are 'homonymous', and the same word goodness is used for them only by accident[2], as glass is for a telescope and a tumbler The point and purpose of a name is, always to refer to the same, it is on this reference that the whole possibility of mutual intelligence depends The connection between isolated reference, or meaning, and reference to a system, or affirmation, has been explained above (sect 2)

[1] A name is speech which has meaning according to convention (*κατὰ συνθήκην*) Arist Περὶ Ἑρμηνείας, sect 2

[2] By accident as regards present use, there may be a common history, but this is rather a source of deception than of clearness

Identical reference or rational convention is thus the root and essence of the system of signs which we call language[1] The act of Naming, i e of establishing such a reference, and of appropriating a sign to it, has been elaborately analysed into a number of processes or aspects In my opinion such an analysis should be regarded mainly as a *mere* analysis—as a distinction of aspects and not as a history of acts Historical conclusions may flow from it, but the analysis is the first thing We are here met by a difficulty which besets all the higher sciences, and which I shall endeavour in the first instance to grapple with in its general form

Meaning of 'implies'

11 The distinction of stages in a continuous growth has always a degree of artificiality It is hard to say precisely when an embryo becomes a chicken, or a boy a man It is impossible to say at what point feeling appears in the organic world, or when a child acquires a will, or a primitive tribe the instinct of religion or of fine art Characteristics which attract general notice only when full-grown, are traceable far back when we come to look for them, and further, they are frequently implied by the nature of an individual long before any scrutiny can detect them It is a cheap and false accuracy to express such a growth in successive stages according to the definite emergence of obvious features, without scrutinising the continuous identity which is present from beginning to end But it is a fatal carelessness, on the other hand, to treat rudimentary attributes as *ipso facto* equivalent to their mature form, or as necessarily identical therewith in the features of chief concern The labour of genuine science is to disentangle the *true* continuity of processes, limiting it only by modifications which are certainly traced or inevitably implied, never assuming the existence of a highly developed attribute or function where it is not seen in operation, or shown to be

[1] I can see no ground for restricting the *logical* conception of language to written or spoken words We must not argue from the possibility of educating the deaf and dumb (cp Lotze, Logik, sect 6) that 'the logical operation in the mind is independent of the possibility of linguistic expression'. It is unfortunate that the German 'Sprache' and 'sprachlich' make this inference appear a truism, while if we ask whether the deaf and dumb can in thinking dispense with fixed signs wholly or in part, the question, though still of interest, assumes a very different complexion

implied to the exclusion of all conceivable alternatives. In fact, if 'conceivable' means 'conceived up to a certain stage of knowledge', the last clause is not stringent enough, a suspension of judgment is often preferable to a conclusion from disjunction which violates general analogy. Many complete-looking disjunctions are imperfect, and unless supported on a thoroughgoing principle, a disjunction is worth nothing at all.

It is hard to escape both the complementary blunders which I have indicated, and the object of this digression is to point out that though we may not escape them altogether, their sting is removed if we are not too ready with our disjunctions, but discuss to the best of our power the principles which underlie functions and attributes, and the consequent limits and laws of their modifications Have such and such savages the instinct of fine art? Probably Yes or No would be no answer. We should find that they had some elements or germ of such a tendency, perhaps the love of imitation, or earlier still, the instinct of construction Then we should have to estimate the value of this and its connection with aesthetic capacity; and the correctness of our whole reading of the facts would probably depend on the rightness of this estimate The act of naming presents such a problem. Does Naming, for instance, imply the processes of Comparison and Distinction? Does it imply a Judgment, such as the Judgment of Perception? If we look at the activity of highly reflective thought, we must unhesitatingly answer both questions in the affirmative It is a serious matter to introduce a new word into language, or to christen a new phenomenon or a new species All that science can do to verify and determine is being done when, and is largely done before, such an event occurs Or if we go to the extension of the individual's knowledge, which is to him a creation of new appellations, the same holds good for kindred reasons The individual profits by the work which language presupposes, and all sorts of apparatus is at hand by which he can put himself through the processes, in learning a name, which the discoverer went through before him in conferring it This is the one extreme of the growth we have to watch, the extreme

at which the function we are discussing has become an instrument of conscious science

iii. In speaking of the opposite and lower extreme, we have to depend on analysis and implication.[a] Let us think of the feature of *reference*, which we found to lie at the root of intelligent speech A name always *refers* to something (I must repeat that the idiomatic 'something' is not to be taken as meaning an individual '*thing*' in space) I avoid saying that it refers to an idea in the mind, because, unless the name explicitly proclaims itself the name of an idea, it does *not* refer to an idea in the mind as such[1] When I use the word 'red' I do not refer to or mean my idea of red considered as my idea, though I do mean red as I understand it by help of my idea When I use the word, I mean a colour, a quality of surface, or at least of light, which I represent to myself by help of one or more reds which I have seen, but which I think of as not dependent either for being or for quality on my happening to know it. The fact that my perception of red may be abnormal does not affect this reference As I pointed out above, if it were not for the identical reference there could be no conflict, no question of normal or abnormal This, then, if no more, is involved in naming. That which is named is recognised as having a significance beyond the infinitesimal moment of the present, and beyond the knowledge of the individual It enters into the 'convention' which he who uses language maintains with himself and with others It is, in short, characterised as an object of knowledge[2] Under this aspect the act of Naming has been well called the act of Objectification. Objectification

Let us further consider what is implied in the act of Naming considered as an act of Objectification

iv. In the first place, then, the matter which is invested A positive content

[a] These sections on naming and objectification would be very misleading if they implied that contents, e g. of sensation, which have never been named or 'objectified', are not objects, but mere subjective states. But it is quite true that a sensation takes a different rank in knowledge when it has been attended to and named For the relation of mental states to objects of thought see p 69 below, and II, chap X

[1] See below, chap I, sect I

[2] Cp Lotze, Logik, sect. 3.

with the attribute of being such as to be known—which is thus 'objectified'—must be something positive, something, so to speak, of affirmative nature[1] It must be presentable to consciousness by help of some actual modification of consciousness I mean that it cannot consist simply and solely in the distinction between itself and something else It is one question how we come to perceive a certain content—contrast and distinction may be essential to perception—another what it has in it when brought into perception It may be—the question is chiefly psychological—that if red had been the only colour we should never have been aware of it *as* red But now, being aware of it, we find in it a positive quality or character which is not exhausted by the distinction between it and all other colours This is the primary condition of the act of naming That which is to be referred to by a name, which is thus erected into an object for intelligence, must be at least a positive content, something with a nature and character It is not safe to clinch the matter by saying '*definite* nature', '*determinate* character', for though scientific naming involves all this, yet it would be overbold, and would beg a question which we shall soon arrive at, to presuppose all this for every act of naming 'Definite' and 'Determinate' introduce negation into knowledge, and so are not words to be used lightly We must if possible keep to one thing at a time, and what we are sure of at this point is this, that the identity in difference which is referred to by a name is something positive, not necessarily a 'quality' in the technical sense of an immediate unrelated matter of perception, but necessarily a something.

It is obvious that for the general purposes of logic such meaning is not confined to substantive and adjective nouns or names. A positive content is also referred to by the material or uninflected element in verbs, if we may for logical purposes distinguish this from the formal or inflected portion, which indicates the connection and function of the verb as such. When I say 'it flows', of course in the element 'flow' I use a significant sign which refers to a positive content. Very probably as we go back into primitive speech the distinction

[1] Cp Lotze, Logik, sects. 10, 11.

between this and the grammatical Name would disappear. Aristotle certainly treats 'is white' as a verb[1]

v Here we cannot escape raising a further question Granting that pronouns used independently rank as nouns, still there remain inflected elements of verbs, the case-endings of nouns, adverbs, prepositions and conjunctions Does not each of these indicate an identity in difference, a positive feature in the world of meanings? Why are not they too to be treated as names? There is no doubt that the words in question are fixed signs which refer to positive features of the world, nor that their contents are such as can have names given them which can be employed in propositions as substantives or adjectives Presence in time or space, Intention, Direction, Proximity, Property, Attribution, Reference, and the corresponding adjectives, these are all intelligible names and designate familiar matters Yet they are but another shape of such linguistic elements as 'Here', 'Now', 'For', 'To', 'At', 'Of', 'About', the *s* in 'flows'. Meaning of verb- and case-endings

It would be easy to answer that these elements are signs whose content consists in the actual operations of thought; that therefore they are not at first names, because we are operating directly and not reflecting on the operations, but that they become names at the stage of reflection in which we become aware of the part played by our intelligence in connecting and comparing the data of sense-perception.

But this would not be true There are names for intellectual acts, such as Comparison, Measurement, Enumeration, but these just show the difference between the operations themselves, and the results gained by them 'Similar', 'Equal', '$\frac{1}{10}$ of', 'Present', 'Near', are not mere signs of operations performed by the individual intellect. We

[1] He distinctly treats the *ῥῆμα* as capable of indicating time. So his idea of it did not quite cover our Predicate, which he would call *τὸ κατηγορούμενον*, though on the other hand we do not recognise 'is white' as a verb It is remarkable that he should have insisted on the indication of time in face of the fact (constantly borne out by his own instances) that the verb 'is' could be omitted in Greek, and predication conveyed by position only. It seems, therefore, that to say the verb was *understood* would have been no empty phrase to him, but would have expressed the fact as he regarded it

unhesitatingly treat them as characteristics of matters which we meet with in the objective world We find them out by combination, comparison and measurement, but we treat them as independent of the acts of our individual mind 'Present' may create a difficulty, if we think of it solely as present to *us*, but it is obviously a relation predicable and constantly predicated of objects or occurrences with reference to any self-conscious subject Indeed, all names are signs of thinking operations, so that this would be no distinction between names like 'equal', 'near', and names like 'warm', 'painful'.

But it is true that the inflectional and formative elements in question have this much in common with mere signs of intellectual acts, that we use them in propositions before we make propositions about them, and that when we come to make propositions about them we still have to employ them in one form, in order to make propositions about their other and abstract forms And further; like signs of intellectual operations, they cannot exist by themselves, they are not intelligible unless put in connection with substantive elements You might say '"Of" is not "For"' in the sense that a man's legal property is not morally for his sole benefit; but then the related points are supplied in interpreting. The proposition is not intelligible in the same immediate sense in which 'Red is not Green' is intelligible

Thus the formative elements of language are not complete names on the one hand, nor mere signs of intellectual functions on the other. They are however significant, and significant of matters which are capable of being named. But the matters or characteristics which they signify are such as to presuppose related terms, and to be incapable not merely of being, but of being understood, apart from those terms We find these meanings or attributes therefore, in the first instance, in explicit dependence on the simpler contents which they imply; and we only find them treated as nameable or isolable contents at a stage of reflection which can supply the presupposed simpler contents in a typical form Thus their apparently subordinate position in the simpler classes of judgment comes not from their being so little significant,

but from their being so much They indicate, not indeed mere acts of mind, but the realised wholes which arise for knowledge through those acts of mind Their names are names for such wholes, and for nothing less, as expressive of a special relation within an individual whole they are not names but auxiliaries [1]

It is quite true therefore that the formative elements of language imply acts of mind, but not true that what they indicate are mere acts of the individual mind, such as Judgment or Comparison

vi So far we have spoken of Naming as involving Reference or Objectification, and of Objectification as involving an affirmative or positive content Are we obliged by these conditions to treat Distinction and Comparison as essential to the act of Naming? When we refused to apply the terms 'definite' and 'determinate' to the positive something which constituted the identity that can be named, we did so in order to avoid begging the question of distinction and comparison We felt sure however of one thing, that a positive content is what makes distinction possible,[2] and cannot itself consist in a mere distinction from something else I am speaking all through of being as it is for knowledge, not of the ontological, and to my mind, fruitless question, how being can be apart from a consciousness What I say is, that we cannot see how the characteristic quality of a colour should be supplied by the distinction between it and all other colours And if the process of distinction does in fact make us notice all the features of something present to perception, this is only because contrast, in this case, invests those features with an interest which they might equally have obtained in some other way

Naming and Distinction

Mere distinction is not the essence of naming On the other hand, distinction is implied in the purpose of naming.

[1] 'Here' and 'now' are peculiar As implying relation to the subject which judges they carry their points of application with them even in their first and direct use, and can be used as logical subjects even in perceptive judgment, while for instance 'at' and 'in' cannot But 'here' and 'now' generalised into 'Presence' drop the special relation to the judging subject, and indicate a relation to any judging subject

[2] Cp Lotze, Logik, sect 11

I do not say that the implication is always apparent But the least reflection, the least practice in the use of language, must bring it to the surface We shall constantly have occasion to speak of the interest or purpose [a] which is essential to all judgment. And it meets us here on the threshold of intelligence Why do we name? Why do we refer to an identity? What do we want with a set of signs? To give voice to our positive wishes and feelings, we may reply Doubtless, in the first instance But this very giving voice, this fixing on a something, *is* selection It answers the question, 'What do you want?' and is meaningless except as an act of choice If there was only one thing in the world, we should not want a name except to distinguish between having it and not having it, or if we wanted no distinction, we should want no name. And when we use a name, we *ipso facto* select, because we omit, and we omit on a plan and with grounds, because a purpose guides us in selecting

Therefore I should state the relation between naming and exclusion or distinction, as follows Reference or Objectification, as represented by Naming, carries Distinction or Exclusion with it *formally* I do not mean externally or explicitly, but just the reverse. The act of Naming *is* in the abstract an act of selection, though we may not at first find it out. It is therefore selective or exclusive in form, but is not so materially, in any special relation, till we use it for that purpose And how soon this happens, how soon and how far a material value is actually given to the implied element of exclusion, is a question rather of anthropological psychology than of logic In the very beginnings of human thought (which I take as equivalent to thought aided by speech) it may be supposed that the sense of distinction would be chiefly represented by the *effort* to identify and fix under a name,

[a] I call attention to this phrase, which indicates the interest proper to judgment—the interest of complete expression It seems to me that the error of not recognising that judgment involves an interest rests with those who assume its subordination to some alien interest, e g that of external action, see II, pp 246, 275 The whole of the present work is based on the conception that judgment develops in accordance with its proper interest and purpose, which of course includes as applications of it all sorts and sizes of partial interests

and by the feeling of *success* when the desired result was attained by such means It is clear that such effort or success would represent a rudimentary work of distinction, which combats the difficulty of fixing the content and of finding and adopting the sign Or if the sign-system grows without perceptible effort on the part of individuals, there is still the interest to which I have alluded The distinction of man and woman or *meum* and *tuum* must, one would think, invest some contents with negative determination from the very first

On the other hand, even our reflective thought is perplexed when its attention is first directed to the mutual implication of very familiar facts, which seem to have independent being because so familiar The essence of objects in space may not be in distinction , we certainly however tend to underrate the importance of distinction in knowing them And when we come to negations such as those implied in consciousness or morality, for instance mind and matter, sin and the law, the degree in which essence seems to consist in distinction surprises us at first Distinction then is formally involved in Naming , but the degree in which it is realised *as* distinction between ' this ' and a definite other, and as essential to the character of ' this ', is not determined by the mere use of a significant sign It may not be so realised at all And when it is so realised it involves logical forms which go beyond the Impersonal or other nuncupative Judgment , forms such as negation, disjunction, and classification

There are indeed facts which are such as to be essentially relative—(*καθ' αὑτά πρός τι*)—a pregnant conception of Aristotle And obviously all facts partake progressively of this character as they are united with the whole of knowledge, and as in this process, their centre of gravity, so to speak, is shifted outside them by their connection with larger systems. We can scarcely understand a curve except as distinguished from a straight line , nor sin except as distinguished from a good will Nor can we know even red light or violet light scientifically without including in the conception of each its wave-length and degree of refrangibility or place in the spectrum—a disjunctive knowledge which in olves a number of precise reciprocal distinctions As thought grasps more of

its object, the object takes more of this coherent character. And in an object thus coherent, it may seem that distinction or negation takes the place of affirmative nature Straight *is* the line which is not curved Sin *is* the will which is not good. Red *is* the light at the other end of the spectrum from violet Here we are in the region of the complicated contents described above as relations Distinction has here a value which it had not before, but it is not mere distinction or mere negation. The distinction is valuable for its positive ground, and the negation for what it affirms The point is not that the essence of a fact can be given by mere distinction, but that positive matter finds it necessary to take the shape of distinction and negation I shall return to this subject in treating of negation and disjunction The distinction from straight is the essence of curve, only because or as far as the positive spatial nature of line and direction is involved in straight', and this same spatial nature is also involved in the opposite curved It is not really that 'Not straight is curved', but that 'Whatever is *a line and* not straight, is curved'. It is the nature of space, as known in line, and in constant and varying direction, that forms the positive content of both determinations

Thus it is never true in the plainest sense that a thing or matter of fact has its essence in mere distinction from another, but it is true, as we shall see more fully, that all reality is so entangled and interwoven, or rather is so coherent in each of its several regions, that in mastering the positive essence of one fact we are forced to master that of many more, all of them being branches of the same stem Even in the simplest cases there is at least an element of content common to the facts distinguished, like the stem up to the parting of the branches, and every distinction made between them has this at least for its positive content, as in distinguishing red from violet we imply that both are light. But no doubt in more intricate cases, the alternative or alternatives may be essential to their subject, and in that case the metaphor of the tree breaks down, for the nature of the whole is such that one *branch* perhaps cannot be seen, or perhaps cannot exist, apart from the perception or the existence not merely of the stem

but of the other branch Still we must think of the positive nature of the whole revealing itself in this peculiar form (say the nature of a moral being revealing itself in the good and bad will), *mere* distinction or negation is no characteristic at all.

Distinction then is involved in the purpose and essence of naming, but primarily as a *consequence* Meaning, or the use of names, is never mere distinction; though proper names are used for the sake of mere distinction, and so with no care for positive meaning except as subservient to that end And though in the deeper grasp of reality Distinction, Exclusion or Negation comes to be an active and prominent property of fact, yet this exclusion depends for its value always on its positive ground or motive, and never, as such, constitutes the essence of anything

Naming and Comparison

VII And what is true of Distinction is true of Comparison, or rather, as I shall use the terms in future, of Identification. Formally, in the light of analysis, and in respect of purpose and interest, to use or give a name *is* Comparison For it is identification, the establishment of an identity which holds good in spite of differences We may illustrate by the German word 'Vergleichung', which is usually rendered 'comparison', and which seems to be used in logic (I am not speaking philologically) to mean the establishment of a 'Gleichheit' or immediate identity between the terms compared This applies—cannot but apply—to the use of names as we have described it But if we take Comparison at the other extreme of its meaning, which 'Vergleichung' shares with it, we must say that it is a distinct reflective operation, which presupposes naming and is not implied in naming Comparison *par excellence* is a process which starts from the content of recognised names, and reacts upon it with a view to the interest which may have provoked the operation. Ultimately as a scientific method, it involves measurement, and is the instrument of classification, whereas the use of names must be prior to number or measure, and classification *as a method of science* arises so late that its genesis is almost within our ken.[1]

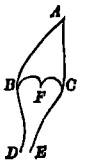
[1] In Western thought, it was probably first analysed and its import pointed out by Plato In practical life it must have arisen in the earliest

It would be easy to say, in the tone which I deprecated above, that Identification as a method of science presupposes Distinction, and therefore is a process naturally later than Distinction, and not to be looked for concurrently with the latter in the earlier stages of knowledge But this would be merely a commonplace blunder It would be as easy to show that Distinction presupposes Identification, as that Identification presupposes Distinction Before you can distinguish colours as red and green, you must identify them as coloured surfaces, and before you can identify the surfaces in respect of colour, you must distinguish them as separate areas in space. We can come to no good result in this way. We are merely pointing out, not any special relation between Distinction and Identification, but that every level of reflective observation presupposes a previous level on which it improves How do we begin then? It may be hard to say how matters get into consciousness, but all that is in consciousness seems to present both difference and identity. And we shall find how closely they are connected, and reveal the true relation of Distinction and Identification in the germ of knowledge—the act of Naming—if we look closer at the nature of Comparison itself, taking this as a process which may end either in Distinction or in Identification

If, in order to effect a comparison, we trace two shapes, say the shapes of two leaves, one upon the other, it is clear that we shall have a repetition at every minutest step of what takes place in the act of naming Coincidence, deviation—curve and straight—jagged or uncut—notched or entire; the discrimination of universal characters like these, with, if worth while, accurate measurement of differences, will mark the process as it goes on In respect of each of these points we may infer an identity or a distinction, we express an identity by a single judgment, either 'The outlines coincide' or 'Both outlines are slightly serrate'; a distinction either by the single judgment, 'The outlines are different', or by the two judgments, 'One outline is serrate and the other simple' We should notice that if any portions of the two outlines

society by the effect of social rules, see above on distinction Exogamy is an instance of an early custom which operates through classification

absolutely coincide, we can only predicate identity of them within that portion by bearing in mind the ideal continuity of that bit of line E C with the two differing outlines E C F and E C A If we leave this out of account, judgment and identity disappear Hence it seems to follow that complete comparison must always resolve the terms compared, in the respect in which they are compared, into cases under a universal, or differences within an identity. Identity without difference, or difference without identity, destroy the meaning of comparison It is for this reason that the single judgment will not contain a complete comparison We shall see that a disjunctive form is really required But it is true that the conclusion in these processes moulds its result into apparent opposition to its starting-point, or rather, causes us to read into the starting-point the complementary aspect to that which is proclaimed in the result This is simply because the result is a modified form of one element in the starting-point; of the identity if it is an Identification, of the difference if it is a Distinction The complementary element is thrown into relief by the explicit exhibition of the new determination, so that Difference always seems to be predicated of Identity, and Identity of Difference But really the judgment has done nothing more than to develope further either the identity or the difference of the datum 'X and Z are like;' here we take X and Z as distinct objects, though we know well enough that if they *are* like, they *were* comparable. 'X and Z are different,' here we take X and Z as instances of some class or rule and so far identical, though we know that if they *are* different they *were* distinguishable. '"Sorrow" and "Sorry" have quite different etymologies' '"Sorrow" and "Sorry" have the same meaning' The former proposition assumes sameness of meaning, in spite of which it asserts difference of etymology, the latter starts from a difference of form, perhaps intensified by a difference of etymology, in spite of which it asserts identity of meaning And yet the former also treats the words *as two*, and the latter as *prima facie* the same in their significant part.



I do not think that distinction can be effected except by

developing differences which are presented, or identification except by developing an existing identity But of course either element may be very faint at first It is of no use to say that we may be artificially set to search for distinctions. No motive will help us in science unless it guides us ; and if it guides us to a distinction, then it contains the distinction in germ The same is true in searching for identities Thus it would be hopeless to distinguish the two sodium lines in the spectrum while they look single as a small instrument shows them And we could never distinguish Ricardian rent from common farmer's rent (e g including interest of capital) except by pressing home the differences which are given with, and in, the two kinds of cases This, like many considerations in logic, will help us to understand the childishly tardy progress of early intelligence, and the cumulative rapidity with which knowledge generates knowledge

Naming then implies some degree of Distinction and Identification These two processes might, as we have seen, conveniently be included under Comparison But Comparison, Identification, and Distinction, as involved in naming, are not the developed methods of which I have spoken.[1] In developed knowledge their *organa* are measurement and counting, in which it becomes mere pedantry to separate Identification from Distinction [2] I shall return to this point in speaking of the value of Judgments, and shall there treat the earliest distinct judgment of Comparison as the transition to number and measurement But the keyword of *mere* naming is *Recognition* , and this is the limiting purpose of all functions *qua* subservient to naming And as regards the affinity between Distinction and Identification, they are obviously two sides of the same process, and it is idle to ask which came first So far as we can see, Consciousness, or at least Intelligence, must begin with both.

[1] See further Book II, chap i, on ' Immediate Inferences '.

[2] I mean that it is pedantic to restrict measurable identity to the case of absolute equality, but that if you do not, you must admit a degree of distinction to be present in all cases A distance of 400 miles and another of 400 miles 1 yard are as we say ' the same to a yard ' , and this is the true way of putting it

6 Enough has already been said to make clear my general view of the growth of logical functions While I would spare no pains to ascertain the precise order *in* which and differentia *with* which logical activities make their appearance, I have never been able to doubt that the central function of the intellect, I would even say of consciousness, is one from beginning to end [1] In speaking, therefore, of the connection between impressions and ideas, and again between ideas or concepts and the judgment, I am obliged to reject the easy partition into distinct operations which finds place in many text-books [a] More especially, I cannot at all follow Lotze in his treatment of this connection, and I select his work, as probably the most permanent in value of those which adopt these views, to comment upon when comment is necessary. Concept and Judgment

As I read Lotze, the act of Naming coincides with that shaping or moulding of an impression which is required to convert it into an idea, as a stone requires shaping to make it fit for use in a building And then, subsequently, the ideas so shaped are fitted together, and the result, I suppose, is a concept, while it is not till the simpler concepts come to be combined that the judgment takes its rise Ideas, it will be observed, are thus subsequent to impressions concepts to ideas, and the judgment is subsequent to the simpler concept

It is worth while to notice the nature of the analysis by which this account is defended Impressions must be shaped like stones before they can be fitted together [2] Judgments must presuppose at least simple concepts,[3] because judgments consist of concepts, and if such concepts presuppose judgments, where are the concepts to come from which make up these latter judgments? It is hard to think that such arguments as these really expressed Lotze's mind; they must rather have resulted from over-eagerness to present a perfectly clear arrangement to his readers. The thought of a germ which unfolds differences, of the elementary sensation as

[1] Cp Bradley, Principles of Logic, p 455 ff Mr Bradley's views on this question have influenced mine, but rather in the way of moderating than of suggesting or intensifying the view adopted in the text

[a] See Keynes, p 8, and my Preface to Ed 2

[2] Lotze, Logik, sect 1 [3] Ibid, sect 8

already containing, in the features which make it a state of consciousness, rudimentary distinctions which are shadowy at first but receive form and fixity by degrees,—such conceptions seem at once to destroy the application of arguments drawn from mechanical processes If metaphors are indispensable, we should rather call to mind such processes as the formation of structure in an embryo, as the separation of a double star by successively higher powers of the telescope, or indeed as the discernment of features in a distant landscape which prolonged attention even without optical assistance has the power to effect

And, though the suggestion is hazardous, I cannot but think that Lotze allowed himself to confuse change as process in time with the rectification of error in knowledge [1] There is nothing whatever in the concept or idea, as Plato thought of it, to interfere with its expressing the laws of process in time. The constitutive equations of curves, read in connection with the law of gravity, have, so far as we can see, precisely fulfilled one of the grandest aspirations embodied in Plato's view of science, the establishment of the true laws of motion as they are in general, and not solely or specially for the heavenly bodies [2] And these equations are the instance which Lotze gives of the highest order *of concept* [3] Such a concept or idea embodies the very essence of process in time, or change. It is true that change is also a principal vehicle of indications that our concepts are erroneous, and therefore often requires them to be changed, but this is not because the concepts are concepts, but because they are wrong. It is wholly an illusion, founded I presume on the doubtful idea that predication involves reference to time, or even that the judgment is a transition in time, to suppose that the judgment as such can represent change or 'Becoming', while the concept cannot. To make this in the least probable it would be indispensable to confine the judgment to narrative judgments which use tense, and thereby to abandon all scientific knowledge

I cannot but think that the reasons alleged by Lotze for

[1] Lotze, Logik, sect 34 [2] Plat. Republic, 530–1.
[3] Lotze, Logik, sect 117.

the transition from Concept to Judgment are wholly visionary; and merely conceal the unreality of the entire arrangement which made such a transition necessary. There is truth, indeed, in the remark that the judgment reconstitutes the concept *with a reason*, but is there any possibility that the act which reconstitutes the concept is fundamentally other than that which constituted it at first?

I will summarise the criticism which I think essential on the whole point of view indicated by the ordinary successive arrangement, and especially by Lotze's form of it.

If a Sensation or elementary Perception is in Consciousness (and if not we have nothing to do with it in logic) it already bears the form of thinking. I will not say that it is a rudimentary judgment; but it is certainly an act, for it is a change within a percipient subject, it has identity in itself, or it could be nothing for consciousness, and difference, or it could not have identity, and it stands out against other elements of the momentary consciousness in a way that approaches to an attribution. An Impression or sensuous idea becomes a logical idea when it is fixed and referred,—fixed and referred if we like to say so by receiving a name, though this is rather a sign of the act than the act itself [a] We have here the explicit form of judgment given to what before must have been a mere actual extension of sensations by idea, depending on a general identity, but not consciously referred to an identity other than the sensation as an object [b]

Judgment is not, in relation either to impressions, ideas or concepts, a mechanical combination of parts which remain outside each other. It is an expression—perhaps at bottom the only expression—of the unity in which consciousness consists. I do not mean that it is nothing more than an idea or impression; but I incline to think that it is better described as an idea or impression writ large than as a combination in which ideas or impressions are units. Judgments may contain

[a] Cp. p 17 above, note *a* We listen at night, half awake, to a sound which we do not at first identify, and then name it by saying, 'Surely, a motor standing still'

[b] In Ed 1 the words ran, 'than the psychical image' But there would be no discrimination of a psychical image at the stage we are speaking of.

complex ideas, but every Judgment *qua* Judgment exhibits the content of a single idea Ideas and Impressions, as I have tried to show above, are not found lying apart as words lie on a page, although, by a reflective abstraction, we can regard them as so lying apart, and when thus regarded they form the world of meanings or of objective references—the identities symbolised by logical ideas.

We have then Judgment or some analogous operation of Consciousness, from the first ,[1] and in naming and all subsequent operations we certainly have Judgment What we are watching all along is the development of an act, a function

Thus Judgment and Idea go *pari passu* An Idea is not presupposed by Judgment any more than *vice versa* And it is, as I have explained, an extraordinary confusion to account for the advance from concept to judgment by the inability of the concept to represent change

And indeed the whole question of advance from concept to judgment is meaningless to me, for I think of the concept as existing only in the act of judgment. I have tried above to explain the deception which language practises on us in this respect The question is not easy, and is all-important. I shall therefore return to it for a moment

If a man were to say in our presence 'The Sun' and no more, we should either suppose that he meant 'The Sun is visible', or, if circumstances excluded this interpretation and furnished no other, we should turn upon him sharply and ask, 'Well, what about it?' This implies that the words have conveyed a meaning to us, but that the meaning is incomplete. I will speak of the second point first. It may be said that our impatience of the incompleteness of the thought is ethical and not logical; that it arises from annoyance at the waste of an intellectual effort, or at the interruption of other thoughts, seeing that nothing is to result from it; and not from any inability to think the thing 'sun' by help of our idea of it, without judging.

This explanation would have much truth, and only needs pushing further We *should* in such a case miss the ethical

[1] Cp Bradley, p 455 ff And see below, Book II, chap. 1, on the lower limit of Inference

purpose which all thought implies. But this defect would have a logical side, which would be this; that we should be started upon an intellectual exercise not only objectless, but also and for that reason endless. Thus the meaning is incomplete because undetermined. We are left to traverse an indefinite series of judgments.

And yet (I return to the first point) a meaning has been conveyed In what shape does it exist? The natural answer would be that an incomplete meaning must exist in the shape of questions or suggestions—of tentative judgments. But a tentative judgment lacks, it would seem, the differentia of a judgment It does not assert, it does not claim truth Therefore we have *prima facie* in the idea or conception something that will not go into the form of the judgment An idea in this stage seems to be in a position corresponding to that of a relative or dependent clause or clauses without a principal clause, a form of language which certainly can exist, but which has not an independent right to existence. Or it may be taken as corresponding to a question 'The sun, around which the planets revolve, which is hot and bright;' 'The battle—lost or won?' These instances give different cases. In the former the attributes are all constant, and we might if we chose say that we first judge, affirm certain attributes of a thing, and only leave it undecided what attribute is in question here If this was so, we should have something like a disjunctive judgment In the latter case, that of the question, the form of sentence is considered absolutely to exclude a judgment, although we have assumed the material for a disjunction to be furnished by the prevalent interest of the moment. What is the thought corresponding to a question? I do not find any sufficient discussion of this subject in the logic books[1] *Is* a question a peculiar act of thought at all, putting language aside? The test for this is to see whether we can genuinely ask ourselves a question, or whether it is,

[1] Cp Bradley, Principles of Logic, pp 13–14 His treatment is definite though brief, and I cannot agree with it I cannot think it possible that the *content* of a doubt or negation should be the same as that of the corresponding affirmation Cp Sigwart on the Question, Logik, 1 231 Eng Trans 1 177

like a lie, only a form of speech which has the object of producing a certain effect on others I am disposed to doubt whether we can interrogate ourselves It appears to me that a question directed to oneself for information which one has is always rhetorical, is a concise summary of the interest which the information has for us But we too often have to ask ourselves questions which we cannot answer, and know that we cannot answer I do not see how these, again, can be genuine questions. While in the former case we know the answer already, in the latter we know that there is at present no answer possible A question addressed to another person in such a case, i e knowing that he cannot now answer, has not the differentia of a question , it is a mere guide to him as to the information which we wish to possess, a memorandum for future use when he may have the information But, if we are speaking with ourselves, this leads to the former ground of rejection ; the question becomes, as before, merely rhetorical And thus there is not even a prospect of genuine self-questioning , to treat oneself as another may react powerfully on the imagination, but is impossible in strict thought [a]

Thus a question cannot be an act of thought as such, just as a lie is not, and for the same reason, that it is not an attitude that the intellect can maintain within itself A question is not merely doubt , nor merely doubt plus the knowledge that the doubt can be resolved in a particular way It is a demand for information , its essence is to be addressed to a moral agent, not ourselves, in whom it may produce action It is closely analogous to the imperative, which also cannot be addressed to ourselves except by mere metaphor. Thus to say that the mere mention of a name leaves us questioning or fills us with questions, is not to say what it does for thought.

I suppose that the thought, on which a question is based, must always partake of the nature of disjunction. Where the interest lies wholly on one side of the alternative this is hardly noticeable, 'Are you going to see Hamlet?' We scarcely think of the possible negation as an *alternative* at all, but rather

[a] I now think this is nearly true, but overstated It seems to me a mistake of principle to deny that a man can have social relations with himself, see p 14, above

as a bare nothingness, a rejection of the idea proposed in the question A more difficult case is ' How much did you give for that horse ? ' In this case, as in asking ' Where ' or ' When ? ' we assign the general principle of the Disjunction under which the answer is to fall, instead of selecting an alternative and demanding information about *it*

I should therefore be inclined to think that when a man says ' The battle '—and then stops, and we ask ' Lost or won ? ' our thought is really a disjunctive judgment with reference to which we express a desire for an action *ab extra* that will enable us to accept one of the alternatives The same result follows if we describe an act of thought as doubting It is impossible to doubt without knowledge, and a definite doubt, apart from a moral or religious sense of the term (for a degree of failure of will may pass for doubt in these spheres) is unquestionably a disjunctive judgment

And if now we return to the case of the sun and its constant attributes, we may find that a similar account is possible. The speaker has uttered what is the equivalent of a dependent sentence , he sets us judging in distinct affirmations about reality which form our resources for estimating what he can mean (or suppose we judge about his mind, it is no less true that we judge), and the indefinite series of these affirmations may be treated as an imperfect disjunction. It makes no difference whether we conjecture as to his meaning or as to the fact which may underlie it ; whether we think ' He either means the sun is just visible, or that it is hot, or — ', &c. ; or again, simply, ' Either the sun is just visible or,' &c.

Such judgments are *prima facie* substantive or independent judgments But if it is our express purpose to regard what has been communicated as nearly as we can in the light of a mere idea, mere concept, or mere possibility, then we must be taken to affirm the universal meaning which pervades these judgments to be true of Reality under specific but unknown conditions, a mode of affirmation which we shall find to be the essence of the problematic judgment Such a point of view as this is rendered inevitable, in the case supposed, by the absence *ex hypothesi* of any ground for restricting our affirmation to any special element of the universal content, unless, as

in the example of the question about a battle, a determinate or partly determinate disjunction is provided by the context. A mere idea then as distinguished from a judgment, but considered as the mere meaning of a name or as an objective reference in the world of meanings—an isolated idea—is the content of a reflective problematic judgment, and is referred to reality as true under unknown conditions or among unknown alternatives But every idea has its existence in the medium of judgment

The judgments which embody ideas may have many degrees of unity. The identity which pervades a set of judgments may be quality, thing, or complex attribute. When the identity is a quality the judgments in which it appears are but slightly connected, and one member of the group will not necessarily be accompanied, opposed, or conditioned by any of the others In the case of groups of not very coherent attributes, such as form concrete things in space, the result is exceedingly curious, and it is very doubtful whether the judgments into which the thing enters should be treated as single rather than as multiple. For the judgment which is made is often related to the others which the identity binds to it, *not* as consequent to conditions or grounds, nor as conclusion to premisses, but as if joined with them either by a copulative or even by an adversative conjunction. 'The strongest men were afraid of him,' i e The strongest men, though much stronger than him, yet were afraid Or, The strongest men were very much stronger than him , and yet, &c. Or, again, The Venus of Milo is in the Louvre , in this any number of judgments may be supplied out of the subject, and linked by a mere 'and' to the one given. There are all degrees of conjunction ; it is well known that even 'and' may carry either adversative or inferential meaning But where it carries pure conjunctive or pure adversative meaning, there must be a question how far the proposition represents a single judgment. It is from this ambiguity that the judgment is freed by assuming the hypothetical form

To analyse these degrees of unity here, under the head of the concept, would be superfluous The whole work of logic is to depict them in the order of judgment and inference.

It should be mentioned that there is unquestionably a reaction of judgment on the actual image or appearance presented to perception. No doubt, *all* arrangement in space has been *learnt*, but I take it that the disposition of points on surfaces perpendicular to the axis of the eye, even if a result of interpretation applied to feelings of motion, is when once learnt an inevitable process, for every detail of which there is a special distinct sense-stimulus On the other hand, in the perception of depth it appears to me that we have a generically different case. The interpretation of certain dispositions of colour, and of certain feelings in the ocular muscles, to mean 'if I want to touch that point, I must put my arm out as far as I can'—this would cause me no surprise, and would simply be knowledge brought to bear on perception, just as it is when certain appearances indicate that one has food or poison before one What is noticeable in the case I now speak of, is that the interpretation reacts on the image, that we *seem* to *see* depth exactly as we see height and breadth ; and that, in learning to draw, the counteraction of this interpretation, and the reduction of objects to their places on a plane surface is a matter of extreme difficulty. The solid images in which a mere interpretation is thus made visible as a fact do realise the popular notion of what I might call a petrified concept, a group of attributes and relations which stands still to be looked at.[a] Ultimately, however, even this petrified concept is a judgment—a perceptive synthesis.

I will recapitulate our results so far .—

1 Naming, or the appropriation of fixed signs for meanings, always marks a first step in the thought which acts so ; scientific naming e g marks a first step in a region of science, though a late stage in the history of the human mind.

11. The formative elements of language are significant, but *qua* formative elements are not names, because their meaning is incomplete without that of other elements At a later stage of reflection names are assigned to their entire significance, that is, to the classes of complex wholes which they imply.

111 A name has meaning only in a sentence or by suggesting a sentence The sentence is the significant unit of language

[a] See further, 11, p 292

This is most easily seen in ancient speech, but is equally true of modern and analytic tongues Dependent or appositional sentences can enter into names It is probable that the thought corresponding to a sentence is always assertory, if not optative, or imperative.

iv. Naming involves Objectification—the treatment of that which receives a name as an object of knowledge, as recognisable, that is, in a world which exists for all thought as such, and is not dependent on the thinking of the *individual* mind [a]

v. Objectification involves being as a positive somewhat on the part of that which is to be treated as an object of knowledge, but this does not amount to the exhibition of a 'definite' and 'determinate' nature.

vi. 'Being as a positive somewhat' includes in a formal sense being known by Identification and Distinction, which are the two sides of Comparison. But as processes of real or material import, these methods presuppose number and measurement, and are posterior to fixed names.

vii. That which is named is always an identity in difference, and must disappear if either element is neglected or removed by abstraction. This is illustrated by the relation of names to sentences.

viii Every name *refers to* such an Identity treated as an object of knowledge, whether thing, quality, or relation.

ix. The meaning of every name is in what it refers to or is meant to mean; but this is represented to the individual intellect by the significant idea which the name causes it to produce.

x. An idea or concept is not an image, though it may make use of images It is a habit of judging with reference to a certain identity.

xi. There is no correspondence between Concepts as such and Quiescence, or between Judgments as such and Change As the fundamental form of Knowledge the Judgment tends to overcome change, and to view phenomena *sub specie aeternitatis*, and is in this respect at one with the Platonic 'forms'

[a] This is not the transformation of a subjective state into an object 'Subjective' states only become 'subjective' as the objective world developes for consciousness

xii. The grades of unity and complication of ideas and concepts are the same as those of judgments and arguments

xiii. The relation of the concept as representative of the meaning of a mere name, to the assertory judgment, is illustrated by the relation of the dependent sentence and of the question to the assertory judgment, and depends upon the possibility of making the identity in a group of judgments the content of a relatively reflective judgment

Logical meaning and the mind

7. It will be observed that having spoken at first of what I may call the *logical* meaning of names, i e. of their reference in the general world of thought, I have digressed in the last few pages into discussing the act of judgment by which the individual mind realises that meaning. The purpose of this was to show that the acts set in motion by the name and by the proposition were the same, and therefore the logical function of these forms could not be generically different.

But before further considering the logical meaning, it will be well to say something on the relation between the universal or logical meaning and the act of the individual mind.

Logical meaning we have treated all along as taken to exist in the world of meanings,[a] the world which is common to thinking beings as such. Not merely London and Mont Blanc, but virtue, redness and pleasure, have their being in this objective world of meanings. And yet the meaning which on the one hand belongs to a world independent of the individual peculiarities of our thought and perception, is on the other *our* meaning. It is dependent on our private experience and our private intellectual endowment in two ways.

First, the psychical 'ideas', the images which our mind generates from moment to moment and which never recur,[b]

[a] It is not admitted, the reader should remember, that the world of meanings is separable from the world as affirmed It is merely the latter looked at in a fragmentary way, see p 5 and p 36 above

[b] See further on 'Mental States', II, ch. x. The view of the present work was from the beginning that a human intelligence has in principle the form of a continuous judgment, in which no psychical elements escape from contributing to meaning, and no meanings are ultimately unaffirmed (see pp 70–1 below, with the observation as to how we are aware of ideas as mere facts) The important point is to avoid the conception that judgment is a transition from mental state to mental state, by distinguishing the use of mental contents from their

are only such as our memory, conjoined with the suggestions present at the moment, will supply Mr Bradley has well explained how, as images in our minds, these are not ideas in the logical sense, not significant, not meanings We *use* these images, make them starting-points of thought, treat them as containing approximations to what we mean, we direct ourselves to omit parts of them, or to note that they require weakening or intensification. We may illustrate this by the way in which we attempt to communicate a qualitative impression; but it is only an illustration, because we cannot employ as an instrument of communication a particular momentary psychical image; it is not transferable, not capable of being reawakened with precision by language in another individual mind, nor in our own We must employ then an image which is already so far universalised as to be subordinate to a meaning; but which may be diverted from its original meaning and applied to another in a way that *illustrates* the employment of a psychical image 'Not quite sky blue, but a little darker;' 'Between pleasure and pain,' 'A baritone is in quality something like a tenor, though with points of resemblance to a bass'

Now though, after this fashion, we can deal freely with our particular psychical images, and make them do duty in very various contexts, yet there are limits to the modifications which can be effected in them To take a well-worn instance, we cannot suppose that a man blind from birth can ever make judgments involving the *quality* of colours, although he can obviously learn the mathematical theory of undulation and refraction. But the whole region of particular psychical occurrences, immediate impressions of colour, which are made

mental existence It does not matter that they are already determined in a context of meaning The point is that in judging we take them out of their context, and identify them with a new one, in which they become part of a new whole in the real world, and give and receive new characteristics I do not mean that novelty is essential to judgment, but when a judgment is well established in our minds it is less easy to see where the predication came from, and what, as a content, it originally was When I first say 'Reality is solid' we see that I am using a piece of my mental equipment, the adjective 'solid', in a special way But, if approved, it grows into its place, and we forget that it was torn from a number of other applications

use of in referring to the recognised colour-system, would simply be absent from his mind. Structure, on the other hand, involves mainly relative conceptions, such as movement and position, in which the nature of the particular images employed is indifferent, or, if we mean the structure of an argument or institution, the notions required are such as condition and consequent, function and purpose Structure therefore can be reproduced by any intelligence furnished with the chief capacities of an intelligence as such

But, secondly, ideas, even in the sense of meanings, are on one side individual and peculiar The intellect, at least the individual mind of which we are now speaking, does not move wholly in the objective world of meanings to which its acts bear reference In extreme cases Content and Reference are in contradiction, in less extreme cases, in veiled contradiction. 'Oh I see, my dear sir,' said a theological disputant, 'your *God* is my *Devil*' Both parties had made the same *reference*, viz to God, neither took what he was speaking about for an idea merely in his mind, but nevertheless, in making the reference, each of them had employed a peculiar and special act of thought, determined by his own intellectual conditions and history The opponent in the dispute maintains that your reference is inconsistent with your content, that one or the other must be wrong But the possibility of conflict is gone if the reference of both disputants is not the same, and the retort quoted above is an ironical suggestion of a basis of *agreement* on the score of *different reference*. The antagonists refer to, or mean to mean, the same thing, but they cannot bring their notions of it into agreement.

If we go lower into mere quality we obtain good illustrations of the line between meaning and psychical idea It is possible, on certain assumptions which do not concern us here, to compare some of the colour-perceptions of individuals, and it appears that there are various degrees of sensitiveness to red light. Now if we take a case, not of absolute red-blindness, but of over or under-sensitiveness to red light; we see that the eye which is thus abnormal can produce, presumably, all the images which the normal eye can produce, excepting only the very weakest in the one case and the most intense in the

other. But these, as we rarely meet with or think of them, we may neglect. Now the mind of a man whose eye is thus abnormal has the same furniture of images as our own, but the meaning in each external reference in which they are used must always be slightly different from ours, though such differences pass undetected in common life If I speak to him of the red of a Doctor of Divinity's hood, he may indeed represent it to himself by any shade of red which springs to his mind's eye; but he will *mean* a weaker or a more intense colour than I mean In this case the abnormal condition has not interfered (as in absolute blindness) with the supply of images, but only with the occasions on which they are produced, and therefore with the meaning attached to each external influence, to each red object.

This paradox—that in using names we refer to matters as independent of our individual thinking which in this very reference are only represented to us by an act of our own individual mind, certainly inadequate and possibly contradictory to the reference—this paradox is inevitable if we maintain the ordinary line between the mind and the world. No doubt the reference demands some one correct or at least recognisable element of meaning, or else we should set down the name employed as a mistake, and thus if the reference contradicts the content, the content must also contradict itself. But this does not alter the fact that what we refer to as independent of our intellectual act exists for us when referring to it wholly in that intellectual act

An effort of imagination might help us to see the real nature of this paradox. We might try to think that the world, *as known to each of us*, is constructed and sustained by his individual consciousness, and that every other individual also frames for himself, and sustains by the action of his intelligence, the world in which he in particular lives and moves. Of course such a construction is to be taken as a re-construction,[a] a construction by way of knowledge only; but for our

[a] This expression was not intended to involve, and I believe does not involve, the notion of an actual external world, similar to, but distinct from, that of our perceptions, by copying which, well or ill, we have truth or falsehood See II, ch IX, below, and compare the criticism referred to in the following note

present purpose this is indifferent Thus we might think of the ideas and objects of our private world rather as corresponding to than as from the beginning identical with those which our fellow-men are occupied in constructing each within his own sphere of consciousness. And the same would be true even of the objects and contents within our own world, in as far as an act or effort would be required to maintain them, of the same kind with that which was originally required to construct them We should know that correspondence * implies a degree of identity, but also that every degree from mere correspondence upwards had to be won and justified by intellectual work, the *onus*, so to speak, of establishing it would be thrown on the intellect, and the progressive coincidence of our separate worlds would be the reward of knowledge The moral of such a view is not a bad one, for it places the solidarity of mankind in the intellectual life

Thus the paradox of reference would become clearer. We should understand that we refer to a correspondence by means of a content We should soften down the contradiction of saying that a name to meet which we have and can get nothing but an idea, nevertheless does not stand for that idea but for something else We should be able to say that the name stands for those elements in the idea which correspond in all our separate worlds, and in our own world of yesterday and of to-day, considered as so corresponding Even when we say, taking the most subjective of feelings, 'Pleasure is the accompaniment of activity,' we refer to pleasure as a point in which all separate worlds correspond, which occupies the same relative position in all the worlds which are framed by the consciousness of individuals, or, what is technically the same thing, a constant position in the world framed by our own. But we should not pledge ourselves to any special

* The unity of elements which correspond of course does not lie in an abstract idea made by eliminating their differences, but in the total structure of which different aspects appear in the different corresponding views A poor man and a rich man may know the same things and places in London, but their ideas of each place or thing would have to be put together if we wished for a really comprehensive account Contrast the criticism in Chicago University Decennial Publications, 1903 (Miss Thompson on 'Bosanquet's Theory of Judgment', p 99)

degree of correspondence or of identity resulting from comparison, only to the bare justifiability of the reference. This suggestion may be considered, if the reader chooses, as a mere simile; but even so it may assist him in seeing the true relation between the idea which a name arouses, and the object to which that idea refers. The distinction between objective reference and actual affirmation depends, as we have seen, on the difference between the analytic consideration of a connected group of judgments, and the affirmative of one among them.

Extension and Intension

8 I now return to some further characteristics of the logical meaning of names, and shall follow Mr Bradley in using 'idea' for a fixed content or logical meaning, not for the psychical images which pass through the mind and never recur—for the signification, so to speak, of the signal flags, not for the particular flags themselves, whose meaning is not affected if different bits of cloth are used on every occasion.

Inseparable. Their nature

1 Intension and Extension are complementary and inseparable. *a* If an idea *is* the meaning or fixed logical content indicated by a name, how does it come to pass that ideas or names are said to have two kinds of meaning, known as Intension and Extension? The meaning proper, the fixed content, is obviously the Intension of the name or idea, sometimes inadequately defined as the meaning which the name *implies*, in contrast with the Extension considered as the whole range of individual objects or instances to which the name *applies*. But it is clear, as Mill has well insisted, that the intension is the primary meaning, or, as we have said, *the* meaning. To speak of it as implied or connoted, or as the connotation, seems therefore to be a terminology which Mill's own view should condemn.

But if Intension is the meaning of a name, or is the idea which is this meaning, what is Extension? How can a name mean anything beyond its meaning, or how can an idea, which *is* a meaning, yet *have* a further meaning? The answer is so familiar in practice that it seemed worth while to observe that it is not free from paradox. Extension or Denotation consists of the instances, ideal or actual, in which any content is considered as realised or realisable. That is to say, extension is the aspect of a content as particular, or as an exclusive unit.

The plural of a noun substantive affords the simplest illustration. 'Men' form the extension or denotation corresponding to the content intension or comprehension of the name or concept 'man' But 'a man', the singular meaning correlative to 'men', is extensive, just as is the plural itself.

If there are two or more instances of the one content, the distinction between these particulars and the content itself is obvious ;[a] if there is only one instance, and still more if there can be only one,[1] the relation is obscured But in every idea the distinction between universal meaning and particular embodiment or exclusive self-identity can be traced, and neither aspect can be lacking in any idea A name or conception without Intension would be a name without meaning, and therefore, also, without Extension , for it is only the meaning that prescribes the Extension And a name or conception that should have no Extension would be one that would not apply to any particular thing or case, and therefore could have no Intension ; for the attributes which are thought of as embodied in particular cases are what constitute Intension.

Fictitious ideas.

β. This latter conclusion might be objected to on the ground that names of fictitious ideas or vain imaginations, or again nonsensical or self-contradictory expressions, have a sort of meaning, or at least find a place in would-be significant speech, and yet apply to nothing, i e. have no Extension. 'Nothing' in this objection must mean nothing *actual*, if the objection is to

[a] We should further note the distinction between what might be called the extension *under* a name or universal, and the extension *of* the name or universal 'Under a universal' would refer to cases like the same colour on different objects, or different coins of the same date and denomination 'Of a universal' would refer to differences which develope in a necessary way the content of the universal itself, such as the kinds of triangles In this latter case the extension seems merely the specification of the intension (Nettleship, Remains, 1 218) But extension cannot really be reduced to intension (Bradley's Logic, 155) There is always a difference between the content and the fact And this is what the difference between extension and intension depends on

[1] Sigwart, vol 1, p 351, gives as an instance 'The centre of the material universe' There cannot be two points, of which this content is true, but the meaning is still distinguishable from the particular instance, and is theoretically capable of having further particulars subsumed under it Of course there may be two such points in succession —the centre may shift

be true in fact; but actual in any determinate sense is a limitation or factor in Intension, and if we introduce it into an imaginary conception we create a contradiction and bring the fictitious idea or name under the head of self-contradictory or nonsensical expressions or conceptions, of which we shall speak directly But if we do not take nothing to mean nothing *actual*, then the objection is not true in fact, and imaginary ideas, the content of absolute fictions, have their extensions in the instances, particulars, or units, or in the aspect of unity which they naturally imply Chimeras, four dimensional space, Gulliver's voyage to Lilliput, have all the same complementary aspects of meaning and particularity that are involved in man, horse, or triangle.

Nonsensical expressions

γ On the other hand, a *word*—for under this head we can no longer speak of an idea—which is unintelligible whether as a mere noise or as a contradiction in terms with no rhetorical significance, is of course not a name, and cannot enter into the discussion, for it has, strictly speaking, neither intension nor extension, so cannot illustrate the existence of the one apart from the other Only it must be observed that even as a name or sign for a certain noise [1] the combination of letters has still its dual aspect of universality as an identifiable sound, corresponding to Intension, and particularity as a momentary and unique utterance corresponding to Extension

Names of attributes

δ Another case is that of abstract names of attributes, such as 'whiteness', 'virtue' It is quite clear that these abstractions are true of particular instances The simplest rule is to adopt as extension the meaning of the plural of the noun; thus virtue becomes a generic conception, and has its extension in the virtues, i e the kinds of virtue, courage, temperance, &c., and its intension in the generic meaning 'a habit of volition directed to distinctively human ends', or whatever our definition of virtue may be Whiteness is not obviously a generic term, but has unquestionably a possible plural either in the sense of kinds of whiteness [2] or in the sense of instances of whiteness In Latin as in English it is somewhat of a rarity to use the plurals of very abstract abstractions, but yet

[1] See Bradley's Principles of Logic, p 157

[2] Cp Shelley's line, 'White with the whiteness of what is dead'

they are sometimes used, and besides, the difference between singular and plural only illustrates and does not constitute the distinction between Intension and Extension As in the line quoted from Shelley, *the* singular whose meaning is on all-fours with that of one case among those indicated by the plural is itself a particular, and accentuates the extensional aspect of the idea I may add that it has been well pointed out[1] that such abstractions are 'doubly adjectival', for they not only apply to real cases or kinds of the abstraction, 'whites' or 'whitenesses', but they actually *mean* the abstraction of a concrete thing or subject that *has* the attribute They imply not merely particular whites, but particular things that are white.

The intension of the simple abstraction 'whiteness' is hard or impossible to state in general terms, if we leave out of account the theory of light, which has not been available for this purpose till a comparatively late date in the history of logic. A parallel difficulty caused Plato to say, at least at one stage of his views, that he could frame no εἶδος of a smell, i e. he could find no general determinate attributes by which to formulate its definition. Such difficulties are plainly matters of the state of knowledge A content which is recognisable and identifiable in different contexts always has a meaning and intension. We are about to turn to two extreme cases, that of proper names and that of number, which will illustrate the lowest grade to which the intension of a significant name can be reduced. No attribute, however hard to define, can be so indifferent to intensional meaning as the significance of a strictly *proper* name and the denomination of a number.

Proper Names

ϵ Proper names have sometimes been pronounced non-connotative, i e. without intension; because their meaning is not fixed and generalised On the other hand, Jevons, rightly rejecting this view, which is absurd because as we have seen intension and extension are inseparable, goes into the other extreme by refusing to distinguish Proper from Singular names, and therefore ascribing to the former a *maximum* of intension By a proper name I understand primarily a name that merely serves to distinguish a place or person, or, in exceptional cases, a thing As a rule, a

[1] Bradley, Principles of Logic, p 156

thing which is neither place nor person has not the individual interest independent of fixed content which is the root of the employment of proper names We name a thing according to its species, its type or function, not with reference to its absolute particularity Cases like that of a favourite animal, e g a horse, to which a proper name is usually given, or even a favourite thing, such as Henry Smith's hammer Samson in 'The Fair Maid of Perth', are exceptions that prove the rule We can see that in such cases as these a special interest has come to be attached to the particular individual independently of its specific nature By a 'singular' name as contrasted with a proper name I mean a name that indicates content as such, but content that is in its nature, or at any rate assuming it to be located in the actual world, unique. Such is the instance given above, 'The centre of the material universe,' or, again, 'The king of England in the year 1832' There is a certain difficulty in finding instances of these names, unless as in the last case we limit them in time, or as in 'The chief murderer of Caesar' confer uniqueness upon them by relation to a true proper name. When we come to speak of the singular judgment we shall see that there is a good reason for this difficulty It is *prima facie* impossible for any content into which time does not enter to stand as the subject of a singular judgment. The centre of gravity of the material universe may shift its place and thus become in one aspect plural, though in another it remains unique No idea can guarantee its own uniqueness, which is only given by reference to a position in the actual sensuous series Still there is a difference between the singular and the proper name; which I proceed to state as shortly as I can

Every name has intension and extension But the extension naturally follows the intension, and the intension attaches to the name, without reserve, that is to say, the name as such has a meaning, and is applied to all objects of which this meaning is true Now the term 'proper', which means in this usage 'peculiar' or 'individual', is in contradiction with the above-mentioned characteristic of significant names, and imposes upon them a function with which the nature

of intension is essentially at variance, viz. the recognition of individuals as such, in their particularity, and without *primary* reference to their attributes. Intension thus becomes a means and not an end A significant general name is used of many objects in the same sense, and a significant singular name is used of one object only, *because there is or can be only one object to which its meaning applies*, but a proper name, though used of many objects, is used of each in a different sense. Its rudiment of general meaning is in such an implication as that John is the name of a man and not of a mountain or a steam-engine, or again, some one of the thousand *different* applications of a proper name may become typical, and so set up a general meaning, which however does not attach to the name in its remaining 999 applications, but only elevates it into a term of ordinary language in respect of one application. I refer to such cases as 'a Daniel', 'a Croesus', 'a Solomon', 'the Rupert of debate', &c.

But these are abnormal uses in which the proper name ceases to be proper. The particular Johns, on the other hand, to whom the name John is applied as a proper name, do not form one extension corresponding to a single intension of this name. Each of them forms by himself a separate extension corresponding to a separate and distinct intension of the name John. The men called John are not related to their name as 'men' to 'man' or as 'towns' to 'town', but as Salviati's glasswork and the Pentateuch to 'Mosaic', or as a human being, and a cairn in the Lake Country, to 'man'. The subject is not without historical interest, but to pursue it would take us too far from logic. No doubt it might be maintained that in early language Intensional and Extensional meaning must to a great extent have coalesced. Whatever sound was appropriated to a sensible incident would at first, very possibly, attach itself only to the concrete or confused perception as a whole, and it might be long before pointing out the occurrence could be in any way distinguished from saying in what it consisted. Thus it might be said that language must have begun with proper names for everything, and advanced to general names, and only then had to face the problems arising from the necessity

of identifying *individuals* by help of symbols whose nature is to be *general*. The problem is now solved to a considerable extent by a peculiar convention as to mode of writing and amount of signification to be expected. We know that to find a town in Ontario called by the name of London justifies no single inference as to points of identity between it and the metropolis of England We must keep etymology out of the question. A word means what it is used to mean, not what it once meant The derivation of proper names justifies no inference at all as to their meaning. The Remington typewriters bear the stamp of Ilion In the same way intensional meaning cannot justly be ascribed to Christian names or surnames, at least in modern England There is now no legal monopoly of such names (though there may be of trading designations), and if we are taken in by ascribing intension of birth and breeding to a particular name, it is our own fault. But probably this state of things is modern, and the existence of proper names of persons in the full sense would in that case be modern also. If legal or social rights depended on bearing a particular name, then such a name had as an element of true intension those general relations—patriciate, legitimacy, civic birth—in which the right to bear it and the incidents of bearing it were involved. 'The art of giving names,' it has been said, 'is lost' It is certain that the purpose of mere recognition, to which all attributes are in their nature indifferent and serve only as a means, tends to destroy the picturesqueness of nomenclature by dissociating it from interest in a general and so significant intension The close relation between mere extensional meaning and the use of number is nowhere more strikingly illustrated than in the custom of numbering not only houses but streets, as in great American cities In the proper name there is still the semblance or fiction of a general Intension—the special name-word *seems* indicative of distinct meaning; in the number even this fiction has disappeared, and nothing remains but the place of the particular in an aggregate of particulars, united solely by a common denomination.

Thus the distinction between a proper name and a significant name (whether singular, as God, or general, as 'man') is

that in the use of a proper name signification is a means to identification, in the use of a singular or general name signification is predicated for its own sake. 'But the identification of a person or thing *is* signification,' it will be said. This raises the question of the nature of personal or individual identity, which is not in place here, it is enough to point out that mere identification is a very barren kind of signification, since there is hardly a single attribute of actual content as distinguished from mere external relations that is necessarily conveyed by it Macaulay after his mind was gone was still Lord Macaulay and his father's son, but what else was he that he had been ? [a]

Names with number attached

ζ. If pure Extension were to be found anywhere, it would be found in a general name or idea determined by number, or of which number is predicated.

An extensional whole is an aggregate of individuals sharing a common nature, but regarded as particulars, i.e as each identical with itself and external to all the others. This is, as we shall see more fully from the analysis of Enumeration (Book I, chap 4), the nature also of a numerical whole. The unit of number and the particular of extension are closely allied Each of them consists in the identity with itself of a concrete thing or discernible particular in spite of differences which it includes. Proper names also depend on this self-identity, but have for their purpose to single it out and mark it apart from the whole universe besides. Number does not seek to single out one such identity *par excellence*, but to formulate the relations which arise between such discrete identities as factors in a sum or aggregate. In the first place then, number, though an attribute and so an element of intension, yet by accentuating the embodiment of a content in units external to one another, demands an extensional rendering of the idea And in so far as depth of meaning is indifferent when we are thinking of aggregate units and not of connected attributes, so far the *intension* of a concept may be reduced to

[a] See Keynes, p 45, note My main point about proper names is that the element of meaning in them is auxiliary to individual identification, and therefore, in principle and not merely in fact (as with general names), arbitrary and variable

the *denomination* of a numerical aggregate But if this remnant of intension, which determines the range of the aggregate, is removed, the thought is destroyed in both its aspects

And further, in the second place, a numerical determination, although itself an attribute, tends to contradict intension proper, and so force the attribute of extension or particularity into importance It is obvious that in every concept the intension dictates the extension And the extension so dictated must as an aggregate of instances be theoretically at least capable of representation by a number, or if not, it must be in conflict with any and every number. We may omit the consideration of parts of space and parts of time, which seem to constitute a series that theoretically defies enumeration; but no actual content of our real world can be thus infinite, so far as we can understand The human race itself must, as we are bound to suppose, have a limited career, and the limitation, however far beyond our knowledge, must be immanent in man's nature in its relation to his environment. Thus the intension even of man, colour, gold, or other ordinary general names, must ultimately and theoretically imply a finite numerical aggregate of instances [1]

This number, which in such cases as the above we can never know, could be of no possible interest to us, were it not that it affects the import of any other number by which any such concept may be determined. In other instances the knowledge, which in the above cases seems not worth serious thought, is actually ours, or treated as being ours. Such instances are the three persons of the Trinity, the three sides of a triangle, the ten decemviri, the 670 members of the House of Commons, the five regular solids, the ninety degrees of a right angle. It will be observed that these illustrations display the number in very different relations to the intension. A member of the House of Commons is no less a member

[1] This is, in so far as the instances are true individuals in a known system. Mere 'observations' on the other hand, successive presentations to sense, must always be taken as entering into an infinite series, for no power can tell how often they may recur, nor what constitutes a single one Nor do they by mere repetition tend to generate a system

if some units are withdrawn from the legal number of 670, as, in consequence of death or resignation, is often, perhaps always, the case The number in which he is a unit does not directly affect his position, although no doubt, if an immense proportion of seats were to become vacant owing to some extraordinary catastrophe, the House would be unable for a time to act as a House of Commons The same is the case with the decemviri or the two Roman consuls, for the authority of one member of these boards was independent of that of the others or other. But if we take the case of twelve English jurymen the matter is altered, for the number is essential, though only made so by specific enactment, and if '*a juror is withdrawn*' the others *ipso facto* lose their powers of trying a case, i e cease to be in the full sense jurymen

And it is possible for the number to enter even more deeply into the essence. Two sides without a third cannot be two sides of a triangle; and an angle of one degree is not, considered in itself, a degree in a right angle The nature of space as an ideal whole does indeed introduce a difficulty here, for it may be said that a degree can only be understood with reference to the circle, and therefore involves the conception of a right angle, and that *in space*, a figure *is* all that it involves. The objection draws attention to a principle which holds good of all units without exception, viz. that every unit ultimately involves the whole in which it is a unit, but it does not alter the fact that we have no right angle unless we have ninety degrees, while we have a House of Commons (unless Parliament is dissolved) so long at least as it has enough members to form a quorum. In other words, the whole implied in the unit in the one case involves a precise numerical determination, and in the other case does not. And it will be observed that as instances tend to approach the former type, the number ceases to be truly extensional, becoming as we shall see (Book I, chap 3) a result due to measurement rather than to mere enumeration. The content, that is to say, no longer falls chiefly within each unit of the enumeration, so as to make the number a mere sign of the repetition of embodiments of the content, but in some essential respects is generated by a repetition of the parts and does not exist in each

taken alone. Thus, as was said, the character of being a right angle is not present in every degree of angular measurement, nor is the character of being a commonwealth present in every individual person—not, at any rate, in the same sense in which the character of being a man is present in every person A name or idea which, while involving a number of identical parts, is not truly predicated of each such part singly, was called in the old logic a collective as contrasted with a general name Army e g is a 'collective' name as regards the individual soldiers in it, but a general-name as applied to the English, German, French and other armies. The distinction indicated by the term was not valuable, for it was not explained But it is obvious that a 'collective' name or concept like 'army' is a halfway house between the mere common nature of units like men, horses, books and the like, in which the enumeration of the particulars repeats the intension in every item of the extension, and wholes like nation, Parliament, triangle, plant, in which the parts are bound together by other relations than that of number, and therefore their number does not form the extension of the whole, so much as an intensional attribute of that whole Thus the enumeration of Englishmen is not so much the extension of the English nation, as it is at the moment an attribute of the English nation to contain such a number of Englishmen The question is whether the content falls within the unit, or only within the synthesis of units

But whether mere units in an aggregate or elements in a numerically determinate whole, numerable parts must have a number, which must directly or circuitously depend on intension And every numerical determination other than that which thus springs from intension has the effect of erecting a mere whole of enumeration which, as regards the intension of the general name, is arbitrary and irrelevant. An exceptional instance will illustrate this Any two sides of a triangle are together greater than the third side Here the two sides are not a mere whole of enumeration, and are not irrelevant to the conception of the three sides of a triangle What is here predicated of them is a result of analysis applied to the triangle with its three sides, and is

a consequence of the three-sidedness of the triangle. The three sides might indeed be named as subject in the proposition. But if we say 'There were 10,000 men in Hyde Park last Sunday' we have constructed a whole of enumeration *prima facie* irrelevant to the concept man and having an extension that conflicts with the extension of that concept, and therefore with the intension that dictates that extension. In other words, we have depressed the term 'man' into the denomination or designation of the unit employed in counting The number may if we choose be stated as a predicate, and the limit 'in Hyde Park' taken into the qualification of reality which forms the subject, all that concerns us here is to point out that we are speaking of an aggregate framed *ad hoc* by enumeration, i e. by taking men one after the other in their particularity up to a limit which does not *prima facie* present itself as implied in their nature We not merely count men as particulars,—we count particulars if we count the angles of a pentagon,—but we divorce them from their natural intension by excluding the greater portion of the extension which it indicates. It is true then that number, *qua* mere enumeration, is, like the proper name *qua* mere identification, in a large measure antagonistic to intensional meaning

Alleged inverse ratio of Extension to Intension.

11 The two cases which have just been discussed are enough to show that not every variation of intension involves a corresponding variation of extension, or *vice versa*. The essence of proper names and of numbers is to mark the same extension or the same amount of extension as persistent through intensions partially at any rate varying Nevertheless, the demand for a formal rule of inter-dependence between these two obviously connected aspects of concept and of judgment was satisfied in traditional logic by the doctrine that intension varied inversely as extension This idea was an early development of the Aristotelian definition by genus and species, from which it obviously followed that whereas the generic attributes were contained together with others in the definition of the species, the individuals belonging to a species must be contained together with others among those belonging to the genus Aristotle noticed this conse-

quence of his own views, but the false accuracy of the traditional rule was a later development of the forms which he established, when their life was beginning to fail Recent logicians have more or less completely condemned the doctrine in question, perhaps the latest well-known writer who ascribed to it first-rate importance was Professor Jevons His account excludes one obvious objection, viz that a multiplication of identical instances cannot affect intension, by a proviso that only *logical* change of extension could affect intension But this makes the view a truism; if *logical* change of extension means admission into extension of a new *kind* as opposed to a mere multiplication of instances, it is obviously equivalent to *some* change, of whatever kind it may be, in meaning as such or intension. But even so, even accepting this proviso, it remains doubtful whether the doctrine of the inverse relation is important in any sense in which it is true I am inclined, however, to think that the recent logicians to whom I have alluded, e g Sigwart, Wundt, and Bradley, and also to a smaller extent Lotze, err by sheltering themselves under a point of form, and avoiding the question of import. It is true, no doubt, that you may have any arrangement of concepts; but it is hard, in view of our gigantic natural classifications with their unrivalled grasp of reality, to place any other arrangement of concepts on a level in real import with that of genus and species

I will begin, however, by going briefly through the weak points of the supposed law that the Intension of a concept or name varies inversely as its Extension.

The mathematical phrase is wrong

a The quasi-mathematical phrase 'inversely as' is wrong. It asserts a ratio, and a ratio is a numerical relation. But in the case before us, one side of the matters compared does not lend itself to enumeration at all The parts of extension may be counted, as we have seen, but it is inept to count the parts of intension For they are not external to each other, and they form a whole such as cannot be divided into units except by the most arbitrary dilaceration. And if it were so divided, all its parts would vary in value, and there would be no reason to expect that ten of them (i e. ten attributes) should have twice the amount or value of

five. We must constantly bear in mind, e g in estimating the false doctrine of analogical inference, that there is absolutely no sense in counting attributes

Assumptions regarding Intension

β. Mr Keynes (p 36 note) admits the objection to a numerical sense But he considers that no error is committed if we speak merely of the enlargement or restriction of the intension of a term

This appears to me to be purely a question of assumptions, and it would not be worth discussing if it did not seem to be the case that the common assumptions, which Mr. Keynes for example embodies in his theory, rule out important elements of our knowledge These assumptions all spring from the belief that the meaning of terms belonging to wider classes is naturally formed by abstraction from that of terms belonging to narrower classes And this belief falsifies our whole conception of knowledge and of the nature of things. The assumptions rooted in it, so far as intension and extension are concerned, seem to me to be three

Neglect of co-ordinate adjectives

(1) The comparison of co-ordinate adjectives, not belonging to the same ascending series of class-ideas, is ruled out Yet it is clear and important that such an adjective as 'visible' has both wider extension and deeper intension than an adjective meaning 'perceptible by smell' (Bradley's Logic, p 158)

Additional Properties.

(2) It is assumed that the terms of the smaller class have properties additional to those of the terms in the larger, and not merely qualities which are variations of those characterising the larger class, each to each This is Lotze's well-known suggestion (Logic, sections 23 and 31) and would overthrow Mr. Keynes's formal argument on p 39.

'Let X be determined by the set of examples $Q_1 \ldots Q_n$, and Y by the set $Q_1 \ldots Q_{n+1}$ which includes the additional object Q_{n+1}'

'Then Q_{n+1} either does or does not possess all the properties common to $Q_1 \ldots Q_n$'

If the former, X and Y have the same comprehensions and the same denotations, if the latter, Y has the lesser comprehension by the properties which $Q_1 \ldots Q_n$ do possess and Q_{n+1} does not, and at the same time has the greater denotation by Q_{n+1} at least. (I compress the statement)

But, I suggest, the fact is that Q_{n+1} possesses in the form a a property which $Q_1 \ldots Q_n$ possess in the form a_1.

Then Q_{n+1} possesses an equivalent amount of properties to $Q_1 \ldots Q_n$, and the comprehension of X and Y is therefore equivalent (more carefully analysed the one would exceed in some respects, and the other in others), while, as there is a distinctive variation between some properties a_1 of $Q_1 \ldots Q_n$ and some a belonging to Q_{n+1}, both of which fall within A, a property of Y, the denotation of Y remains greater than that of X by Q_{n+1} at least

Or consider Mr Keynes's sentence, p 36 note 1, 'There can be no doubt that intension is increased when we pass from animal to man, or from man to negro' Does not common sense receive a sharp shock at the second instance, even though unable to escape from the belief that every sub-class has additional intension, on the top of the whole intension of its over-class? Surely it is better to take it that the negro has qualities which are distinctive variations within the qualities of man, and which are compatible with his falling short of the full intension of man as such For minute analysis, it might be said, he would have excess in some respects and defect in others; but that is only what every variation in quality would exhibit So as regards the relation of animal and man.

It is easy to say that animality is common to men and beasts, while rationality belongs to men only, and in place of it animals have either instinct or nothing, and that therefore animality is the intension of the class which includes beasts and men, while each of these sub-classes has a separate and additional intension But in fact the animality of men is quite different from the animality of beasts, and is not an attribute common to both in the sense in which a tree-trunk is the common support of two of its branches While on the other hand the thorough modifications which distinguish the intelligence of man from that of animals do not suffice to dissociate them beyond identification, and the class-conception which simply omits all reference to intelligence is an inadequate class-conception for men and animals Therefore the notion or abstraction which is to include both men and

animals must on the one hand provide for a variable animality; must be considered, that is, not in the light of a fixed mark, but as a scheme of modifiable relations, and must, on the other hand, find room for some reference to intelligence, and not simply strike it out as a mark in which the kinds to be classified are not the same *Prima facie* then the content of the superior class-conception is made up of the very same elements as those of the conception nearer to individual reality, only that it must represent each attribute schematically, by limits of variation, instead of embodying a fixed system of amounts or values

Concrete intensions unaccounted for.

(3) Either the increasing concreteness of intension due to inter-relations among individuals is wholly ruled out, or it must be assumed that such intension can be reduced to common predicates of terms in classes, and ought, like such predicates in general, to be presupposed in the intension of every term which possesses them But the fact is, that the meaning which accrues to a term from its functions or relations in a whole cannot be represented as a class-predicate which it shares with other members of that whole As Lotze pointed out with regard to humanity, that in it you have inter-relations which make it more than the class concept 'man', so it is with the state in regard to the citizen, or the army in regard to the soldiers

The old logic would retort here that the extension of 'state' is made up of particular 'states', not of persons, or that that of 'nation' or 'army', consists in the several nations and armies, not in individual men and soldiers, or that the extension of God is in his particular existence (as we do not here admit a plural) not in elements within God's being But this would only meet the objection at the cost of narrowing the idea of universality to that of mere abstraction, in contrast with the sense—synthesis of differences—in which we have taken it throughout Moreover, even the aggregate of men, nations, or animals which is indicated by an abstract universal name has in virtue of that universal a common nature which is a germ of concreteness A crowd is not an army, but it has in it always the elements of a mob As we saw above, collective names mark a mere half-way house from aggregation to

individuality, and it is a purely arbitrary procedure when examining the nature of universals, to restrict our notice to such as have attained to no higher embodiment than an aggregate of particulars But in fact our prejudices would cause us to neglect a concrete nature if any such were apparent within the aggregate We should insist that the idea which should include the states or nations of the world *must* have less import than the idea of England or France, and should *therefore* look for this idea in the abstraction 'state' or 'nation', neglecting to consider whether, e g as the source of international law, the aggregate of nations has not in it something beyond the elements common to various peoples, or whether, if this is not so, the absence of such a central unity is not at least a defect which we might hope to see amended.

So with the conceptions which might be supposed to be the emptiest of abstractions because the widest of ideas, Being, Reality, the Whole, the Universe. The entirety of our profoundest science and speculation attaches to them, because, in their enormous extension, all kinds of problems and interdependences present themselves You might urge formally that as the widest classes they should have an intension prior to and included in that of all common objects And no doubt, if we completely knew the flower in the crannied wall we should know the universe But no sane man surely would maintain that the profoundest thoughts about the universe are presupposed in the comprehension or intension of every flower, because it is a member of the universe, or of the class of existents or reals Plainly these thoughts arise with the increasing extension, as we pass from the plant to its functions and relations in ever wider provinces of the world

Subsumption. (4) The whole idea that judgment and reasoning naturally or exclusively depend upon subsumption, i e upon taking attributes as connected simply within and by the unity of individual subjects, has of late been rejected, and with good reason. And with this idea disappears any formal or universal necessity that may have been supposed to attend the arrangement of attributes as designations of successively wider aggregates of individual subjects It is important that we should dismiss the notion that the higher degrees of knowledge

are necessarily and in the nature of intelligence framed out of abstractions that omit whatever has interest and peculiarity in the real world. Nothing has been more fatal to the truth and vitality of ideas than this prejudice, which I do not admit to be a just representation of the principles of Plato and Aristotle, although certain salient features of their doctrine gave it an unfortunate advantage If the present reaction against formal logic should end in establishing a more vital conception of universality than that which sets it down to mere abstraction, a fundamental reform will have been made in philosophical first principles

Truth of 'inverse ratio'

III. Nevertheless, subsumption and abstraction play a part in knowledge. It is not quite certain that there is any judgment or argument which is incapable of being exhibited in subsumptive form It is certain that to abstract and to distinguish—to know what belongs to one relation, and what, again, though conjoined with that relation, yet does not arise out of it, but out of some other condition or cause —is the first duty of the scientific intelligence In consequence of this activity, arrangements of individual objects under a series of abstractions, each applying to a wider aggregate than the last, meet us on every hand, and most obviously of all in family relationships as estimated among civilised nations The question before us cannot be dismissed until we have treated it from this more real point of view. Is the 'pyramidal' arrangement of concepts, subordinating the less to the more abstract, scientifically convenient, or, what in an ultimate sense comes to the same thing, is it true? In answer to this question I will mention two further points of interest.

Alternative classifications

a. Sigwart reminds us that every concept may be regarded from different points of view, and classification or abstraction may therefore have different lines open to it. e. g. do we class a square first with four-sided figures, and then, subject to that limitation, with equilateral figures, or *vice versa*? It is obvious that we put the square in quite different company according to the order in which we apply these points of view. The point is that in this and similar cases we seem to have *bona fide* alternatives. No serious attention would be needed

by the mere fact that we can class a copy of 'Paradise Lost' either as a black or as a rectangular object But if genuine alternative classifications are possible, it is clear that we may have a hundred objects before us, and being forced to divide them into classes from each of ten unconnected points of view, may be left with ten different classifications for each object, and apart from some peculiar ground there will be no reason for subordinating each object decisively to one classification rather than to another

The first remark that such a suggestion invites is that the idea of alternatives only touches the subordination of every object to one class or series of classes, but does not touch the alleged necessity of successively emptier abstractions within any one point of view which may be selected (as some one must always be selected at least *pro hac vice*) and adhered to And, secondly, the mere difficulty of alternatives is inherent in the nature of intelligence, meets us in the problem of giving names, and is overcome in some degree whenever we venture to affirm a fact. In its most genuine form it is met by the theory of the relation between different sciences, to each of which the same individual thing obviously presents a different aspect. Sigwart's suggestion[1] of an inference to Pantheism from the doctrine of essence combined with that of subordination—because only the essence of the highest abstractions is in no relation accidental—seems to me perfectly wild We should by this time be well aware that all essence is relative, but that relativity does not exclude absoluteness One set of attributes are a man's essence *qua* citizen, and another *qua* parent. I may add that his argument depends on assuming that any class may be regarded in its turn as genus and as species But this is an obsolete conception belonging to purely formal logic. The successive abstractions of classification have distinct characters expressed by definite titles and not interchangeable. A genus is not a species of an Order, and a Class is not the genus of a Division

Inverse ratio partially justified.

β. Thus I cannot think that Wundt is right in tracing the relations between class-idea and individuals to the mere effort

[1] Logik, 1 355 E Trans 1 274

of language to economise its store of words. The whole fabric of the organic and even of the inorganic world creates *prima facie* an overpowering impression that natural classification can correspond to reality. The perceptible fact of graduated affinity has in all ages taken precedence of its causal explanation. The facts of human or animal descent, so far as immediately observed and as embodied in systems of relationship, supplied a name, if not a thorough-going explanation, for the affinities observed in nature [1] The degrees of family connection, at least in mature European society, are the simple prototype of the ordinary process of classification , and the analogy has extended since the earliest days of Logic to the inorganic as well as to the organic world Now the alleged relation of Intension and Extension may be simply illustrated by the characteristics shared by a group of first cousins, i e. persons descended from the same grandparents, compared with those shared by second, third, or fourth cousins, whose common descent is more remote by one or by several generations In the human race, indeed, individuality of mind and disposition has so much to feed it in special knowledge and experience that the phenomena are but irregularly observable , but in the evolution of plants and animals their characters have the same graduated identity without the same deductions on the score of special training and mental development. And it is not only evolution by descent that will produce these pervading affinities Cognate processes of causation result in kindred formations all through the world of inorganic substances ; and even in the sphere of necessary ideas, of number and geometrical figure, identical relations, under varied conditions, produce the appearance of graduated kinship or affinity.

Granting, therefore, in accordance with what has been said above, that the work of abstraction should be represented *not* as selective omission *but* as constructive analysis, and that the blanks in a schedule of attributes common to a large range of existence are not *mere* blanks, but zero values that may become positive in some cases , still we cannot on the whole deny that

[1] See Lotze, Logik, sect 30, on the probable original meaning of *γένος* and *εἶδος*

the graduated operation of natural causes is very conveniently embodied for some purposes of knowledge in a hierarchy of abstractions But it must be remembered that the working idea of the evolutionary science of life is not represented by the classified organic world, but by the interdependent play of creatures upon each other and their environment in a concrete universe—a conception which, as explained above, cannot at all be represented in the form of a classified system And the largest conceptions of thought will not be the *summa genera* of a hierarchy of abstractions, but ideas of a self-determining totality, comprising an enormous intension corresponding to their immense extension

So, again, as has been shown above, where the higher concept is not a mere law, but a concrete real whole, the idea of diminishing Intension has no application whatever. It is for special knowledge to determine how far these different points of view are respectively to be taken. Psychology subordinates the human soul to a set of laws which include, as they grow more abstract, a larger range of animal and organic existence.[a] Political science treats the same spiritual being in its concrete relations within an actual community of such beings—not as a mere member of a class To which of these two sciences that of Ethics ought to assimilate its procedure is a vexed question which illustrates the problem of deciding in what regions the rule of diminishing Intension applies.

One more suggestion may be ventured which aims, it will be said, at rehabilitating an old fallacy. I am unable, however, to persuade myself that it does not appeal to an unquestionable truth I assumed above, for the sake of simplicity, that regressive abstractions such as figure in classification could represent in respect of their abstractness no character of

[a] Yet even in this case a deeper intension tends to present itself along with the larger denotation We can hardly argue (cp p 60) that the wider intension belonging to the word soul in the universe falls within the class-predicates of the human soul, as a stratum below its distinctively human properties You may urge that if it is true we might have known it from the human soul alone, but the same argument would require us to take the metaphysical theory of the universe as a prior basis for the intension of every flower

reality. The grandparents from whom a cousinhood is descended have often a more real, marked, and individual character than their grandchildren, though what these latter retain of that character can only be represented by an abstraction, much of it having been lost and supplemented from other sources The grandparents are represented by an abstraction, in the existing first cousinhood, but were not themselves in any way more abstract than their descendants But when we look at long intervals of evolution the matter undergoes a change, as is easily verifiable in the case of human character and intellect A savage has not the individuality of a modern European , he *is* more abstract , his nature includes fewer differences, less profound feelings, less grasp of fact, and less definiteness of imagination Or to take a more tangible instance (for the facts of savage life are always subject to dispute if not to doubt), even the Greek intellect in its prime, or Greek art at its best, has not the many-sided concreteness that belongs to Shakespeare or Goethe, Raphael or Turner And indeed, if we go to natural history, and say that ' Organism in the abstract ' never existed, really the Amoeba or the white blood corpuscle seem to contradict us It is hard to see how every successive generation or epoch of evolution, so long as growth is not counterbalanced by decay, can avoid adding import and significance to the content of things With the mind this is admittedly the case, and the course of evolution seems to show it in nature too It may be said that the antecedent is no more abstract than the consequent, and that the universe as a whole cannot grow either more or less concrete. But it may be doubted whether this formal argument applies to a system which has individuals within it In bringing to bear its total content upon such individuals there seems to be scope for infinite grades of concreteness In this case the advance from abstractness to concrete individuality would have grounds in historical fact In one form or another this idea has often been maintained, and I think that it bears witness to a truth

We have now considered the primary aspects of the materials or instruments of logical thought—the idea and the name. This discussion seemed appropriate to an Introduction, be-

cause it is impossible to admit that the name, concept,[a] or idea, is a portion of the content of Logic, in the sense in which such a position is assigned to the Judgment and to Inference. We do not enter upon logical development proper till we come to deal with the evolution and affiliation of judgments.

[a] Nettleship (Lectures on Logic) and other writers have insisted that the concept, as the point in which judgment has rest and unity, ought to be definitely recognised in Logic I see that we possess a relatively solid world of our own construction But I do not see that this can avoid being maintained within the general form of judgment by being referred to its place in our universe of experience See II, p 292.

BOOK I

OF JUDGMENT

CHAPTER I

OF JUDGMENT AND JUDGMENT-FORMS IN GENERAL

1 JUDGMENT is coextensive with affirmation and denial, or, which is the same thing, with truth and falsehood True and false are not indeed terms applied exclusively to judgments, but yet in all their applications their essential meaning depends upon judgment The sensations of a diseased organ may be abnormal, but cannot possibly be false unless, on the strength of them, we judge erroneously A false note is a real sound, a false man is an actual individual It is not their existence, but a judgment implied in their nature, that gives meaning to the censure of falseness The musical note is not what its place demands, the man is not what he pretends or aspires to be; it is the demand or pretension, ascribed actually or by metaphor to thing or person, which condemns them as false in as far as it is unrealised

1 The Nature of Judgment as such

Thus truth and falsehood are coextensive with judgment, and depend on the fact which is its primary condition, the fact that a thing may have an ideal relation to reality over and above its own particular existence, so that its existence, though in itself real and actual, is empty and valueless in the absence of the further reality that such a relation demands. Truth must belong to something whose unreality is not simply non-existence, or how could falsehood exist? The essence of falsehood or fiction is that there should be an actual something that pretends to be something else

Thus if we describe Judgment as the act of thought which is capable of truth and falsehood, the description, although tautologous, is not unsuggestive. It tells us that we are to

look for the differentia of judgment not in a mere mental fact, but in some further value with which the mere mental fact is invested

Symbolic ideas

1 This primary condition of judgment recalls us to the subject of the Introduction In judging, we use ideas, but the ideas which we use are not mere particular mental images, the perishing existences which pass through consciousness, and which, *qua* particular psychical states on a level with mere sensations, never recur Ideas,[1] according to Locke, though particular in their existence, are general in their signification In judgment, ideas are employed solely for the sake of their general signification, and without reference to their particular existence An idea, considered as a general signification, is what was described in the Introduction as the meaning of a word. It is not without effort, as we all know, that we can find in our consciousness any intermediary between the word on the one hand, and the reality on the other, and when it is brought to our notice that a reality cannot as such be a state of our individual consciousness, we are sometimes tempted to deny that it has any representative there beyond the name We see from this how utterly the symbolic and secondary employment of psychical images obliterates all consideration of their particular existence as mental occurrences We no more take note of them than, on meeting a welcome friend, we give ourselves a detailed account of the peculiarities by which we recognise him The word and its reference—a reference to some continued identity in the world of meanings[2]—are inextricably welded together It is only by reflective analysis that we discover, within and auxiliary to the meaning of a word, the particular psychical images by help of which we symbolise it.

And the meaning tyrannises over the psychical image in another respect Besides crushing out of sight its particular and exclusive existence, it also crushes out part of its content. The psychical images that pass through our minds might be compared to a store of signal flags Not only is it indifferent whether your signal flag of to-day is the same bit of cloth that

[1] Essay on Human Understanding, Book III, chap 3, sect 11

[2] See Introduction, sect 7

you hoisted yesterday, but also, no one knows or cares whether it is clean or dirty, thick or thin, frayed or smooth, so long as it is distinctly legible as an element of the signal-code. Part of its content,[a] of its attributes and relations, is a fixed index which carries a distinct reference, all the rest is nothing to us, and, except in a moment of idle curiosity, we are unaware that it exists The well-known difficulty of detecting misprints arises from the same despotism of the meaning Let the main index-letters of a word be correct, or even the main index-words of a sentence, and we are oft at once in thought to the word or sentence which is indicated, and remain unaware of minor variations in the content employed as index

Thus the idea, as used in judgment, is a general signification, or in other words, a fixed reference And because fixed, it is limited, limited to portions of content which serve as indices of the reference, and are compatible with psychical accompaniments that vary with the series of images I will give another instance Some one speaks to me of the Aegean Sea, which I have never seen He tells me that it is a deep-blue sea under a cloudless sky, studded with rocky islands The meanings of these words are a problem set to my thought I have to meet him in the world of objective references, which as intelligent beings we have in common How I do this is my own affair, and the precise images at my command will vary from day to day,

[a] I can see no difficulty in this conception, if we bring in the acquired content, e g of a word Critics might say that this equals meaning, and that the text explains meaning by meaning But here, perhaps, they supply the answer For a word, they point out, has a psychical fringe, which, they again point out, is not meaning (cp Stout, Ar Proc, 1903, p 13, and Hoernlé, Mind, 01, p 81 ff) For my own part (see p 40 above, and II, p 298), what I take to matter is the use In an original or acquired content (though already symbolic, as in my view, all ideas are, see p 70 below), when not being used in definite judgment, the aspect of mental existence becomes prominent The use in judgment I take to be a question of degree (p 5 above) I may add that I am aware of the passage about Mr Bradley's phrase 'acquired content' in Professor Stout's paper above referred to But I continue to think that the term is explained by the words 'The meaning may be part of the original content, or it may have been discovered and even added by a further extension' (Bradley's Logic, p 3) And it appears to me clear that with this explanation it is true that the content of a sign, though not, of course, the whole content, always qualifies for us the thing signified See Nettleship, Remains, I 23

and from minute to minute It sounds simple to say that I combine my recollections of sea and sky at Torbay with those of the island-studded waters of Orkney or the Hebrides Even so, there is much to adjust and to neglect, the red cliffs of Torbay, and the cloudy skies of the north But then again, my recollections are already themselves symbolic ideas, the reference to Torbay or the Hebrides is itself a problem set to thought, and puts me upon the selection of index-elements in fugitive images that are never twice the same I have *first* to symbolise the colour of Torbay, using for the purpose any blue that I can call to mind, and fixing, correcting, subtracting from, the colour so re-called, till I reduce it to a mere index-quality; and *then* I have to deal in the same way with the meaning or significant idea so obtained, clipping and adjusting the qualities of Torbay till it seems to serve as a symbol of the Aegean [a]

Here then we have the first essential of judgment The ideas used in judging are not particular existences but general significations, or objective references No mere mental occurrences as such, no series or combination of particular images, can by any possibility be a judgment It is the essence of judgment to claim a value which is beyond the mere mental act itself, and which is therefore liable to be divorced from the mental act, and this divorce, as we have seen, is what falsehood means That is false, which *is*, but like a false coin, has not the significance which it claims In judgment, then, all ideas are symbolic, that is to say, have a constant reference

Can ideas be symbolic apart from judgment? [b] If no ideas in a human consciousness *are* apart from judgment (see Introd 2 11) this question falls to the ground so far as concerns

[a] Then, it has been said, my test of truth is correspondence, i e how far my idea is like the original I reply, to me this is meaningless There is no given original Sense-perception gives no original How can it, when its main task is self-amendment? A man may have seen the Aegean, and yet have a worse idea of it than a man who has not The ultimate 'original', got by amending our perception till it could be amended no more, would involve the Absolute, and would be like nothing in our world of percepts But the working test for us is what idea does most justice to all the experiences concerned

[b] For the significance of this paragraph see p 39 above

that consciousness But the discussion referred to made it clear that *apart from ultimate analysis* we do entertain ideas without judging them true, as in the question and in the negation, and that these ideas are symbolic The further problem may then be pressed upon us 'Are there at all ideas which are not symbolic?' In identifying the *human* intelligence with a continuous judgment we seem to have denied in advance that non-symbolic ideas *are* for that consciousness The answer is that α In judgment itself the idea can be distinguished *qua* particular in time or psychical fact, and *so far* is not symbolic and β In all those human experiences from which we draw our conjectures as to the animal intelligence, when in languor or in ignorance image succeeds image without conscious judgment we feel what it is to have ideas as facts and not as symbols

Reference to reality

11 Granting that symbolic ideas cannot ultimately be entertained without judging it does not follow that to judge is merely to entertain ideas In what does the act of judgment consist? An act it undoubtedly is, an act which is as certainly present, and which we find as hard to describe, as the much disputed act of volition itself

I shall attempt in the first instance to make the essentials of the matter quite clear in a simple case with which we shall afterwards find that all more elaborate instances agree in fundamental structure

If I say, pointing to a particular house, 'That is my home,' it is clear that in this act of judgment the reference conveyed by the demonstrative is indispensable The significant idea, 'my home,' is affirmed not of any other general significant idea in my mind, but of something which is rendered unique by being present to me in perception In making the judgment, 'That is my home' I extend the present sense-perception of a house in a certain landscape by attaching to it the ideal content or meaning of 'home', and moreover, in doing this, I pronounce the ideal content to be, so to speak, of one and the same tissue with what I have before me in my actual perception That is to say I affirm the meaning of the idea, or the idea considered as a meaning, to be a real quality of that which I perceive in my perception

The same account holds good of every perceptive judgment, when I see a white substance on a plate and judge that 'it is bread', I affirm the reference or general meaning which constitutes the symbolic idea 'bread' in my mind, to be a real quality of the spot or point in present perception which I attempt to designate by the demonstrative 'this' The act defines the given but indefinite real by affirmation of a quality, and affirms reality of the definite quality by attaching it to the previously undefined real Reality is given for me *in* present sensuous perception, and *in* the immediate feeling of my own sentient existence that goes with it The real world, as a definite organised system, is *for me* an extension of this present sensation and self feeling by means of judgment, and it is the essence of judgment to effect and sustain such an extension. It makes no essential difference whether the ideas whose content is pronounced to be an attribute of reality appear to fall within what is given in perception, or not We shall find hereafter that it is vain to attempt to lay down boundaries between the given and its extension The moment we try to do this we are on the wrong track The given and its extension differ not absolutely but relatively; they are continuous with each other, and the metaphor by which we speak of an extension conceals from us that the so-called 'given' is no less artificial than that by which it is extended It is the character and quality of being directly in contact with sense-perception,[1] not any fixed datum of content, that forms the constantly shifting centre of the individual's real world, and spreads from that centre over every extension which the system of reality receives from judgment

Waiving then this distinction, though as a matter of degree it may find a place in the enumeration of judgments, we find the same general features in all judgments of perception There is a presence of a something in contact with our sensitive self, which, as being so in contact, has the character of reality, and there is the qualification of this reality by the reference to it

[1] This has sometimes been remarked on as if it implied a sensationalist view of Reality But the point is merely that in the 'this' of sense Reality appears in a quasi-satisfactory form, having a sort of self-completeness When we begin to explain and define it, of course it breaks down, and demands the effort to complete it by way of thought

of some meaning *such as can be* symbolised by a name It cannot be alleged in theory that a name is essential to judgment At least for 'name' it would be necessary to substitute in such an allegation 'some symbol' The spatial order of things which we see whenever we open our eyes, is, *qua order of things*, the content of a perceptive judgment, in which universal ideas are presented through sensuous symbols.

The subject in every judgment of Perception is some given spot or point in sensuous contact with the percipient self. But, as all reality is continuous, the subject is not *merely* this given spot or point [1] It is impossible to confine the real world within this or that presentation Every definition or qualification of a point in present perception is affirmed of the real world which is continuous with present perception The ultimate [2] subject of the perceptive judgment is the real world as a whole, and it is of this that, in judging, we affirm the qualities or characteristics

The claim to be true, which as we saw belongs primarily to judgment, indicates the same relation In every judgment, as Mill incisively contends, we profess to speak about the real world and real things 'The Sun' means 'the Sun', and whatever that may be, it is not anything *merely* in my mind; not relative purely to me as a conscious organism; not a psychical fact in my individual history Every judgment, perceptive or universal, might without altering its meaning be introduced by some such phrase as 'Reality is such that ——', 'The real world is characterised by ——'

Thus in the Perceptive Judgment at least we find the meaning or objective reference of an idea—such a content as is indicated by a name—affirmed to characterise some reality present in sense-perception, and through it, reality as a whole We shall find that all Judgments of every kind share the main elements of this description; only that the reference to an indeterminate element of present sense-perception is gradually displaced by the introduction of explicit ideas to describe

[1] See Introd 2 1

[2] See Introd 2 1 Analysis and cross-examination readily verify this as a fact After admitting any judgment to be true, you cannot deny its modifying effect on any portion whatever of your real world, i e it has been admitted of the real world as a whole

the immediate subject Such ideas disguise but do not remove the reference to Reality as the ultimate subject in every judgment, they have, however, important effects in modifying both the act of affirmation, and the nature of what is affirmed When I come to examine the chief types of judgment, I shall have to consider the nature of these effects But I intend in the first place to say something of the proposition, from the analysis of which many current views about judgment are derived

Judgment and Proposition

111 The enunciative sentence—the unit of language which represents a judgment—is called a proposition Language, as we saw, supplies the fixed symbols which stand for ideas It would be rash to say that there can be thought without language—if language includes every possible system of recognisable signs—and wholly perverse to imagine that the ideal of intelligence lies at all in the direction of a severance of thought from words The Introduction, in dealing with Names, showed us the absurdity of any such conception But yet the spoken or written proposition differs fundamentally from the judgment

I do not think that it is convenient to rank the narrative or temporal affirmation as a 'proposition' (German 'Satz'), and reserve the name of judgment for an act of thought which has some purpose in the way of classification or definition. To do so is in English terminology at least to confuse a distinction of degree with one of kind, but it is worth noticing that such a nomenclature has been proposed,[1] and that according to it judgment proper would begin at the point where inference and necessity become explicit For against any *doubt*, judgment maintains itself as an inference,[2] and this is exactly the test that has been held to distinguish a judgment from a proposition. To affirm that a carriage is passing the house, Hegel says,[1] is not a judgment unless there is a question, e g whether it is a carriage or a cart; i e, I suppose, unless some general connection of attributes is intentionally affirmed Now a general connection involves a ground, and so an inference Thus the classification in question would

[1] Hegel's Logic, Wallace's Translation, pp 258–9
[2] Bradley, Principles of Logic, p 404

have the merit of suggesting that judgment begins with inference But the point of commencement taken is really arbitrary, though judgment and inference begin together, yet both begin before this point.

I prefer then to take the proposition all through as the actual spoken or written enunciative sentence, while the judgment is the intellectual act which depends in various degrees upon words or other symbols, but is different from any mere combination of words or symbols whether heard, read, or remembered

The essential differences between judgment and proposition may be arranged under two heads, which cannot however be wholly separated from each other I shall first speak of the so-called *parts* of the judgment, the current conceptions of which are derived from grammatical analysis of the proposition, and then pass on to consider how far the idea of a transition in time, which is inseparable from the apprehension of a sentence, is applicable to the judgment as such

a The division of the Judgment into Subject Copula and Predicate is obviously derived from the analysis of the enunciative sentence The finite verb, which is a proposition in miniature, contains all these elements within itself, and the history of their being distinguished within the sentence is the history partly of linguistic evolution and partly of grammatical or quasi-logical analysis Even the separation of the substantive from or within the verb, is, I suppose, an early analytic development of language; and it is the tendency of modern speech, no less than a supposed convenience of thought, that has finally transmuted Nominative and Verb into Subject Copula and Predicate The Copula in the modern sense was unknown to Aristotle, although the use of the predicative Verb 'to be' attracted his attention and drew from him a somewhat inadequate explanation If we return, however, to Aristotle's main position, and, in agreement with students of English grammar,[1] regard the Judgment as made up of Subject and Predication (ὄνομα and ῥῆμα), we have got rid of one fiction in the separate Copula, but the distinction

The parts of the judgment.

[1] Cp Mason's English Grammar, Jones's Analysis of English Sentences, and Wrightson's Functional Elements of the Sentence

which we retain may still be challenged It is plain that the judgment, however complex, is a single idea The relations within it are not relations between ideas, but are themselves a part of the idea which is predicated In other words, the subject must be outside the judgment in order that the content of the judgment may be predicated of it If not, we fall back into 'my idea of the earth goes round my idea of the sun', and this, as we have seen, is never the meaning of 'The earth goes round the sun'. What we want is 'The real world has in it as a fact what I mean by earth going round sun'.

This view stated thus extremely would not only annihilate the copula with separate content but also the whole distinction of Subject and Predication, and it is an argument in its favour, that, in doing so, it would seem only to lay bare in all judgments the elementary type which forces itself on our observation in the simplest perceptive apprehension, at a stage, that is to say before the grammatical subject, which creates our present difficulty, appears explicitly in the proposition But we shall see in tracing the evolution of judgment, that it is impossible to dispense with the distinction of Subject and Predication, and that the appearance of contrast between propositions which have and which have not grammatical subjects, is caused by the necessity of representing immature thought in developed language, so that the thought in which distinctions are rudimentary must either be mutilated by the omission of an element, or transformed by explicit articulation It is impossible to *represent* a judgment by a single noun belonging to a modern language, though such a noun is often all that we utter Such a judgment should really be represented either by a rudimentary sentence, that is, by some element of language not yet reduced to the position of a part of speech, *or* by a miniature sentence, i e by a verb

In other words, although the ultimate Subject extends beyond the content of the judgment, yet in every judgment there is a starting-point or point of contact with the ultimate subject; and the starting-point or point of contact with reality is present in a rudimentary form in the simplest perceptive judgment, as it is explicitly in the later and more elaborate types.

Then it would come to this Subject and Predicate in the actual judgment are really distinct, as a real identity from or in its differences The relation of their contents is itself ideal, and not a relation between ideas, but nevertheless the judgment demands this relation, for the judgment is my consciousness *qua judging* and my consciousness in judging identifies the ideal or symbolised reference which constitutes the predication with its own construction of the Real world [1]

The difficulty is that you cannot affirm without introducing a distinction or reference into the content of the affirmation; and yet such distinction or reference being part of what is affirmed, and not a relation between what is affirmed and something else, cannot it would seem be the essence of the affirmation What is the connection between the two things, between the reference of 'is-building a-wall' to 'Balbus', and the affirmation that the *whole* idea 'Balbus-building-a-wall' is true of reality? What has the action of Balbus to do with my affirmation that Balbus acts? The latter seems wholly unconnected with the former and yet they are inseparable.

And the answer is that the real world is primarily and emphatically *my* world I take it to be real in virtue of its contact with me Therefore though the ideal relation within a judgment is not a psychical fact in my mind but a fact affirmed objectively of the real world yet, the real world for me being the world that hangs from my present perception, I identify my assertion about it with its assertion of itself In every judgment the ultimate subject Reality is represented by a *selective* perception, or idea, which designates a something accepted as real This something taken as standing for reality, is the actual subject of the judgment, and is qualified by the ideal content which forms the predication No judgment can be found in which Subject and Predicate are not apparent Reality is one, but its presentation varies, and it is impossible to judge without explaining where and how Reality accepts the qualification which we attach to it The presentation of Reality qualified by an ideal content, is one aspect of Subject and Predication; and my individual per-

[1] Cp Introd 2 1

cipient consciousness determining itself by a symbolic idea, is the other That the latter is identified with the former follows from the fact that in entering upon the world of thought my consciousness enters upon the experience of logical necessity—the *nisus* towards complete expression

Thus I am of opinion that Subject and Predication are essential elements in the Judgment But whereas in the judgment they are differences within an identity, in the proposition they are isolated parts of an extended whole, and the copula, which in Judgment is merely the reference that marks predication, and has no separate content, becomes in the proposition an isolated part of speech When therefore the analysis of the proposition controls the interpretation of the judgment, each of these parts of the sentence is treated as a separable content, and perhaps as a separate psychical existence, and we are told of two ideas or two ideal contents, and a variable copula, itself also an ideal content, which indicates the varying relations[1] between them In this sense Subject and Copula and Predicate are mere fictions The judgment is not a relation between ideas, nor a transition from one idea to another, nor does it contain a third idea which indicates a particular kind of connection between two other ideal contents.

The real nature of the copula we have seen already It is the mere sign of affirmation, and, though usually conveyed by a finite verb in languages which possess one, does *not* depend on tense Aristotle, who was inclined to include 'indication of time' in the differentia of affirmation, was nevertheless aware that judgment could take place 'absolutely' (ἁπλῶς) as well as with note of time (κατὰ χρόνον). Moreover a verb can exist without definite tense, and predication can exist apart from a distinctly formed verb

The reason why the verb, where there is a verb, is appropriated to the act of predication, is not that the verb signifies time, change, or action, but that it is, as has constantly been repeated, a miniature sentence Not merely does the verb 'agree' with its subject—the adjective also agrees with its

[1] Lotze, Logik, sect 52 It is very doubtful whether in this passage Lotze escapes the error which he imputes to others, of 'reducing a logical operation to a mere psychical occurrence'

substantive—but by convention, or explicitly in the person-ending, it includes within itself a reference to given reality, and can therefore stand alone as an enunciation which no other part of speech can do[1]. In other words the verb is *prima facie* a content referred to a real individual subject, and though the subject may be by the help of additional phrases defined, set down as imaginary, or even denied, the verb has always in itself the force of this demonstrative reference. An adjective implies a reference to something else, but the something may be a mere idea, it is only the verb that professes to select an element directly related to the speaker's apprehension, and to attach a significant content to that element.

It is in the demonstrative force of the verb that we must look for its fundamental predicative force. I suppose that the collocations which in Greek and more or less in many languages, have power to turn the epithet into a predicate, owe their significance to a quasi-demonstrative emphasis. In 'the white horse' (ὁ λευκὸς ἵππος) there is nothing that can be taken as a reference to a special point in reality, no indication of a real existence, either in the ideal content or out of it, which we propose to qualify by its meaning. In 'the horse (is) white' (ὁ ἵππος λευκὸς) there is an indication of a line between *α* an individual that may be real and *β* a content that may be attached to him or it, and therefore the instinct of reason which sees a judgment wherever a judgment is possible, takes the individual named as if it were an appeal to perception, i. e. a demonstrative reference to reality, and the content as a quality ascribed to the real subject so obtained. In the universal judgment this demonstrative reference becomes merely formal, but it continues in all language to supply the symbol of judgment.

Judgment in relation to Time

β It has been proposed to distinguish Subject and Predicate simply as earlier and later in time, and the above instances of demonstrative reference appear to support this notion. But rather than admit it to be correct I should surrender the distinction altogether and adopt the view that there is no subject in the judgment as such. For it is absolutely impossible that priority in time should subsist between the parts of a com-

[1] See, however p. 99 below.

pleted judgment. But if not, the priority of the subject would exist merely in memory, and an act of thought cannot be characterised by a mere recollection of the process that generated it

In what sense is it true that the Judgment is in time, and in what sense not? It may be convenient to distinguish between arriving at the judgment, and subsequently modifying the judgment, although the two processes are, as we shall see, really continuous

In what follows I do not identify the aspect of Judgment as in time with the series of images *qua* psychical occurrences that pass through the mind while we judge It is probable that the view which defines Judgment as a change is influenced by the particularity of ideas *qua* events of consciousness as well as by the constant transition from judgment to judgment But the former element ought to have been eliminated by what has been said above

Arriving at Judgment

1 In arriving at a Judgment, as when we hear a sentence and 'wait for the verb', or scrutinise an approaching person until his name comes into the mind, we undoubtedly appear to begin with a ready-made Subject, to which a Predicate is added by a subsequent transition. But closer attention will show us that this is not the case We have always some anticipation of the meaning of a sentence, and this anticipation takes the shape of a provisional judgment or judgment in outline, very probably disjunctive in type, the shape of which becomes more definite as we follow the sentence, until the final clause determines its ultimate content In the first place, consciousness, when any ideal content whatever is presented to it, absolutely refuses to abstain from judging, and in the second place, what comes first could not have (as it undoubtedly has) the significance of a subject, unless with reference to something already referred to it in the way of predication

The case of perception leads to the same conclusion You can come to no judgment by help of perception unless you interrogate perception, and you cannot interrogate perception unless you have in the mind some general idea as a basis for further specification

Thus, in reaching a particular apprehension or perception,

there is a transition that occupies time But the transition is not from Subject to Predicate which we will call S and P respectively, but from s–py (where y indicates superfluous detail, which is omitted when the perception becomes clear) through Σ–Π, to S–P To speak of a transition from S to P is wholly false We never have an S first and then tack a P on to it, we have always an inchoate judgment or a choice of judgments The process is not like adding one piece in a mosaic to another it is more like enlarging a hole, which has centre and circumference from the beginning

The complete Judgment

2 What has been said of the transition by which we arrive at a judgment cannot but apply to the judgment when arrived at It is clear indeed that we are thus led to regard the completion of the judgment as an arbitrary distinction, dependent solely upon our momentary interest The completed judgment, like the process by which it is obtained, obviously possesses duration It is absurd to suppose that a judgment cannot be dwelt upon, and only exists as a momentary transition from S to P Such a conception arises from the confusion of two points of view, either of which may be taken as a presupposition, and reconciled with the other by a mistaken compromise It may be assumed that the judgment as such, is not in time, and then this assumption has to be reconciled with the obvious fact that judgment as an intellectual process is a transition that occupies duration, or it may be taken as certain that the judgment is a transition in time, and then we have to face the experience that its essential parts do not fall outside each other in succession To treat it as an instantaneous transition is a ridiculous attempt to combine the character of occupying duration with that of not being in succession It recognises both principles, and satisfies neither.

As we have seen in the process of arriving at the judgment, the act of judging as an occurrence in consciousness presents itself in the aspect of an interval of consciousness extended in time, and therefore including successive differences within it. But it does not include succession because the nature of the judgment is to be successive, but because the flux of sensations and ideas is always pressing new material upon consciousness, and a perception, once attained, satisfies no interest by

being further dwelt upon unless it gains in content from moment to moment Thus the duration of the judgment as a transition in time is, so to speak, its external aspect, the aspect which, as a whole, it presents when compared with other occurrences in consciousness, and this duration is theoretically capable of any degree of extension On the other hand, as between Subject and Predication, that is to say, within the judgment, there is no transition at all S and P are modified *pari passu*, and so, as a relation between them, the judgment is not in time This relation is a continued identity S–P which includes within it the differences s–p, Σ–Π, and so on The transition is not from S to P, but from s–p to Σ–Π within the general signification S–P. The idea of mere momentary existence has therefore thus much truth, that if you cut across the interval of consciousness occupied by a single judgment at any point whatever, you will always find in the plane so laid bare both S and P in one or other of their forms They are in every minute part, but they are not confined to such a minute part Judgment breaks up into judgments as rhomboidal spar into rhomboids, but nevertheless it is one through its whole extension

But if a judgment can be thus extended, what do we mean by *a* judgment, and how do we know when we enter upon a new one? The question is in each case a material one, being in fact the question of continued identity, and it is impossible to give it a formal answer As a first approximation we might say that a single judgment is any extent of judging activity that can be summed up in a single proposition But as the proposition takes its value from the judgment, and not *vice versa*, this is no more than an appeal to the fact that we succeed in distinguishing single judgments The question is one of continued identity, and therefore must be dealt with as concerning organised wholes or systems A mere extension of a system, or a mere omission within a system, does not bring us to a new and different system The clearest cases of transition from judgment to judgment are those in which language uses a mere conjunction When, on the other hand, we have propositions united by the inferential particle, it is a matter of degree how far they stand for separate judgments Ulti-

mately perhaps every inference may or should be represented as a single judgment, as being a mere extension of an existing whole of thought, and not a transition to a different one Such an idea conflicts with the traditional differentia of inference, that it *should* lead to a new judgment, but this only means a *bona fide* extension of the previous whole, such that, *if taken in abstraction from the process that generates it*, it *would* appear a perfectly new judgment At least in elementary cases it is easy to see how inevitably inference shrinks up into single judgments, if we look at the actual life of thought

Take such an every-day judgment of mixed perception and inference as 'He is coming downstairs and going into the street'. It is the merest chance whether I break up the process thus, into two judgments as united by a mere conjunction, or, knowing the man's habits, say, when I hear him half-way downstairs, 'He is going out.' In the latter case I summarise a more various set of observations and inferences in a single judgment, but the judgment is as truly single as each of the two which were before separated by a conjunction; for each of them was also a summary of a set of perceptions, which might, had I chosen, have been subdivided into distinct propositions expressing separate judgments, e g. 'He has opened his door, and is going towards the staircase, and is half-way down, and is in the passage,' &c If I simply say 'He is going out' I am not a whit the less conscious that I judge all these different relations, but I then include them all in the single systematic content 'going out'. 'Cromwell Road runs east, and the Brompton Road north-east,' are two judgments; but if the road happens to be thought of *qua* continuous, one would say, 'Cromwell Road turns from east to north-east, where it becomes the Brompton Road.' Again, 'Knightsbridge and Kensington Gore run east' may be generalised as 'The street from Kensington Church to Knightsbridge Barracks runs east'.

Thus a judgment is one in respect of the continued identity of its Subject and Predication, and this identity cannot be defeated by the inclusion of difference, but only by the failure to recognise continuity. It is obvious that the generalised forms in the above instances presuppose a work amounting to

colligation of facts, if not to elementary induction and analogy. If the included judgments were never separately made, the inferential work of colligation has not been explicitly done, but it would be found absolutely impossible to draw the line between cases where it has been done and those where it has not. Every judgment would on scrutiny reveal differences which had more or less been absorbed into its formation. It follows inevitably that every systematic inference considered as a judgment is single and not multiple. However this may be, it is clear that extension in time is no bar to the unity of judgment.

Scheme of arrangement of Judgments

II. Judgment, as we have seen, is primarily the intellectual act which extends a given perception by attaching the content of an idea to the fact presented in the perception. The whole of consciousness, in as far as it is the consciousness of a single world that shares the reality of our waking self, may be regarded as a continuous judgment, which qualifies our present feelings and surroundings by the knowledge of what is more remote in space and in time. From the point of view of common logic, that is of individual knowledge, the intellect sustains its world by continued effort, as Atlas held up the sky.[a] Every judgment is an effort of this kind, affirming on the one hand that the same reality which we touch in the present is rightly described by such and such an idea, and on the other hand that such and such an idea is real with the same reality as that which we touch in the present. The 'idea' of which I am speaking is, of course, not the particular existence or single occurrence of a psychical image; it is the general signification for the sake of which we use the psychical image.

The object of this Book is to analyse judgment into its principal kinds, and, as a necessary consequence, to trace their

[a] This seems to me *prima facie* a plain and obvious fact. Every one's world is relative to his capacity and energy of mind at a given moment. If you ask 'What, then, becomes of the real world?' the answer is that that is a metaphysical question, but by no means specially hard to answer from this point of view as compared with any other. The real world lies, I suppose, or rather lives, in the efforts of all spiritual beings to sustain a unity of experience, taken in their completeness as successful. Of course it does not live merely in judgment.

affiliation We shall find that no linear arrangement will represent these affinities Judgment, as the effort of thought to define reality, must vary with the kinds of reality to be defined no less than with the degree of its success in defining them. An equation is to one kind of whole what a definition by genus and species is to another, or an appreciation of aesthetic value to a third , the function of judgment is present in each of these activities, and the difference between them is the difference of the wholes which they respectively analyse They are divergent developments of the same relation, in each of which an aspect has become predominant that remains subordinate in the others

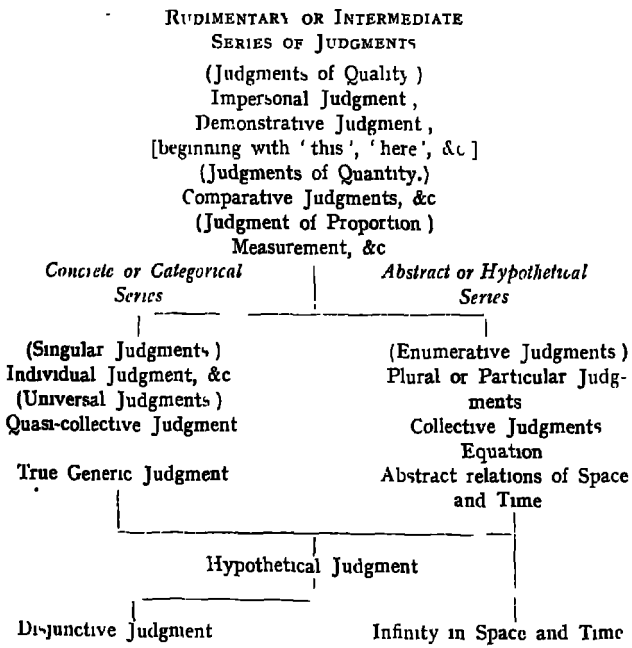
But we shall have to deal with convergence no less than with divergence The unity of the judgment excludes no complexity of synthesis, and in determining the species of a plant or the character of a man we may be obliged to employ, among others, accurate determinations of number, time, and space. That is to say, the treatment of a content by abstraction as a spatial or numerical whole may be re-absorbed in a more concrete treatment of it as an organic, aesthetic, or moral whole

It might indeed be urged, from the point of view of metaphysics, that every kind of judgment must have its value and no more as a contribution to the whole of Reality, and that therefore the series of judgments, arranged according to the degrees of their significance for knowledge, must after all be linear. In other words, if a whole in number or a whole in space is not final in itself, but demands something further to complete its significance, this might be enough to show that we ought not to represent it as the goal of an independent series, but rather as a stage or phase of construction, subordinate to the more concrete forms of knowledge [1] I should not greatly object to such a view and shall endeavour in some degree to meet its requirements by exhibiting the more complete and concrete syntheses as involving the reunion of aspects which have been developed in the abstract But though the forms of space and time are involved

[1] Cp Plato's arrangement, in the Republic, of the mathematical sciences in an order proceeding from abstract to concrete

as a fact in the perceptive construction of the world of individual things, yet the mathematical analysis of these forms is an effort of the same scientific spirit which recognises the principles of order in the world of things itself The two aspects of constructive science are co-ordinate and complementary activities of reason, and it would be ridiculous to treat geometrical analysis as prior to the perception of characteristic size or proportion Intelligence is many-sided, in spite of its unity; and its aspects, which are correlative to each other, lose their true interdependence by being drawn out into a linear series

SCHEME ILLUSTRATING AFFILIATION OF JUDGMENTS AS DESCRIBED IN BOOK I.



Purpose of scheme

1 I subjoin a scheme of the arrangement which I propose to follow in the remainder of the present Book It is simply intended to assist the reader in apprehending the views which I submit, and is not meant to be a bed of Procrustes for the

facts of logic I take it that variations of arrangement and nomenclature are as inevitable in logic as in botany, and are not undesirable in either science, for they force upon our minds the truth that species are but sections of evolution, and that their arrangement is merely subsidiary to a correct apprehension of the process which we divide into such intervals

ii I will begin with a few words in explanation of the scheme which I have adopted. Explanations of scheme

There is no need to apologise for describing some types of judgment by appellations which are not to be found in Mill, Whately, or Hamilton Recent attempts to restore to logic its hold on living concrete thought, a direction in which Mill was himself an able and adventurous pioneer, have made us familiar with a whole chaos of psychological, grammatical, and quasi-mathematical titles applied to phases of the judging activity. All that I have done has been to concentrate in a single review the best estimate that I could make of the typical character and true affiliation of such phases, attempting to give each judgment its appropriate place in relation to *all* the principles employed in the classification, and eschewing the too common habit of adducing various groups and appellations in two or three successive chapters, without any distinct reference of the one grouping to the other

But besides adding to the traditional scheme of judgments, the arrangement suggested involves a dislocation of its parts, and the omission of one familiar antithesis. These innovations, though by no means original,[1] may conveniently be indicated and justified in a few introductory remarks

a. 'Categorical' and 'Hypothetical' are taken to designate fundamental characters of knowledge, and not mere grammatical appearance It follows that the natural series of judgments commonly known as singular, particular, and universal, cannot wholly fall within the genus categorical, but at a certain point and in a certain degree assumes a hypothetical character More especially, the formal equivalence of Categorical and Hypothetical

[1] The treatment of the universal judgment as fundamentally different from the singular in categorical character was really initiated by Mill in so far as he analysed the content of affirmation into coexistence of attributes, and has been adopted and accentuated by Lotze, Sigwart, and Bradley

the singular to the universal judgment is replaced by a profound distinction of kind between the two The disjunctive judgment again has a place and value of its own, and is not a mere proposition or grammatical form

Divergent species

β. The true quantitative judgment—not the 'universal' judgment of ordinary thought, which derives only its name from quantity—reveals itself as a divergence from the central development of the judgment by reason of its extreme abstraction, in which one element of the relation essential to judgment almost disappears

Analytic and Synthetic

γ. The familiar terms 'analytic' and 'synthetic' are not made use of in the classification, because they belong to the theory in general and are not distinctive of any particular types

I subjoin a short explanation on each of the above subjects

Categorical and Hypothetical

α A categorical judgment asserts an actual fact absolutely. A hypothetical judgment asserts only the consequence that follows on a supposition The distinction between the two seems clear It is the difference between 'There is a bad smell in the house' and 'If there is an escape of gas there will be a bad smell'. But when we come to the 'Universal' Judgment the line of demarcation is at once blurred Hamilton gives 'Rainy weather is wet weather' as an instance of a categorical proposition, and 'If it rains, it will be wet' as an instance of a hypothetical In the former, according to him, 'rainy weather' is 'unconditionally thought to exist' But is it? *Prima facie* the two propositions represent *the same* judgment, that is to say, their difference is grammatical only, and their meanings are identical It may be that the categorical shape conveys a presupposition which is absent from the hypothetical enunciation—the presupposition that rainy weather exists *in rerum natura*—and into the question thus raised we shall have to enter at length later on But it is clear at all events that the 'categorical' form conveys in this case a meaning which is in a large measure, if not entirely, hypothetical

By referring to our account of judgment as such we shall see that the distinction before us involves a fundamental difficulty Every judgment affirms an idea of reality, and

therefore asserts the reality of an idea Now an idea is necessarily abstract, because determinate , and therefore all judgment involves abstraction And abstraction is the essential element of hypothesis ; it consists in taking up into an idea some elements out of the content of experience, for the sake of consequences which attach to the elements so taken up Therefore it seems, as the real world *for us* is maintained and extended by Judgment only, that in all extension and even maintenance of the given reality there is involved an element of abstraction, which is the same as to say that in all categorical judging there is an element of hypothesis The relation of these characters to each other throughout the history of the judgment will be the main principle of the evolution which I shall attempt to describe But we must accept as the usage of thought, which we are to explain, that the assertion of actual fact coincides as a rule with the individual or singular judgment, and that the universal affirmative of formal logic, on the other hand, *may* in every case be taken as purely hypothetical

The categorical character of a judgment in the above simple sense may be tested by the possibility of expressing its meaning by an impersonal proposition, however awkward may be the necessary grammatical transformation For 'Caesar was crossing the Rubicon' we may substitute 'There was Caesar crossing the Rubicon', but for 'All men are mortal' it is impossible to frame such an equivalent, for the reference to given reality in the impersonal expression would be at once contradicted by the abstract 'all', which = 'any' or 'if a–', and so points, in the absence of any more effective assertion of the limited unity of the race, to an infinite series If it is possible to say 'These are all the men who, &c , &c', the 'all' cannot be the true generalising 'all', but must indicate a sum of known individuals It will be necessary however to point out hereafter that the distinction between these two senses of 'all', or in other words the limit of individuality, is not absolute, but is a matter of degree

These instances suggest the principle to which we shall adhere, viz that every assertion is as absolutely categorical as the nature of its elements will permit , and that demonstrative or individual judgments are in the plainest sense categorical,

because the realities indicated by their subjects are of a nature that can be given, in a way in which the realities indicated by more definite abstractions cannot In every case the *real* subject is the reality indicated, in every case this subject is alleged to exist, but the question is how and in what way it is capable of existing, in other words, what is the kind and degree of its individuality For only what is individual can have actual existence as a whole An infinite series cannot have such existence,[a] and therefore cannot be taken to have it. The 'all' in this latter case remains a demand with which we cannot comply

Divergent species.

β. The content of a judgment is always a significant idea, that is to say, a recognised[1] identity in differences The varieties of judgment correspond to the forms which identity in difference is capable of assuming

An identity in relation to its differences may always be regarded as a whole in which they are parts An expanse of the same colour includes the changing lights and several points of space through which the one identical colour extends, the policy of a government includes the details into which its principles are developed, the moral character of a man is a whole in which his several acts of volition are the variously dependent parts In this wide sense, as a synthesis of differences, not as a sum of units, the relation of whole and parts is a fundamental relation of all judgment It is only when the differences or parts assume the maximum of homogeneity, and conceal, so far as is possible, the individuality of their relations to the whole, that the parts become *units*, and the whole a *total* or a *sum* The relation of unit to sum total, that is, of quantitative part to quantitative whole, is thus obtained by abstraction. It is not the complete natural relation of concrete identity and differences, but is a device of knowledge which by sinking all other aspects of a given content is enabled to regard it as a sum of units, that is to say, as a whole of magnitude.

[a] See p 163 below

[1] 'Recognised' is necessary to restrict the description to *significant* ideas For a sequence of images in elementary reproduction, such as we ascribe to the animal mind, is an identity in difference, though we do not suppose it to be an objective reference, i e a *recognised* identity See Book II, chap 1, sect 3, The Reproduction of Ideas

For this reason it seems right to consider the judgments of number, with the kindred judgments of magnitude in space and duration in time, as belonging to an outgrowth of thought which diverges from the complete evolution of judgment All these judgments begin as qualitative, but become quantitative by intentional abstraction, and end in the creation of ideal totalities (abstract number, abstract space, abstract time[1]) which we are unable to think of as complete, and therefore are debarred from treating as actual totalities This, I may point out, is a case of the connection between individuality and actual existence, which I spoke of under the last head

Analytic and Synthetic

γ Every judgment is both analytic and synthetic. This would not by itself be a sufficient ground for refusing to employ these terms as heads of classification, for it is more or less the case through the whole of Logic that terms must be employed to mark predominant aspects rather than exclusive characters Nor do I find a sufficient ground of objection in the psychological comment that the judgment which adds a fresh predicate to a subject to-day must become tautologous or analytic if repeated to-morrow, and that therefore it merely depends on individual knowledge and memory whether a given judgment is synthetic or analytic Any conception of dominant quality, function, or essence, is enough to make this comment futile, and without such a conception it would seem that science is impossible It is a superstition to suppose that the progress of theoretical explanation in terms of general law threatens the doctrine of essence, form, or function However clearly an individual thing may be explained as a section of evolution or a meeting-point of forces, there will always be a definite continued identity conferred by characteristic form or function. No explanation can destroy the actual relations of whole and parts which form the essence of everything that is real. Knowledge has quite enough fixity to give meaning to the contrast of analytic and synthetic judgments wholly apart from the progress of individual minds

The reason why I no longer care to lay emphasis on the

[1] Cp Locke's Essay, II xiv 21, on 'Duration'

antithesis in question is not that it is purely 'subjective'—for this is not the case—nor even that it is only a distinction of degree—for that is the character of most distinctions in Logic, but simply that it is not a sufficiently specific antithesis to be of practical value in classification I suppose that if the terms were to be employed, we should call those judgments analytic which attain to an adequate explanation or appreciation of a complex whole The best instances might be the definition or the disjunction, the equation, and judgments passed upon moral and aesthetic value In all these cases we have the whole completely given in its parts, the identity in its differences, and therefore we are entitled to consider not so much the nature of the whole reconstructed, as the exhaustiveness of the reconstruction But, as the above instances show, adequacy or exhaustiveness exhibits itself in contents whose nature is wide apart, and therefore it has no convenient place as a general character in a classification

On the other hand, as terms belonging to the general theory of judgment, analytic and synthetic are of profound significance I said at the beginning of this section that every judgment is both analytic and synthetic This assertion demands no explanation, if we remember our account of judgment as always involving identity in difference But I will attempt to illustrate its meaning more fully

If I say 'Caesar crossed the Rubicon', I start with an individual Caesar, whose continued identity extended through a certain space of time and revealed itself in a variety of acts, and I exhibit his identity in one of the acts and moments—its differences—through which it persisted What I mean by the affirmation is that he, *the* Caesar[1] who had before conquered Gaul, and who was afterwards murdered on the Ides of March, displayed his character and enacted part of his history by crossing the Rubicon This is a clear case of exhibiting an identity in difference But the process has inevitably two aspects On the one hand, I analyse the individual whole that is called Caesar by specifying one of the differences that may be considered as a part within it,

[1] Contrast Lotze, Logik, sect 58

on the other hand, I construct or make synthesis of the individual whole in question, by exhibiting it as a whole that pervades, and absorbs in itself, each or all of its differences It is only an *individual* whole that is obviously present in *each* as well as in *all* of its differences, as the individual Caesar in the act of crossing the Rubicon A totality whose unity is incomplete, such as 'all men', is only *implied* in *each* of its differences, and is not *given as* a whole in anything short of *all* But this does not alter the fundamental nature of assertion Every judgment exhibits a whole in its parts, and parts as contributory to a whole

Much has lately been said of Kant's celebrated instance, the equation $7 + 5 = 12$ We have here a total, twelve, which can be compounded in an immense variety of ways, and we display this total as identical, whether expressed by its place in the series of numbers (which implies one and the simplest mode of its formation) or treated as the sum of two other totals, each of which is expressed in the same simple way It is obvious that if analytic and synthetic were reciprocally exclusive characters, the question 'Is this equation an analytic or a synthetic judgment?' would be wholly unanswerable If 12 were not the same number as $7+5$, the judgment would not be true, if $7+5$ gained nothing by being defined as 12, the judgment would cease to be a judgment at all $7 + 5$ is one of the differences which constitute the nature of the total 12, and by *constructing* 12 in this way we ipso facto *analyse* it

The relation of these two processes, or rather two aspects of the same process, is so fundamental in all knowledge, being in fact *the* relation which especially characterises knowledge as such, that I may be pardoned for continuing to insist on it by help of another set of considerations The notion of a plain difference between taking to pieces and putting together arises from actual operation on material things. This origin of the metaphor involved in 'analysis' and 'synthesis' has reacted and still reacts injuriously on our conceptions of intellectual processes In mechanical operations we cannot pull to pieces and put together the same thing by the same act, and which of the two we can do is

determined by the material handed to us If a thing is complete already, we cannot put it together any further, the only alternative then open to us, as between these two processes, is to pull it to pieces; and so *vice versa* But this feature of material operation cannot be transferred to thought, and for this reason, that the essence of thought is to show the process in the result, and exhibit each as necessary to the other. Therefore, if we construct in thought, the materials out of which we construct have not lost their separateness when the fabric is finished, the fabric as it is still issues from them as they were, if not, we have dropped a link, and our construction is unwarranted The synthesis, one might say, is based on the analysis, but this would ascribe a false priority, because the fragments supplied to us only *become* an analysis as the synthesis, which relates them to a whole, progresses Apart from the synthesis they are mere fragments, and therefore are not an analysis of anything The workman who puts together the parts of a watch has first the handful of wheels and springs, and then the completed watch, he cannot have both at once, and in as far as he has one he has not the other. Moreover, when he has made the watch the wheels and springs are together and are not separate, nor are they separable consistently with the existence of the watch Synthesis in this sphere is incompatible with analysis, and *vice versa* But a man who wishes in thought or calculation to construct any instrument out of parts has a very different task. Every element of the handful of parts must have its place and functions clearly retained in the intelligence which constructs the whole, for the whole, as a whole of intellectual synthesis, exists no longer than its parts are clearly apprehended in their relations. 'Yes,' it may be said, 'but the distinction must remain that even in thought you may *either* begin by considering detached wheels, &c, and finding out how they must act in the watch, *or* by looking at a watch and detecting, within its completed system, the separate parts and their relations. The former process is synthesis, the latter analysis'

This is true so far as judgment or inference is an activity in time, and includes within itself a transition in time In

so far as it has this character, the process of thought can simulate or share the characteristics of material operation But this does not affect the internal nature of judgment, as I have pointed out in discussing its temporal character. The question is not whether you begin with the whole or with the parts, but merely what sort of whole and what sort of parts you begin with Given an escapement wheel, I may chance even to be ignorant that it belongs to a watch at all, but none the less I judge of it as a part in a whole, which whole I can at first only think of, perhaps, as 'some piece of mechanism that depends on a catch playing into a delicately toothed wheel'. The further intellectual construction of this mechanism and the ultimate definition of it as a watch, is, according to the views of the passage just referred to, not a transition from S to P, but a transition from the judgment s–p to the judgment S–P We therefore find analysis no less than synthesis to be the internal essence of every minutest section of the judgment or inference in question. In the same way, if a watch is put into my hand with instructions to find out what makes it go as it does, I have primarily a thing in space as the given whole, and indefinite wheels, springs, &c (which as yet I cannot distinguish by position or characteristic shape) as given parts No doubt in space all the parts which I shall need to learn are given in position within the whole, and so we tend to describe the problem as one of analysis, in contrast to the other (in which I had to find out or imagine the position of the parts in the whole) as synthesis; and these titles serve well enough as superficial descriptions of certain cases to which judgment and inference are applied—not of *any* judgment or inference as such. But the whole is not, in the latter case any more than in the former, given as an intelligible machine, nor are the parts given within the whole of knowledge because they are within the whole of space. In other words, to see the escapement wheel lying inside the watch does *not* 'give' me this wheel as a part of a mechanical arrangement; to know it as a part of *such* a whole I must understand it; and in understanding it, i e in my analysis, perform the synthesis of the watch as a definite mechanical contrivance.

Therefore not only is every judgment both analytic and synthetic, but it is analytic only as far as it is synthetic. It can only be called analytic or synthetic *par excellence* if, by the same confusion that causes the judgment to be regarded as a transition from S to P, we consider the joint analysis and synthesis of one whole as the analysis or synthesis of another, because in that case we seem to have a fixed and given whole, and to predicate of it nothing but parts, or *vice versa*. In this confusion there is an element of truth. Though s must become S when p becomes P, yet s has continued identity with S and p with P, and therefore the transition in time from s–p to S–P does all that could be done by the unreal transition from s to P. Present me with a pattern s which is a tissue of intersecting curves p, and when I have analysed it into the thistle design P, the pattern s is transformed to my eye into a distinct and beautiful design S, but S is the same that was s, and in that sense we have connected s with P, and we may represent P as the analysis of s, only not forgetting that it is the synthesis of S which is the same as s, and that therefore in predicating P of s we *ipso facto* transform s into S. And thus the complete understanding of a watch as a mechanical system, expressed in the joint analysis and synthesis S–P, may be accepted as happening to involve, *par excellence*, *either* the intellectual analysis of the watch as a given whole in space s, *or* the intellectual synthesis of the watch out of given separate parts in space s_1, s_2, s_3, &c. The given whole, or given parts, can be thus allowed to pass as merely whole, or merely parts, because they are not respectively whole and parts in the sense contemplated by the judgment S–P, and therefore it does not press home their relation to one another. The watch seems to be from the first a ready-made whole, a round thing s in space, which can only be analysed, and not constructed, by the judgment S–P. But it *is* further constructed, not as a round thing in space, but as a mechanical system, by means of that judgment.

CHAPTER II

Quality and Comparison

I now turn to examine specific types of judgment, but in doing so, I must beg leave to remind the reader of the principle which I have laid down as governing any enquiry into a continuous development.[1] What we wish to master is the nature of a process, the scientific history of a function. To do this, we must of course study and arrange its detailed manifestations, mere generalities are valueless. But we need not be disheartened if our subdivisions and specific names are different here and there from those adopted by better authorities, nor even by the possibility (which can hardly be absent from a highly detailed treatment) that we may misinterpret some phase of evolution, or lay down some inconvenient demarcation. If the main problem is thoroughly faced, and the analysis of some chief typical forms accurately conducted, the reader will be in a position to correct blunders and to supply omissions by the light of the knowledge so gained. If we can help him to sound principles and practice of morphology, he will make short work of particular varieties of type

Judgment, we have seen, is, speaking generally, the intellectual function which defines reality by significant ideas, and in so doing affirms the reality of those ideas I use the term 'define', because to define implies something given which is to be defined, and it is an essential of the act of Judgment that it always refers to a Reality which goes beyond and is independent of [a] the act itself

1. We will now look at the judgment in its simplest form, which I have ventured to call the Judgment of Quality, and which, with its immediate sequel, the Judgment of Comparison, finds linguistic expression in the Interjectional or Impersonal Proposition

The Judgment of Quality.

[1] See Introduction, p 15

[a] On a criticism which this expression has attracted, see vol II, ch IX

Meaning of Quality

i. By Quality I understand, not all attributes without distinction, but the unanalysed content of any idea, when treated, in its unanalysed simplicity, as a feature of reality. It may be that all qualities are capable of being analysed into relations, but for our present purpose the question is not whether a quality *can be*, but whether it *is* so analysed. Even the diagrams familiar to us in Euclid, which exist for the very purpose of being analysed, have each its peculiar look or effect, lopsided or symmetrical, solid or slender, circular or bristling with angles Qualities of shape, however, are as a rule quickly analysed into relations of proportion, so that the commonly cited phenomena of colour and sound give better instances for our purpose, while perceptions which are named, like softness and sweetness, with some reference to pleasure and pain, are the best instances of all. It might indeed be suggested as a definition of quality that it is that aspect of any perception or idea in which it gives rise to pleasure or pain

Judgment of Quality proper

ii. The Qualitative Judgment proper affirms a nearly simple content directly of present Reality An absolutely simple content is indeed an impossibility, every 'red' or 'sweet' or 'pleasant' belongs to some context and includes some differences But, as I have attempted to explain in the last paragraph, a qualitative content is very nearly simple The context which makes its difference is the context for the sake of which we affirm it and is thus presupposed, and not itself affirmed as a further complication. If I exclaim 'How hot!' I do so because the weather or the room is hot, or perhaps if I am feverish, because I am hot for no obvious reason In all these, no doubt, the content 'hot' belongs to something; it is not isolated from the varied surroundings of my position, but exists in and extends over some of them But I affirm it without specification, or rather as a first effort to make specification, of my position and surroundings in general, and neglect to analyse its relations, or only analyse by implying that it has some relation or other

Thus much for the Predication As for the subject of a pure Qualitative Judgment, there is *prima facie* no assignable subject within the judgment. No ideas are employed

to limit the aspect of reality to which the predication refers The whole of what is perceived at the moment, or more probably some unspecified aspect of it or element within it, is the subject, and it is of this that the content is affirmed By ' unspecified ', I do not merely mean unspecified in words spoken aloud, but undefined by any such act of consciousness as employs symbolic ideas and tends to call up words. But if thus unspecified, how can the subject be indicated, limited, or selected ? I answer, simply by the concentration upon it of perception or attention, the exclusive gaze which might be represented by pointing with the finger, and which, though it *has* limits, is definitely aware of none. In this sense, confining what is ' present to perception ' within the limits of what more especially arrests attention and is taken as ' This ' *par excellence*, we might say that the subject of the Qualitative Judgment is always the whole of what is present to perception

The best illustrations of this rudimentary Judgment are drawn from significant Interjections, or from significant phrases used as Interjections A distinction must be observed between the true Interjection,[1] the outcry which relieves the urgency of feeling, and the affirmation which refers the content of feeling as a quality to the surroundings that evoke it The cry of an animal is often significant for us ; it tells us what the animal feels, and why , but we do not therefore reckon it as the expression of a judgment What makes the judgment is the idea that exists in our minds and yet that only has truth as referred to Reality And therefore if we doubt whether we have a judgment before us, we should ask if it is anything that could intelligibly be denied. It is impossible to *deny* the animal's mere expression of its feeling ; and although we may see a reason for its feeling which we know to be an illusion (e g if a dog barks joyfully in expectation of being taken out, when he is not going to be), yet there is nothing which we can deny unless we can suppose that this illusion exists in the animal's mind as an idea distinguished from and referred to reality. *If* we could believe this to be so, we should have to admit that the animal

[1] Cp Lotze, Logik, sect 48

judges But short of this, there is nothing to deny. The dog sees me take my hat and stick, he has a set of mental images connected with going out, and he expresses pleasure. I cannot deny that he has the images or that they give him pleasure It would be only if he could take these images as standing for something other than themselves, and so distinguish their meaning, a future event in reality, from their existence, as present images in the mind, that anything could possibly arise which I could intelligibly deny. The ideas used in judgment must exist before they can be denied, and therefore their existence cannot be denied, but only the affirmation of their meaning [1]

Coming to human interjections, we might think that 'Alas!' characterises the present as grievous; and it is certainly sometimes *answered*, though hardly denied. But the answer, even if it takes the form of a denial, is usually rather prohibitive (imperative) than negative; though if self-deception or hypocrisy are suspected, 'alas!' may be interpreted as 'I am sorry', and in that sense denied The distinction to be kept in mind is here one degree more subtle than in the last case, for the objective content of the judgment is the fact of a feeling in the mind of the person who judges Thus we have, if 'alas!' is a true interjection, the emotion of grief present in the mind of the person, which merely forces him to utterance by way of relief. But if it is to be taken as a judgment, then we have in any case the symbolic idea of sorrow, *existing* in psychical images, but having its meaning beyond them, and a reference of this meaning to the present perceived being of the person in question And above all this, if the judgment is true, there must be, as before, the actual felt emotion of sorrow, though if it is false, this element of the complication is absent It is possible that this peculiar complication, of the idea with the actual feeling behind it,

[1] These remarks are made purely for the sake of illustration I have not the least prejudice against admitting that animals can judge, if the admission can be warranted by fact, and does not involve truncating the theory of judgment Domestic animals certainly *seem* to use the imperative, i e to insist on the realisation of their ideas Cp Bradley, Principles of Logic, p 33 On an idea without forethought or image, see Bradley, Appearance and Reality, p 607

is responsible for the curious duplication of personality which is sometimes experienced in protracted pain or anxiety. The person whom we analyse and judge seems other than the person who all through the process has the feelings which are being analysed, or rather are forcing their disagreeable peculiarities on our attention. 'If *I* were suffering so, how horrible it would be,' we repeat to ourselves Just as the complementary image comes between our eyes and the sun, so the idea of our feeling comes as the object of knowledge between us and the feeling itself, which remains in the background and resists our successive efforts to include it in an adequate idea The sufferer remains to us distinct from the person whose suffering we conceive and affirm

However this may be, when we come to such quasi-interjectional expressions as 'Bad!' 'How ugly!' 'Such pain!' 'Oh horrible!' we are unquestionably dealing with judgments

The Impersonal Proposition is also a suggestive counterpart of the judgments in question, which have even been treated as Impersonal Judgments But the impersonal form of sentence has become in developed language so purely a grammatical fiction, that it no longer illustrates with special appropriateness any one type of assertion, although it exhibits a certain coincidence with the range of the concrete existential or singular categorical judgment

A few instances however occur to us at once—and philology might be able to furnish more—of impersonal sentences that really seem to have stood for judgments whose subjects were not especially designated by means of ideas, but were accepted as merely the given in perception. Such may be 'Methought', 'Him list', 'Mir traumt's',[1] 'Es trieb mich', 'Taedet', 'piget', &c, 'It rains', 'es macht heiss', *νέι*. As,

[1] In Greek, a conservative though flexible language, 'I dreamt,' is never, so far as I know, impersonally rendered but on the other hand, in Homer the dream is personified The principle is the same in so far as my dream is not referred to me as my act Indeed the coincidence is curious with the view of those who have held that the subject of an impersonal proposition is the content of the verb itself as in 'the rain it raineth every day' It is much the same to say 'Es traumte mir' and to say 'A dream came to me'.

in the plastic Greek imagination, Athene may suggest a man's thought, and the actual Dream stand over him in sleep, so, it would seem, in the beaten track of language, the thought and the dream are simply referred to 'it' or to 'something', to the present reality or to an indefinite element within it. Though the formed verb presupposes a distinct reference of ideal contents to real subjects, yet the habitual use of expressions in which this reference is blunted and neutralised, testifies to a survival, in certain preeminently obscure relations, of a rudimentary type of judgment This, I think, is the only connection that we can safely assume between the Judgment of Quality and the Impersonal Proposition

The Qualitative Judgment is the germ and simplest case of the Perceptive Judgment Perception is a wide word, including, as frequently used, any so-called immediate apprehension; even that, for instance, by which we are supposed to see the necessary truth of one of Euclid's axioms. But if we are to give the term a distinctive logical meaning, we should do well to restrict it to so-called immediate apprehension when dealing with the portion of reality which is in contact with the individual through the senses Perception thus defined deals primarily with what is present, but extends it by ideas which go beyond the present When we recognise a man and call him by name, we are said to 'see who he is', i. e. to perceive. In this case our perception is expressed by an idea that goes a long way beyond the present and brings in the man's entire personality.

The Qualitative Judgment does *not* differ from the judgment which recognises an individual, by being shut up within a minute interval of present space or time It is not minuteness of extension or of duration that distinguishes the reference of this simplest case of the perceptive judgment; the qualitative affirmation may deal with what *is* really a considerable area of space or interval of time. The distinction is not one of magnitude, but of definiteness. There is always a risk of construing the absence of quantitative determinateness into determinate minuteness of quantity This is just what we want to avoid The qualitative judgment knows nothing at all of duration or of extension, and can have no

specified individual for its subject. It is thus confined within the given presentation in so far as the universality, whether abstract or concrete, is absent that alone could extend it beyond. The Reality which is the subject is the given as given, not as a universal that reaches before and after, the content of the predication includes no negative element, summarises in itself no diverse manifestations, and thus neither refers to anything beyond the present, nor in any specific way to the present itself It is attached to the present, by the mere fact of its actual reference to presentation, not by anything within its explicit content The first specification, the first establishment of an identity that can be called by a name, is the work of this judgment, and is not presupposed by it. We must take it, I think, after the discussions of the Introduction, that the establishment of a name—a permanent identical symbol of a meaning—must *on the whole* have coincided with the establishment of meanings as such, distinct from psychical occurrences, and capable of being referred to reality. I have attempted in the discussions in question to qualify the rashness of this unverifiable[1] historical allegation, by pointing out that the two co-ordinate processes—the constitution of symbolic ideas and of linguistic symbols—must be regarded as processes of gradual and unconscious adaptation, widely differing from the methodical extension of nomenclature according to modern ideas But the logical track is the same, whether the historical evolution is quick or slow, conscious or unconscious.

Perception as above defined would include two species which have been called respectively the 'Analytic' and the 'Synthetic' judgments of sense If these distinctions are to be seriously treated, and we are to speak of any judgments as merely analysing a presentation of sense, without going beyond what is given within it, then, I think, we must identify the Analytic Judgment of sense with the Judgment

[1] Of course there can be verification by analogy, and it is hard to draw the line between this and actual verification The characteristic cries of animals and children have significance for us, and we cannot suppose that they do not react upon the intelligence of those who utter them It would be far easier to understand how animals should acquire language, if, like children, they did so in fact, than it is to understand how they stop short of it

of Quality It appears to me quite idle to treat a description of an ordinary scene, such as 'The blacksmith is at his forge mending the ploughshare', as a case of a judgment confined within *present* perception Every element of the description is a concrete individual, including innumerable differences, involving elaborate categories, and extending indefinitely into past and future. If ever there was a constructive or synthetic judgment, this is one It would surely be more appropriate to treat these common perceptions, which we deal with lower down, as Synthetic judgments of sense, because they interpret what is given by ideal contents that go beyond it. And then we might reserve the title 'Analytic Judgments of sense' or 'Judgments of Quality' for the activities represented by the Interjectional and Impersonal propositions of which I have spoken, and the true Demonstrative propositions of which I shall speak below

The difficulty of identifying simple forms of judgment is intensified for modern reflection by the definite and diverse articulation of the elements of modern speech The languages which embody the reflective thought of Europe, both ancient and modern, resist the expression of elementary perception in two ways. First, for a germinal thought we need a germinal word But the languages which mould our ideas have no germinal words Every word, in the languages of European culture, is a particular 'part of speech'. That is to say, it is adapted to fulfil some one function in a sentence, whether substantive, verb, pronoun, or conjunction, and, if used alone, has an air of incompleteness which forces us to 'understand' supplementary words The existence of the Judgment of Quality is but slightly corroborated by the fact that a single word often conveys a judgment. Many such single words are conventional symbols for quite definite sentences But the judgment of which we have been speaking corresponds to a whole sentence in the bud, with its differences unevolved How far philology could furnish authentic representations of the sentence in such a stage—words which are not '*parts* of speech', but entire though undifferentiated *units* of speech—it is beyond the limits of the present work to enquire Even the verb of ancient Greek or Latin, which

required no supplementary pronoun to represent its subject, is one degree more appropriate for the purpose than any element of modern speech.

And the second difficulty is really a case of the first. Most judgments are expressed by help of a verb, and if we employ a substantive or adjective alone, it urges us to 'understand' a verb But a verb is in our languages above all things a *tense*, and for a rudimentary judgment like the judgment of Quality, a tense is exactly what we do not want, least of all the elaborate duration-tense of the present (τρέχει, amat, 'he is acting'). We want to affirm neither duration nor yet point of time; we simply want to qualify the given by a content, without specific limitation or extension. It is true that the logical present, the absolute present of the universal judgment, marks no limitation of time, and it depends on the nature of the content involved whether universality of time as an infinite series is asserted in such a case But whether it claims universality in time or by negation of time, such a judgment implies the conception of time as an abstract whole and is posterior to this conception, while the reference of which we are speaking is prior to the origin of the systematic idea of time

These properties of developed language,—and prior to developed language it would scarcely be possible to have analytical reflection,—may be compensated, but cannot be cancelled. The fact that sometimes thought is behind language, and at other times struggles to pass it (it would be hazardous to complete the antithesis by saying that thought can really outstrip speech) is a fruitful source of misinterpretation. Children learning to speak, or savages learning a European tongue, are like the wizard's apprentice uttering a spell; they are incapable of grasping the significance or controlling the effect of their words And all human beings perpetually oscillate between limits, different in every case, on the scale of intelligence, for not only is the student's or politician's world of thought very different from that of an illiterate man, but every man varies in the level of his intelligence according to momentary conditions of interest and capacity. The same difficulties of interpretation which are found by a student in

the speech of a child or Anglicised savage, subsist in a less degree as between every man and every other, and as between every man and himself.

Up to this point the constituent elements of the Judgment have been naked before us. There was the Subject,—the actual contact in which reality pressed upon our sense-perception,—and there was the significant idea by which we defined it It is this case that affords the strongest support to the view which denies all meaning to the distinction of Subject and Predicate, as a distinction of elements within a judgment. The Reality to which we ascribe the predicate is undoubtedly self-existent, it is not *merely* in my mind or in my act of judgment; if it were, the judgment would only be a game with my ideas It is well to make this clear in the case before us, for in the later forms of the judgment it will be much disguised Still the reality which attracts my concentrated attention is also within my act of judgment; it is not even the whole reality present to my perception; still less of course the whole self-existent Reality which I dimly presuppose The immediate subject of the judgment is a mere aspect, too indefinite to be described by explicit ideas except in as far as the qualitative predication imposes a first specification upon it *This* Reality *is* in my judgment, it is the point at which the actual world impinges upon my consciousness as real, and it is only by judging with reference to this point that I can refer the ideal content before my mind to the whole of reality which I at once believe to exist, and am attempting to construct The Subject is both in and out of the Judgment, as Reality is both in and out of my consciousness.

The demonstrative Judgment

iii. We have now to consider a slightly more definite type of judgment, which we may still rank among the judgments of quality, although we can detect in it the beginning of a further growth

When we say, '*This* is hot,' '*Now* it is raining,' '*Here* it is dark,' the demonstrative pronoun or particle designates the point in given Reality to which the affirmed content is to refer The point is designated, but seems, *prima facie*, not to be described. The demonstrative has a meaning,

no doubt, but its meaning seems to consist of a mere reference to what is presented before perception, and therefore does not seem to introduce any abstract limitation that qualifies the given subject. Compare, for instance, 'This is hot' with 'This metal is hot'. The latter judgment may possibly be met with 'This is not metal at all', and by such a reply the judgment is cut in two, and the more significant half becomes a conditional assertion whose condition does not apply to the 'this' in question. But 'This' alone is on a different footing. You cannot say, 'There is no *this* at all' There is always a *this*, as there is always a *that*; and the same applies to *here*, *there*, *now*, and *then*.

One of these demonstratives indeed appears at once to take us over a boundary. 'Then' requires a past or future tense in the predication; and in referring to the past or future we get beyond analysis of the present, which is the province of the analytic judgment of sense The problem is one of real importance, but its point is not where we are most likely to look for it It is not that the Judgment of quality refers to a point of time, and that 'then' takes us outside this point, it is that the Judgment of quality is prior to the idea of duration, and that we have now introduced the idea of duration definitely into the subject The effect on tense, which happens to be the vehicle of predication, produced by taking 'then' as the subject, calls our attention to the fact that every present includes a past. The contrast of 'here' and 'there', not happening to affect the verb, did not force on our notice the equally real universality of the present in space Every 'here' is made up of 'there's' as every 'now' is made up of 'then's'. And thus in designating a given subject as 'now' or 'here', we have unawares included in the subject a 'then' and there', and by introducing universals of space and time have set our faces to leave the region of the qualitative judgment The demonstratives stand for ideas, and it is therefore *through ideas* that in the judgments now in question we refer to reality But the demonstratives have the peculiarity that *their application* cannot be denied, as can that of a determinate idea such as metal, and therefore, though they characterise the given

Reality as appearing in space or time, yet they can specify no nexus, introduce no condition, which may be absent in fact, and through its absence may save the judgment from falsity by rendering it inapplicable If heat is not present in the 'this' to which it is ascribed, the judgment 'this is hot' is false without reserve, but if to 'this' we add 'metal' then the absence in fact of the condition 'metal' makes the whole judgment ambiguous and inapplicable. Thus we have in the demonstrative judgment of quality, as in the simple or pure judgment of quality, a perfectly categorical judgment; the Subject must, in its nature, exist, and the Predication must therefore be alleged to hold good of actual existence. It is noteworthy that of this perfectly categorical judgment we cannot say whether the existence of the Subject is affirmed or presupposed. Where we are dealing with the given *qua* given, the difference between affirmation and presupposition has not emerged

The Judgment of Comparison

2 The contrast between 'now' and 'then' suggests to us the consideration of such judgments as are expressed by 'Now it hurts less than then', 'This is redder than that', 'Here it is hotter than there'. The form of these judgments indicates as their appropriate title the Comparative judgment, or the Judgment of Comparison. They arise naturally out of the Demonstrative judgment of quality, because, as we saw in the case of 'now' and 'then', it is impossible to prevent the present subject from revealing differences within itself. 'This,' as more clearly defined, will display itself as a part, 'not-that' within 'this and that', 'now' as a part 'not-then' within 'now and then,' 'here' as a part 'not-there' within 'here and there'. The whole, when thus resolved, displays differences of quality between its parts, or rather the given reality reveals itself as a whole for the first time when it breaks up into parts united by an identical but varying quality Even if we forget that the 'this' and 'that' ever entered into a single whole, yet the identical quality because of which we compare them contains in itself the essential of the comparative judgment, viz. the *explicit recognition* of difference in identity. 'Redder,' 'hotter,' 'less painful,' are terms that go beyond mere

quality by introducing the conception of more and less, that is to say, the beginning of quantity. I give instances, bracketing the explanatory words which would be superfluous in presence of direct perception, and which belong to a higher level of judgment than that which we are discussing. 'This [paper] is green, and this [part of it] is lighter green than that' 'Now [all to-day] it[1] hurts less than it did [yesterday], but [just] now it hurts more than it did [a moment ago]' But in cases like these we are apt not to notice that we are predicating differences within a single identity, the green paper, or the whole of to-day, though we must be aware that we imply them in the comparison of quality. And therefore, having pointed out the underlying character of such a simple analysis as the above, I will pass at once to a case which is one degree more complex, but which displays the essence of comparison beyond possibility of mistake. I refer to the case in which a given whole of perception designated by one demonstrative has parts distinguished within it by means of the others, and differences assigned to it conditionally upon these distinctions There is no difference of principle between defining 'this', within 'this and that', and defining it within 'here and there'. The only advantage is that a demonstrative of another kind is more readily taken as a condition, while one of the same kind is apt to be understood as a jump to a wholly new subject. We will therefore merely change the *parts*, which might be new and substantive wholes, into *conditions* '*This* is redder now than it was then' 'This is hotter in this part than in that.' 'This [green paper] is lighter here [in this part of it] than there [in that]' These instances clearly show the primary datum revealing itself as a whole with parts distinct yet bound together by a common quality

Quantitative Comparison.

1. Let us now examine the essential nature of the comparative judgment in one of the above instances It is not my intention to enter upon the niceties of quantitative comparison at the present stage

[1] The 'it' in this instance is on the verge of introducing an identical subject I did not, however, mean it to stand as a definite subject, but merely as the grammatical complement of the impersonal verbs

We will take the instance, 'This is redder now than it was then,' or in the simpler form, 'This is redder than it was' We will take the Predication first, and then consider its reaction upon the subject.

That which is redder is also red The red and redder are both red, and yet differ from each other, not, or not merely, in other ways, as in time or place, but in respect of their redness It has been sufficiently insisted on that there cannot be difference without some identity, as for instance a red and green surface are identical in respect of reflecting light But these, though the same in as far as they reflect light, are not the same in the light which they reflect There is a break between the two colours, considered as colours,[1] which nothing can bridge, and the immediate perception of their discontinuity supplies the terms which indicate the difference between them The surfaces which are both red, but one redder than the other, are separated by no such break If one changes into the other, it does not cease to display the same quality that it displayed before But a quality that changes, and yet remains the same quality, has passed into quantity, which might be defined as difference, not merely *in* identity as its meeting point, but consisting *of* identity as its material

Thus the fundamental identity and difference of judgment are specified by the comparative judgment as whole and parts in the simplest form of that relation viz the form in which the whole differs from any part by an interval which consists of other homogeneous parts Parts in this sense differ from units only by lack of precise comparison, but *precise* comparison is posterior to the conception of a whole, of which we are just considering the first establishment. We do not measure or count until we know of some totality that requires definition by these processes

The reaction of comparison upon a simple subject indicated by a demonstrative, that is, on a mere spot or point

[1] It is possible that, considered as amounts of light, red or green may share a continuous element and so have quantitative relation If so, this is another case in point For the difference is then in respect of the characteristic which forms the identity

upon which perception is concentrated, is an essential step towards the recognition of an individual totality The present of space or time is as we saw in its nature continuous. Therefore the spot or point on which perception is fixed, and which we indicate perhaps by 'this', will undoubtedly exhibit differences under analysis Such analysis is brought to bear by the judgment of Comparison The spot or point[1] in which a change of degree is observed forces itself on by that fact from being a mere spot on which the eye is fixed to the first stage of individuality as a synthesis of differences Change is not necessary to this result, though negation in some form is The observation of parts differing in degree within the spot which we have in view is as effective for the purpose as the detection of succession within the time which we call 'now' The mere spot fixed by perception begins under such analysis to assume the character of a Thing, and by a parallel process, the distinctions of Time and Space begin to emerge as parts within homogeneous systems

It is obvious that such a point in the evolution of thought would correspond to the first distinction of Noun, Finite Verb with Tense system, and the more elaborate spatial and temporal adverbs and prepositions But it would not be fair to test the correspondence strictly by negative instances, Language fits thought as a very loose glove, and if it were the case that we could find several languages in which our familiar parts of speech, more especially the tense system, do not exist, we should still no doubt find that the distinctions, whose origin we are examining, are represented in some other way than by linguistic signs, or are thought even if not represented It would be ridiculous to contend that the Chinese do not think of self-identical and independent things,

[1] It is hard to escape the dangerous pitfall of speaking as though there were no perception but sight, and therefore as if the germ of the judgment were always fixation of the look in space The focus of attention may operate through any sense, and is characterised at any moment by that identity which the judgment makes explicit But the identity is referred in rudimentary judgment not to a special content as subject but to what could only be paraphrased as 'That which engrosses my attention', the present feeling which the judgment determines into thought

even if it is true that their language has no special class of nouns substantive Nevertheless, the contrast of development between different languages has or has had its meaning; and it appears to me absolutely impossible that a people whose sole language was Hebrew could have had the accurate consciousness of time as a system which came easily to the Greeks and Romans of the classical age I believe, indeed, that the origin of the Aryan tense-system is not beyond the ken of philology, and that its probable history reveals an evolution much like that which has been here suggested—a transition from simple unspecified reference, to reference differentiated by a temporal system

'This,' then, as the subject of a continuous Quality including differences tends to acquire an individual name I have pointed out in the Introduction that the process of Naming in a world distinctly organised by knowledge cannot be that which belongs to the unreflective epochs of thought A natural name must be a petrified description The linguistic element which stands for the content of the Judgment of Quality is already a name And some such element, in the simplest case perhaps that element itself, will emerge in later forms as a description of the newly distinguished individual, which in the Judgment of Quality is only known as 'this'. 'This red (leaf) is redder than it was' And when the individual is once revealed as a whole with parts by this judgment of Comparison or synthetic judgment of sense, the ascription of other differences to it cannot but follow It seems obvious that an adjectival appellation, or at least an appellation of unspecified grammatical class, would come first, and the hardening into a substantive be a later process I incline to think that the hardening of a description by usage, and the isolation of its elements by employment in different judgments, must have been the real and natural process of naming

Comparison in Space and Time

ii. It is evident that judgments which assert distinctions of Space and Time, without proceeding to measurement by units, must be ranked among comparative judgments, or, as these might otherwise be called, judgments of continuous quality It is not the business of Logic to analyse the means

by which the consciousness of extension or succession is obtained Logic only deals with the nature of such a consciousness, and not with its psychical genesis. But we cannot entertain a doubt that position in Time or Space can only be indicated to consciousness by qualitative marks that fall outside the content which is perceived as in Time and Space. Our inability, in many or most cases, to detect these marks by immediate observation (I have never been able to analyse my seemingly direct perception of the quarter from which a sound comes), cannot, I think, outweigh the possibility of showing other means by which the eye can judge distance, the ear direction, and the memory recall a series in its serial order, and no other

The only logical importance of this psychological analysis lies in its confirmation of the idea, suggested by the facts of language and the very nature of quantity, that Space and Time must imply qualitative discrimination as an element of quantitative comparison Nearer and further must be qualitatively distinct spatial perceptions, as red and redder are qualitatively distinct chromatic perceptions. Thus, the abstract totalities of Space and Time have their germ in comparisons effected by perception co-ordinate with the perception of continuous quality and of its differences

But the demonstrative judgments have forced upon our notice a further and peculiar distinction within space and time as continuous qualities, which is known in space as difference of direction, and in time as difference of past and future. 'Nearer' and 'further' are different spatial perceptions; and it is possible that 'this' and 'that' may be naturally equivalent to 'nearer' and 'further', as 'here' and 'there' must be, or must soon have become[1] But besides 'this' and 'that' we have in space the distinction of 'that' and 'that', 'there' and 'there', just as in time besides 'now' and 'then' we have 'then' and 'then'—a distinction which may apply to points equally removed in past and future, and therefore cannot be reduced to quantity, i e degree of remoteness. 'That' and 'that', 'there' and 'there', imply

[1] Even if, prior to a spatial distinction recognised as such, there may have been a less defined distinction as between 'by me' and 'not by me'

no difference or remoteness, nor indeed do 'this' and 'that' necessarily do so

All these may just as well mean on the right hand and on the left as nearer and further They must indeed *be* comparable in distance, but it need not be distance that furnishes the distinction between them With other qualities the case is different There cannot be two different reds that match, all reds that match, i e that are 'equal', are the same. And if there can be two or more different musical sounds that have the same pitch, this is because the distinction between them is one of kind, not of simple quality, because, that is, they are composite perceptions which are estimated with reference to one element within them taken as dominant or essential.

Thus it seems that (1) Space and Time appear in the germ as mere qualities whose continuity is displayed in the judgment of comparison like that of any other qualities If apprehension of Space and Time really depends on 'local' and 'temporal signs', we must suppose that the peculiar definite externality which characterises extension could only appear by degrees, and that perception must have been transformed from perception of contents *plus* local signs, to perception of contents *arrayed in extension by means of* local signs, that the *logical* character of spatial perception, its continuity and homogeneous differences, must have been present from the first in any system of apprehension which could develope into our spatial world The judgment of continuous quality admits of this[a] And (2) the distinctions which first present themselves within a given spatial or temporal perception are not simply and solely differences of quantity, though in time more so than in space, and in both capable of quantitative expression They are more analogous

[a] See Professor Stout, Mind, 77 7, 'The essential fact is that colour presentations and touch presentations, both as actual sensations and as images retained or revived, are diffused in a continuous quantum within which it is possible to distinguish parts outside of each other, beside each other, and between each other' I am sure this is right Then the question is whether the local sign theory is consistent with it All that concerns us in logic is to maintain that the quantitative differences have a qualitative basis, both of identity and of distinction

to differences of kind We must therefore take the spatial and temporal demonstratives, apart from explicit quantitative comparison in space and time, as in themselves comparative contents involving continuous quality

The divergence from the main progression of the judgment, by which space and time are erected into totalities having a special structure of their own and a peculiar mode of existence, must be taken as beginning with the resolution of a given 'now' and 'here' in the judgment of comparison. In this resolution we have the two grades, corresponding to distinctions of direction, or to the distinction of past and future, and to quantity respectively, first that in which 'this' and 'that' may be e g to left and right (not nearer and further), where the spatial comparison is implied rather than expressed, corresponding to the judgment, 'This is red, and that is green,' and, secondly, that in which a true spatial comparison is introduced into the content, as in 'This is nearer than that', which is analogous to 'This is redder than that'.

So-called Qualitative Comparison

III. Under the head of Comparison it is usual to treat of like and different in kind, as co-ordinate with more and less. But it is to be observed that such a co-ordination is not accurate Mere qualities, as such, are disparate, incomparable with each other The judgment of quality pure and simple, as we have seen, excludes comparison This is red, this is black, this is golden, this is sweet, this is sour. These are successive and isolated judgments of quality, and the semblance of comparison which they now bear is due exclusively to the advanced point of thought at which language places us to begin with

Comparison of degree, as we have seen, includes difference or elementary negation within the limits of a single quality, but the differences themselves, in regard to the aspect which makes them distinct, remain disparate or incomparable. In other words, every part of a quantitative whole is distinguished by a peculiar quality as well as united with the rest by an identity of quality Every shade of red, besides being a degree of red in general, is also a particular hue and produces a distinct impression Every perception of warmth is qualitatively peculiar, and often it is not without an effort

that we can recognise the character in respect of which it can increase or decrease Every inch in a yard measure, every cannon ball in a heap, in thus distinguished; and if it were not so, the parts would have no stability and the quantitative whole would cease to exist

We therefore are driven to conclude that quantitative comparison is not *prima facie* co-ordinate with qualitative, but rather stands in its place as the *effect of comparison on quality*, which so far as comparable *becomes quantity*, and so far as incomparable furnishes the distinction of parts essential to the quantitative whole It is with this latter aspect in which qualities are incomparable that qualitative comparison as such must be connected It is thus, *prima facie*, comparison of the incomparable. Any two shades of red, regarded as shades of red, are respectively more and less But they must also be, as we have insisted, *different reds*, and if regarded simply thus are pronounced incomparable as a result of comparison. If we pass, as we can, by slow transitions, along the complete solar spectrum, comparing each colour with that which formed our starting-point, we shall arrive first at differences which may, and then at differences which must, be thus regarded The difference between red and green, for instance, is not to ordinary perception a difference in the same quality, and if it can become measurable, can only become so by reference to an identical quality, such as brightness of illumination, which falls outside the peculiarities of red and green as such

I believe that it is futile to attempt the measurement of difference except in respect of a continuous quality And the mere affirmation of difference, without the attempt to measure, appears to me absolutely devoid of meaning. The mere judgment, 'These two colours,—these two sounds,—or, these two perceptions (a colour and a sound), are different,' is an imperfect and unreal judgment, which in this form, and apart from a meaning which I shall explain below, is as I believe to be found nowhere but in logical text-books. It may best be considered as an incomplete quantitative comparison, in which the parts are distinguished, but their place in a continuous whole has proved impossible to determine In

this sense, the mere judgment of difference would mark the initial effort of quantitative comparison Such a relation is illustrated by the well-known fact that qualitative difference, e. g between two musical notes, is perceptible before its quantitative nature (their relative pitch) is ascertained *Such* a judgment of qualitative difference may be regarded as a first determination of quantity, for its point is merely to deny identity of quality, and in matters of simple quality to be identical is to be equal and *vice versa* In this sense the judgment of qualitative difference, such a judgment as we make when we see that two colours do not match, is an aspect of the initial stage of quantitative comparison

But another judgment of sameness or difference, which it is almost impossible to avoid confusing with the above, has its true place in classification and analogical inference, and, if explained as mere qualitative comparison, is an unreal fiction Such judgments are —'These two instruments are not in tune,' 'Gladstone and Chamberlain are very different men;' 'That victory is uncommonly like a defeat,' 'The globe-flower is just like a hellebore, only it is yellow' Here we are not speaking of an immediate qualitative identity and difference, but of essential and dominant qualities or rather attributes, in other words of differences valued by a presupposed standard or purpose The idea of a standard involves the idea of kind, and kind goes beyond quality Therefore, we come to a conclusion which I think frees us from much sham accuracy and pretended precision. The judgment of difference is never made apart from a standard of difference The apparent exception, when such a judgment denies identity of quality, is simply the first step in quantitative comparison, and it is by quantitative comparison that precision must in such cases be obtained. But the class of judgments from which our later instances are drawn do not refer to identity of *quality*, but identity of *kind* They presuppose classification, and affirm difference or likeness with reference to this classification All attempts therefore to introduce a quantitative estimate into these generic judgments of difference are founded on a confusion between judgments of quality and judgments of kind, on an

attempt, that is, to reduce the latter to the former. This does not deny that the latter may imply the former in addition.

It is a futile introduction of psychology into logic to speak of measuring difference by the difficulty or duration of a psychical transition, the measure of the difference is what we mean by the difference, and what we mean by it depends on the series or classification within which we affirm it. Apart from such a standard the judgment of difference is nonsense; it becomes like 'The soul is not square' How idle to inform us that Gladstone and Chamberlain are different! How superfluous to affirm that one plant is like another! Any assertion like these, if it is not referred to a ground of distinction, in these cases to political and botanical classifications respectively, is as destitute of content as a bare negation. The instance of two instruments pronounced out of tune with each other may seem not to be in place under this head, and to be a judgment of true qualitative comparison I inserted it expressly to indicate the line of demarcation Different colours are such as do not match, i e are not discernible in simple quality But in comparing musical notes we have not to do with simple quality,[1] but with dominant quality,[1] i e. kind 'A note' is identified by its pitch, and different notes are sounds differing *in pitch* Therefore in pronouncing notes to be different we do not merely deny that they are indiscernible, we deny something further than this, we deny that they are indiscernible *in their dominant quality*, viz their pitch That the pitch is itself a quantitative attribute makes no difference; for it is a quantitative attribute, which has become characteristic, and therefore stands logically in the same position as any other basis of classification

In short, then, we must not confuse quality and kind. Kind is dominant or characteristic quality and involves a series of ideas which we have not yet discussed And whereas comparison in respect of simple quality is prior to and absorbs itself in quantitative comparison, comparison in respect of kind is subsequent to quantity and involves

[1] I use quality in the logical sense in which it includes timbre (musical quality) pitch, and loudness

other ideas. An *isolated* judgment of difference can have no meaning except as the first stage of quantitative comparison, the negation of identity (= this and that are unequal). The attempt to assign gradations to the mere judgment of difference rests on a confusion between quality and kind, each of which has in itself an adequate and objective principle of measurement independent of psychical transition, and in the case of kind, incapable of reduction to quantity.

CHAPTER III

Measurement—Quantity and Proportion

Measurement and Individuality

1. **Measurement** is the equation of any whole, by comparison, to a numerical aggregate of determinate parts The parts may be determinate through reference either outside or within the whole to be measured; but if the reference falls within it (as when we say a man's whole height is so many times the length of his head) the whole must be complex and contain subordinate systems The reference may also take the shape of relations which are not purely quantitative (as a tone or semitone in music, apart from its physical cause, is simply a difference between two peculiar sounds), but in that case we are passing out of the region of pure measurement. Some reference, however, there must be in measurement beyond that to the *simple* whole which is to be measured It is no measurement of a line to divide it into 100 or 1000 equal parts We must know *what* else they are parts of, besides being parts of the line to be measured The length of a line is measured when it is equated to feet and inches—to the length, that is, of some actual piece of metal agreed upon as a standard—the pitch of a note is measured when we have determined its place in the scale or the number of vibrations per second that enter into it, the specific gravity of a substance is measured when we have stated the ratio of its weight to that of an equal volume of distilled water at a certain temperature Here a verbal difficulty may be cleared away If the weight of the substance before us is twelve times that of water, our definition of measurement applies straightforwardly We equate the whole substance in respect of its weight to a numerical aggregate of twelve parts, each of which is determined by equation to a known volume of water[1] But if the substance is $\frac{1}{12}$ of the

[1] Practically of course we do not heap up volumes of water in one scale till they balance a substance in the other scale The process

weight of water, we seem not to be breaking up the whole which is being measured into an aggregate of parts, but to be representing it as a part within another aggregate. This is a mere matter of practical convenience The equation of a whole to a numerical aggregate is as much involved in the expression $\frac{1}{12}$ (a twelfth part), as in the expression 'twelve times'. In measuring, we bring two terms into precise equation, and the entire relation of whole and part is involved in each The numbers in which the same part enters into two or more wholes are in every case the organon of measurement

1 Measurement is to begin with Simple Measurement, resulting in pure Quantity

Simple measurement

Quantity

Simple Measurement consists in judging of a perceived object that it is a whole containing a certain number (one or more) of a determinate unit. Measurement is thus a development of comparison, which is the first revelation of the unit, or equal part—the result of successful equation—within continuous quality Simple qualitative identity, for instance, such as that of colours which match, may be set down to comparison or to measurement according as it is or is not ideally referred to a scale of degrees In pure qualitative identification we have sometimes no idea of possible degrees, and such identification must be regarded as the earliest germ at once of measurement and of comparison Thus 'The taste of this is the same as the taste of that' is mere identification or comparison if it only means that the tastes of the two things are indiscernible, but is measurement if we are considering whether the one taste is sweeter than the other. We have seen that qualitative identification *is* only the germ of quantitative comparison, and that the two are not co-ordinate [1]

How is the unit fixed? It is fixed, as we have seen, by equation or identification of it as the same throughout the various wholes or aggregates into which it enters as a part.

employed is equivalent to weighing first the substance, and then the water which it displaces, against known standard weights, and taking the two results as a ratio, i e by help of the balance we state the two things to be compared in terms of an aggregate of already known and determinate parts, one thing will = 12 oz and the other 1 oz

[1] p 116 ff

This process of equation tends to repeat itself *ad infinitum* Pure quantity is an essentially relative attribute. Hence in Simple Measurement the paradox of knowledge takes an extreme form, for every measurement presupposes and provokes others *ad infinitum* The tables of weights and measures of our arithmetic books are enough to illustrate this They are long lists of equation after equation, by means of which all objects that are measured or weighed are ultimately equated with some single portion of matter or relation in nature,[1] arbitrarily selected as a basis of the division and multiplication that facilitate comparison. The substitution of a determinate physical motion supposed to be constant for a particular portion of matter makes no change of principle so long as it is taken *qua* a term in a fixed ratio or mere ratio, and not *qua* a term in a generalised ratio or proportion. But in fact the two ideas are at bottom inseparable Everything fixed is *qua* fixed, potentially generalised The wave-length of what is a particular red to a normal eye does not vary (so far as I know), but this is so to speak an accident , and if we take in non-normal eyes, all that is certain is that this wave-length preserves its place in the colour-series above some wave-lengths and below others Its fixity for red judged by the healthy eye has however caused it to be suggested for an unit of length Weight—the relation of a portion of matter said to weigh 1 lb to all other portions of matter in respect of their gravity—becomes a generalised relation or proportion the moment we consider distance from the earth's centre '1 lb. at earth's surface 2 lbs at earth's surface 1 lb five miles up' 2 lbs five miles up' When a thing is described generically by the number of its own parts this is more obvious, for the ratio is then *ipso facto* generalised by the mere recurrence of individual things But simple measurement of perceived objects gives simple ratios expressed in singular judgments with

[1] Such as a fraction of the earth's circumference But any such relation is likely to be variously determined at different times, while it is not convenient to alter the basis of a system of measures I believe that practically all systems of measures depend upon the actual material standard, which as a mere piece of metal, not capable of being tested by any general relation in nature, must be reckoned as a purely arbitrary standard

external reference As the judgment becomes general the ratio becomes first formally proportional, because the ratio is generalised as against instances in which it occurs, and then really proportional, because this generalisation comes to apply to cases in which the corresponding terms are *different magnitudes* The ratio between measures[1] is *formally* proportional; that between weights, explained as above, is *really* proportional

Complex or mediate measurement Proportion

11 Thus measurement necessarily becomes complex, ideal, or mediate, i e in short generalised In this aspect it first appears within the singular judgment, and then breaks loose from it

Every relation established by measurement is a ratio, or relation between magnitudes, and as incommensurable magnitudes are *for logic* a contradiction in terms, every ratio can be expressed in so far as it is a true ratio (in so far as its terms are magnitudes) by a relation between two numbers Number refers the relation to an abstract whole of quantity, and therefore determines the identity of the relation by its place in a self-identical articulate system But the effect of such expression is to generalise, while the results of *simple* measurement can warrant no generalisation, and therefore are not spoken of as a ratio, and often not reduced to their simplest numerical expression Nothing would be gained by saying that a given plant 5 ft high had a height which was to 1 ft as 5 1, nor by judging that *this* piece of lead is to *this* piece of gold in weight as 11 to 19 The ratios so affirmed would still be destitute of general significance, would be mere facts, alleged of a particular reality present in perception, and would therefore gain nothing, but might lose their truth, by abstract expression

But thought always tends to coherence and necessity, and we cannot even employ a determinate idea to assist us in pointing out an object of perception without creating the impression that the ideal content which we use will be charac-

[1] By ultimate refinement, this too is *really* proportional Suppose all measurable things to expand and contract preserving their ratios, we could never know it Given a foot-rule, we could still construct a yard-measure, however the absolute length of the foot-rule might vary

teristically connected with the content of our predication. When this occurs in the judgment of measurement, ratio passes into proportion, that is to say, the ratio enunciated as true of the particular given subject becomes a universal rule applicable to all variations compatible with the determinate idea which conditions the subject. Such measurement is complex, because the unit on which it is based is no longer single and fixed, but variable in absolute magnitude, though determined by a condition. It is ideal, because no longer a mere fact of sense-perception, but enunciated as flowing from a content intellectually defined. It is mediate, because the reference to reality which constitutes this as every judgment, is not direct, but has to pass through a condition before it can attach to reality.

Therefore if we take one of the above simple measurements, and even without removing the demonstrative 'this', insert a determinate condition into the content, we shall find that the whole affirmation is greatly modified in its nature. Let us judge that 'This piece of lead and this piece of gold, *being of the same volume*, are found to be in weight as 11 : 19'. Then we at least suggest the erection of the ratio 11 : 19 into a law of proportion. 'The weights of equal volumes of lead and of gold are as 11 to 19'.

So again, we may have simply counted the leaves on a plant-stem, going round the stem in the same direction, till we find a leaf immediately over or under that from which we began. If we do not know that these ratios are characteristic, or if we suspect the plant to have been injured so as to make its ratio undiscoverable, we may simply judge, 'On this plant I find five leaves in going twice round the axis'. Whereas if we insert the name of the plant or tree, and use an abbreviated expression for the ratio of divergence, we at least suggest the idea of a characteristic law of the leaf spiral. 'This oak shows a divergence of $\frac{2}{5}$' (i.e. the sixth leaf is directly over the first, and in counting from 1 to 6 you have gone twice round the stem). It is obvious that here generalisation and characteristic attribution have begun. Granting that we have not yet any assertion about the *genus* oak, or even about the species in question, for the 'This' hinders such an interpretation, yet

we unquestionably are awakening to the expectation that the tree before us will present the ratio in question in all its different parts and from time to time, and to the problem whether and how far we may drop the 'This' which indicates particularity. When the 'This' is dropped the judgment ceases to be singular, unless it is attached to a proper or in some way individual name Whether and how far, failing these refuges, it ceases to be categorical and becomes hypothetical, or how far individuality continues or even revives after the loss of particularity, as a factor in the evolution of thought, is a question which will frequently occupy us in the sequel

Characteristic ratio, or proportion, may refer to standards external to the whole which it qualifies, or to relations within that whole. In the former case the qualitative quantity remains subject to relativity and eked out by equations *ad infinitum*, hardly less than in the case of simple measurement.

In any table of specific gravities, for instance, we have a number of substances each severally characterised by the ratio of its density to that of distilled water at a temperature of about 39·1° Fahrenheit. Now in the first place, each of these several ratios may obviously be regarded as a proportion in so far as it applies without variation to any volume of the same substance, in the sense that the weight *of any* volume of any substance is to the weight of the same volume of water [1] in the ratio of the specific gravity of the substance. Proportion is defined as 'equality of ratios', and equality of ratios obviously is identity of *the ratio*, and exists between every ratio and all cases in which it applies

But in the second place, although in these cases we have proportion, yet we have also relativity *ad infinitum*. Select the specific gravity of some one body, and suppose that of all the rest except water itself to be erased from knowledge; the significance of our one fragment of information would then be all but gone, were it not for the accessory idea, which we cannot now get rid of, that the rest could easily be recovered.

[1] I omit for the sake of brevity in this and parallel cases to repeat in every sentence the precise determinations by which the standard unit is made a standard But it is all-important to remember that there are such determinations, and that they in turn need re-determination *ad infinitum* There is no ultimate unit

Apart from this accessory idea, the supposition makes it plain that the ratio between the density of silver or flint glass and that of water is not sought out as valuable *per se*, but is valued as a means of equation with all measurable densities In such instances as these we have the first grade of characteristic quantity or proportion, still subject to an external relativity which extends into an infinite series

Now though this relativity never disappears as an aspect of human knowledge, yet characteristic quantity can assume a more self-sufficing position than that which has just been described Instead of developing the *this* and *that* of the comparative judgment into separate units connected only by an abstract identity of quality, we may consider them as structural elements within a concrete whole As before, we shall find ourselves at first in the stage of simple measurement and pure quantity But the tendency to advance to proportion is in cases of this type much more pronounced, because the parts of an individual whole are more likely to vary in connected ways and therefore lend themselves to proportion, than the elements of wholes external to one another. Still we begin with pure quantity '*This* plant has petals exceeding the calyx segments,' or ' has radical leaves half the height of the stem ', or ' has twenty-one carpels and forty-two stamens ', or, as we said above, ' has a divergence of $\frac{2}{5}$ between its stem-leaves ' In the first instance such judgments as these are mere judgments of perception, or at the outside of direct historical fact, and the ratio which forms the content of predication is therefore not a proportion, because it has no extent of application. In proportion, the ratio is to the cases of the ratio as Intension to Extension

But the moment thought has seized a significant[a] idea, it is committed and must go wherever the idea carries it, in despite of the demonstrative ' this ' And the moment that any such internal ratio as those which I have suggested is taken to be characteristic, i e to be involved in the significant idea, it becomes a *proportion*, i e a law of structure which holds in spite of varieties of size, shape, and number, although, at least

[a] I have here altered the term 'designative' into 'significant', because of the contrast between designation and ideas See II, pp 260 ff

in natural objects, always subject to limits which as regards the proportion itself are arbitrary and external

Proportion is the simplest expression of individuality [a] All intelligent recognition of individual objects depends either on proportion or on some principle which involves proportion It is in this that the truth lies of the well-known Pythagorean doctrine that all things are embodiments of number All things have aspects and effects which find generalised expression in number Shorten a snipe's beak, take one from the divisions of the horse-chestnut leaf, or misplace the accent (a variation of loudness and duration) in an English word, and recognition falters or fails Even a human character or an artistic inspiration, though not in itself susceptible of numerical expression, leaves traces in all its acts and products of an individuality that takes shape in proportion or qualitative quantity An exhaustive statistical treatment of a man's life in all its tangible aspects would give, by the graphical method, not indeed his character, but a set of proportions penetratingly significant of his character

It may seem indeed that in common hurried thought recognition simply attaches to some pre-eminent quality, a bright colour, a marked outline, a peculiar movement, and that such elements as these, without extension into proportional systems, furnish the practical meaning of words in ordinary life But in the first place, this is perhaps a superficial analysis of perception I very strongly doubt if the element of proportion, both external as in size compared with surroundings, and internal as in shape, symmetry, or harmony of sound or colour, is ever absent in a recognitive perception of an individual thing. A really abstract quality would hardly mean anything ; we should be able to place it nowhere in our world ; and if we even recognised its degree of intensity, that would at once constitute a quantitative element. It has indeed been observed that a familiar scent (one of the least articulate of qualities) has a notable power of stirring associated memories But this seems so noteworthy just because the scent does *not* recall any individual thing, but rather brings

[a] That is to say, when once we have abandoned or transcended the unanalysed ' this ', which has individuality in a different sense

back a general state of feeling connected perhaps with entire scenes and incidents, but especially with emotions.

And in the second place, if an abstract or mere quality were used designatively [a] in judgment, it would not grasp or enter into the nature of a real individual; it would simply be a falling back towards the demonstrative affirmation with its 'This', which may on occasion be eked out by any element that draws attention.

But all ordinary recognition of individuals undoubtedly depends on the judgment of proportion. We cannot indeed tell the specific gravity of a metal by the mere sense of pressure or of resistance, but we know how a sovereign ought to feel when we lift it on the palm of the hand, and though we may call this effect on the hand a quality, it is plainly a quality pervading differences and so quantitative, and moreover taken as characteristic and so proportional. Consider once more the effect of altering an accent in English (I do not speak of languages in which accent depends on pitch) as exemplified in the change of 'conquer' into 'concur' by transferring the stress from the first to the second syllable, or the utter unrecognisability of such a term as 'sleeping-car' when pronounced with a heavy stress on the second syllable,[1] and a light stress on the first. Here it is the internal proportion that is modified, with the result of destroying the peculiar rhythm by which in a great degree the ear instantaneously recognises a word. The more marked an individuality is the more it depends on internal proportion. Every instrument fitted for a purpose has internal proportions dictated by its purpose, a knife is sharp at the edge and blunt at the back; the thickness of the blade in its transverse section depends on the requirement of strength on the one hand and on that of dividing without displacement on the other, and these requirements together dictate a certain set of proportions characteristic of a blade suited for some particular purpose. The length of the blade compared with its width depends on such another

[a] See notes on pp. 126–7. The argument here amounts to saying that a quality cannot be used designatively.

[1] The author once heard the words thus mauled at Calais, and could not imagine what was being said, though no elements of the sound were omitted.

set, and its temper on a third All of these being on the one hand relative to each other and on the other hand relative to a purpose are internal proportions subject to limits prescribed by external proportions It is by acquaintance with the perceptible character impressed by such proportions as these that we readily pronounce on the use of objects made by the hand of man, and that we detect, somewhat less readily, the *actual* purpose served by adaptations in the organic world Such attributes as are expressed in these proportions form, for perception, the content of individualities

It follows from these considerations that the question of individuality in contents the main attributes of which arise from external proportions is not an easy one. Such are nearly all inorganic substances, except where adapted to a purpose by man. External[a] proportions *per se* produce no effect of individual unity, and though it is true that all substances occur in particular fragments which have definite characteristic forms, yet the ratios which would express these forms are not absolutely typical or essential (though perhaps all substances prefer some shapes to others, and have typical fractures, &c), nor do the particular masses of substances or volumes of gases demand or receive individual names Professor Jevons has called attention to this curious fact, which goes deep into the nature of individuality Below the level of organic form, or form given by human interference, what do we mean by a *thing* ? Of course we may take a lump of metal or an ounce of water, a handful of sand or a jarful of chlorine, and speak of it as a thing , but we shall be puzzled to find any name that recognises its separate identity as ' lion ' or ' spade ' or ' house ' recognise that of the contents that form their meaning Is gold a class-name, say a specific name, and are the actual pieces of gold individuals under it ? This does not seem to be right , a class-name is true, without further determination, of the individuals under it ' This is a lion,' ' This is a spade,' &c But you cannot say ' This is a gold ' you can only say ' this is gold ', where ' gold ' almost = ' made of gold ', i e. is

[a] There seems no meaning in this word except in contrast with the inner proportions of an organism It must be understood of a difference of degree between organic and inorganic structure.

adjectival, and has no plural, or if it has one, uses it for different kinds, not for different pieces, of gold

It is enough at present to call attention to this difficulty as illustrating the place of structure in individuality It should be noticed that a structure however complex which repeats itself homogeneously throughout all atoms of a certain substance tends to confer individuality, if at all, on the minute units in which the complex structure exists, but neither on the substance as such nor on its larger fragments, the supposed minute structure is not the structure *of it* or *of them*, but only a structure repeated *within* it or them A heap of corn is, *qua* heap, no more organic than a heap of sand But it may be for instance that, in virtue of a common structure, all the iron that anywhere exists is united by reciprocal reaction in a common magnetic world, if so, it is then up to a certain point single and individual The further consideration of this difficult subject belongs to the discussion of the Individual and Generic judgment

Qualitative unity of Individuals

iii The relation of Quantity to Quality results at this point in a further problem Assuming that a thing which has marked individuality has always a number of pervading qualities, each of which contains gradations and a distribution expressible by a ratio or proportion, what are we to say of the interconnection of these various systems of proportion with one another? Is it necessary that there should be a general proportion of proportions which, whether our actual apprehension of it be rough or exact, must be taken as capable of expressing the various systems of attributes as gradations within one and the same totality? Is there any sense in talking of the proportion not only of length to length and of colour to colour, but of length to colour, or of the proportion not only between rhythm and rhythm or pitch and pitch in a melody, but between rhythm and pitch as elements of the musical effect?

The view which we took above of the effect of comparison upon quality forces this suggestion upon us *If*, in other words, a single qualitative effect pervades any and every totality which we apprehend, and *if* within such a quality there are parts perceived as differences *of it*, then these

differences must in respect of that quality be regarded as gradations. It is not necessary to press this conception home at present It is possible that there may be individuals whose unity lies in an idea only and not also in a quality, and that an idea may hold together without crushing them into gradations, antitheses which a quality could only admit as quantitative But it is worth while to bear in mind[a] that there may be a quality of 'effect', or secondary quality, within which even form and colour or pitch and rhythm may take their place as degrees, just as the repetition of analogous though different forms in a picture or design gives the impression of a pervading character which is more and less intense in different parts of the work Take for instance a picture about which there is a question whether Turner painted it, or a song which is ascribed without certainty to Shakespeare In such cases we point to this and that characteristic as more or less Turneresque or Shakespearean, and the elements so designated need not be in actual sensuous quality comparable with each other. It might even be suggested that the exhibition of such a pervading quality was a condition of æsthetic though not necessarily of actual individuality, a suggestion which would raise the fundamental problem whether all actual individuality has, for those who have eyes to see, a thorough characteristic unity It would seem not improbable that *true* individuality is attained by *actual* individuals in very different degrees

Change and motion in reference to Individuality

IV In explaining the apprehension of individual things,[b] which I have set down to the sense of proportion, it is usual to lay stress on the fact of change, including motion Change in time and place is no doubt a primary instrument in revealing the fact of individual existence The moveable and modifiable Thing proclaims itself unmistakeably as distinct and permanent. Nevertheless, for logical purposes change is only a case of difference or negation, or if in fact the two are coextensive, yet change is not the essence of all difference. It may be that

[a] I have here cancelled a sentence which implied that zero of a quality was a degree of that quality See Bradley, Mind, 74, 184

[b] All the 'individuals' here discussed are of course only *prima facie* individuals See II, p 253

every apprehension of difference requires lapse of time, but this is only because our activity is in succession, and does not mean that the differences themselves must be (though they may be) successive

What the whole matter comes to is this Difference is the principle which when generalised is known as Negation, in as far as contents merely differ, they are merely not each other. Every continuous quality includes negative determination, i e differences, elements which are not each other Among other continuous qualities, duration and extension characterise a large part[1] of the world of sense-perception, and duration characterises in one aspect everything that comes before the human intelligence It is not, therefore, surprising, that not only the parts or elements of individuals, but the individuals themselves as parts or elements of our perceived world, should bear a negative relation to each other and to themselves in time, in space, or in time and space together.

Now if we contrast change and motion on the one hand with mere perceived difference on the other, as influences bearing upon the apprehension of individuality, the distinction between the two cases amounts simply to this, that in the latter we have a single set of differences, which can go but a short way, as a rule, to exhaust any identity, while in the former we have at once a summary of innumerable differences or negative relations. These differences, in virtue not of their mere spatial or temporal distinctness, but of their positive content, are read off in judgments which may[2] coalesce into one or may be distinguished into several determinations of the individual. These judgments may have a negative or positive form, but must always express a partly negative element of apprehension, viz. that 'what is not B is not A'.

When we see a moving animal against a variously coloured background, different elements of its outline and colouring are successively thrown into relief by successive contrasts, and perception traces its form with increasing completeness by

[1] I cannot think that sonorous bodies appear to be clothed with sound as luminous bodies are with colour To me sound is an unextended perception, though referred to a cause in space

[2] See the general theory of judgment in time, chap 1, above

the negations which these contrasts furnish, i e by the correction which is effected when a line or colour, which the moment before ran into the background, is sharply defined by a change of contrast The perceptive judgment resulting from this change will take the shape ' That (the dubious border of colour) was not B (a particular part of the animal's outline) and therefore is not A (does not belong to the animal) ' More elaborate interpretations than this may of course attach themselves to the motion of a separate material thing ; thus, for instance, that which moves all together must in all probability have mechanical cohesion, for it is unlikely that in presence of varying friction at its different points the parts of a moving appearance should keep together apart from such a condition. But cohesion is for our present purpose merely a definite content assigned to unity, and the negative relation of the moving *thing* to what does not partake of its cohesion is established by just the same logical process as its relation to that which does not partake of its distinctive shape or colour Change in time without motion has the same effect , nothing is more readily detached from its background and apprehended as individual than an object whose transformations take place before our eyes in surroundings which do not share them. The judgment or judgments [1] ' What is not a_1, a_2, a_3, &c is not A ' seems to form itself in such a case almost without an effort of mind. We shall be told indeed that ' What is not a_1, a_2, &c. is not A ' is a mere inference from ' A is a_1, a_2, &c ' This objection would raise questions which cannot be dealt with till we treat of negative and of inductive inference Here we need only insist that however we may elect to describe the process of the negative instance, it is easily seen to be the most effective instrument of definition When we inject a system of vessels with coloured fluid, in order to observe them under the microscope, it is not the particular colour, red or blue, that we look for, but the contrast between the artificial colour and the dull grey or yellow of the background Even granting that we start from ' The vessels which are to be traced (A) are the red lines (a_1, a_2, &c),' still this judgment cannot have scientific precision apart from the determination of the

[1] See the account of judgment as an act in time, p 79 ff.

detailed not a_1, not a_2, &c , and when the not-a's assume a positive character, the negation ceases to be an inference from the affirmation A is a And *change* is an infinite succession of such contrasts, that gives every element of the complex individual before perception its chance of being sharply defined, and by successive negations defines each of them both against its own permanent elements and against the background Instead of the simple change of colour by injection, let us think of the effect produced by rotating the polariser or analyser while observing an object that modifies polarised light The successive but gradual changes of colour, illumination, and background which are thus obtained bring out the details of a structure as clearly as if we could handle it and move it freely in space Change and motion merely do for a single individual identity what a comparison of instances does for an abstract identity That is to say, they show through what contrasts the individual can pass, to what negations it can be subjected whether within its content, or between its content and the background from which it is distinguished, without losing characteristic identity Change and motion have their logical value simply as embodiments of difference

Abstraction and Necessity

v. In the Judgment of Measurement we find ourselves face to face with the element of abstraction and necessity, the medium in which exact science moves, and the occasion of the most fundamental *crux* for logic as for ethics We are no longer, as in the Judgment of Quality, simply ascribing the meaning of an idea to an unspecified reality given in perception We are indeed, as always in judgment, defining the reality which perception presents to us , but we find that in trying to define any special feature or element within it, we are under constraint, not merely as always from the pressure of perception, but from the inability to select and connect at pleasure within the presented content Judgment—so far as it escapes from the distractions of mere association—proceeds in grooves or along threads which are always leading it across and out of the picture It cannot, in the present stage at all events, simply characterise a given identity by differences related to it and to nothing else Such differences, as we have seen, would

have no stability, and could characterise nothing, although the more highly organised and individual the identity the more capable it is of prescribing a necessity to subordinate wholes which appear as differences within it The course of judgment within the present whole of perception is determined by connections which refer beyond that accidental whole, to other more comprehensive totalities, and ultimately, in every case, to the system of the known world The connections thus prescribed between part and part within some systematic whole are *necessary* connections, and judgment, in so far as it is controlled by them, is abstract or hypothetical judgment

But the appearance of this element in the judgment of perception makes it simply self-contradictory The specification of a subject by means of an idea, which is only meant to point out a feature in present reality, brings the judgment into a groove of necessity, and all but makes its affirmation conditional A speaker who has affirmed that 'This execrable ruffian should be hung' will probably, if convinced that the man is not a ruffian at all, consider that the non-existence of the condition precludes the application of the judgment, i e in spite of the 'this', he will maintain that his judgment was essentially conditional We shall not find it easy to decide whether the idea in question was really a condition or a predicated content In the latter case the judgment is falsified by the non-existence of the fact indicated in the subject, in the former it is not There can be little doubt that we must follow the analogy of 'this', 'here' and 'now', by treating ideas, which characterise[1] the subject, as presupposed rather than affirmed, but seeing that in this case the idea which forms the presupposition is such as may or may not be realised, and yet is welded together with a presupposition—the 'this'—which cannot but be realised, there is an inevitable ambiguity in the judgment,—the ambiguity between absolute and conditional assertion

Absolute and Conditional Affirmation

vi. In logic as in ethics, Individuality or Absoluteness is at first sight opposed to Necessity or Relativity That which is individual or absolute claims to be self-sufficing, that is to say, to be an Identity which determines and is determined by

[1] See note on p 126

its own differences, but is not dependent on anything outside itself. Every content partakes of this character in so far, but in so far only, as it has a unity or an interest for its own sake or in itself A material 'thing', an organism, a work of fine art, possesses such unity in a degree that forces its individuality upon perception and ensures it universal recognition in language. But every content without exception that is exhibited in judgment has such unity or interest in some aspect or to some degree. Even the abstract idea that qualifies 'this' in a perceptive judgment such as 'This cold is intolerable' is taken as the key to the interest of a presentation, as a predominant feature that arrests attention in our momentary surroundings That the distinctive character which makes the unity of the presentation is abstract and indeterminate follows merely from the judging function being in a rudimentary stage to which a concrete synthesis is still unattainable.

On the other hand, every judgment may also be regarded under an aspect of relativity or necessity In so far as a content is necessary it is not self-sufficing, but is a consequence of something else, and in so far as it is relative it fails to explain itself, and refers to something else for explanation. Every content, every identity in human knowledge is on one side wholly of this character How the two sides, the absoluteness and relativity of the objects of knowledge, can coexist without interfering may be considered if we please, though in my judgment erroneously, as a desperate problem But that they do coexist we may convince ourselves by the evolutional history of any flower, by the analysis of any ornamental design, by the study, in its genesis and with a view to the influences that conditioned it, of any human mind And, in one sense, necessity is more universal than what I call absoluteness, or if I may coin a phrase significant of the sense in which I speak of absoluteness, morphological unity. Morphological Unity has degrees, but relativity or necessity has none The only escape from relativity is in the exhaustion of relations If, for instance, we can intelligibly speak of the universe as a whole, we must take it, I presume, as the totality of relations, and therefore as bearing no relation to anything outside itself. But this speculation

is unprofitable, because what is out of relation is out of knowledge, or it has at most a negative value as against doctrines which extend the relativity which holds within the totality of relations to the ideal totality of relations itself, and so discuss its origin or the possibility of it not having been. This is futile, from the very nature of explanation. All explanation is *within* the universe, not *of* it. Therefore every content that qualifies a subject invites consideration as an antecedent in necessity in a judgment '*if a* is, *then b* is', while it is *not* every content that has morphological unity and so is given as a whole in each and all of its differences And only such a content as this is adequate to reality, or can stand, without special symbols of reference, for an individual reality What is not individual can only be a fragment of the real

The above considerations were touched upon in chap. I, in explaining the pregnant distinction between Categorical and Hypothetical judging, and are to govern, as was there indicated, the application of that distinction in the remainder of the present Book. We start from the principle that all judgment whatever is an attempt to make explicit the nature of Reality, and is directly or indirectly attached to the reality which is presented through perception The ultimate subject of all Judgment is the Real, and any idea which appears as characterising or even as in lieu of the subject of judgment must be taken as simply indicating or calling attention to some aspect of the real world That is to say, such an idea must be taken as morphologically corresponding to the 'this', 'here', or 'now' in the demonstrative judgment, to the unnamed direction of perception in the pure judgment of quality, or to the significant ideal content which expands the 'this' or 'here' in the elementary judgment of measurement. It follows from this that in every judgment the immediate subject is *prima facie* taken to be real, and therefore every judgment is *prima facie* taken to be categorical This does not mean that in the strict sense it *asserts* real existence of the subject, for its real existence is presupposed,[1] but rather that it defines the reality of a real existence presupposed as subject.

[1] See last section

But this *prima facie* semblance of the judgment must be qualified The explicit content which characterises the real subject may be inadequate to the nature of reality In 'this red thing', 'this metal', even 'this man', the explicit contents 'red', 'metal', 'man' are typical and general, not single and individual; and still more is this the case if we think of such judgments as 'Red is a colour', 'Metal is lustrous', 'Man is mortal'. These contents stand for imperfect and incomplete realities; realities that could only be completed in an infinite series of time and space The difficulty is not that they go beyond present perception and beyond our knowledge Caesar is not in present perception and we do not know all that he did, yet in 'Caesar crossed the Rubicon' no one doubts that Caesar is a reality Caesar is an individual,[a] and his entire identity is present in his every act and attribute Man is no doubt a definite concept, but its instances or manifestations have not *prima facie* individual identity one with another The centre of morphological unity is in each separate human being, not in the idea of the race as such The concept, as we commonly think it, is an abstract idea, and the reality that corresponds to it is a series of individuals, which not merely is *not yet* actual as a whole, but is not in our predication treated as an actual whole The reciprocal relations which bind together say the English nation or the Roman Empire into a historical individual may be present also more or less in the case of humanity, but when we say 'Man is mortal' this is not the light in which we look at the Subject; we are speaking of individual men whom we designate by help of the concept man, not of humanity or mankind as such, for which it might be maintained that morphological unity is possible No doubt however, if we push the matter home, even the predicate mortality is affirmed of all individual men in virtue of a oneness of nature running through them all, and therefore we must, as I have said, take the individual unity to be a matter of degree, and to be wholly absent in no content that can be presented to thought as characterising a subject of judgment.

When, in view of cases like these, we qualify the principle

[a] See note [b], p 131

laid down above that in every judgment the subject is taken as real, we must do so by the emendation, 'The subject is taken as possessed of that kind of reality of which it is capable, subject to any alteration which the predicated content may declare in such reality'. The first part of this sentence provides for judgments introduced by abstract ideas and not referred to actual individuals, the second for all kinds of judgments which formally affect reality and which are alleged [1] as proofs that content and reality cannot depend upon one another. First among these come the Negative Judgment which sets up an aspect of Reality in order to demolish it, and then all such peculiar cases as 'The chimera is a fiction', 'Nothing is here', 'A wise knave is a contradiction in terms'.

The view to which I have just alluded might raise an objection at this point, which I will only mention in this place, as the real answer to it, if any, can only be found in the whole conception of the judgment which we have adopted 'Whether an idea stands for a reality or not does not depend on its content, but on that content being recognised as somehow and at some distance or other belonging to the world continuous in quality with the object of present perception—i e to the actual world' My answer to this would be that I have accepted this identification in quality as the abstract logical or rather as the psychological condition of all judgment, but that this identification is empty apart from the specific kind and degree of reality assumed or alleged, and this, as it appears to me, is a matter of content and of content alone

The Hypothetical Necessary or Relative aspect of judgment is a consequence of the characterisation of the subject by any *determinate* ideal content It is the universal connection of attributes within systems, as opposed to the morphological unity of individual systems upon which that connection rests. This aspect is undoubtedly perceptible from the moment that the immediate subject is made explicit by help of ideas, but as long as there is a gulf between the ideal content and the latent reality which it designates—the 'this'—the necessary aspect of the judgment is absolutely dissociated

[1] Bradley's Principles of Logic, p 14.

from its categorical aspect, and the divergence almost amounts to a duality of the judgment. In some cases indeed the contents employed to designate the subject will have only a partial connection with the predicate, as in 'This flower is a rose'.[1] Such a judgment has been called a double appellative judgment. But I incline to think that affirmation of this type is always on the road to necessity, 'flower' does not indeed tie down the predicate to 'rose', but the thread of botanical classification runs through both. If it were not a flower it could not be a rose. So the two designations are undoubtedly chosen with reference to one another, and the true duality of the judgment is not in christening the rose twice over as rose and as flower, but in the ambiguity between 'This is a rose' and 'This, *if* (or 'in as far as') it is a flower at all, is a rose', which might well be said of the Tudor rose as it appears in some decorative designs.

The judgment whose subject is designated by a proper name is at first sight devoid of necessity. For in it determinateness of content is sacrificed to the indication of actual continued individuality, and therefore the relation of necessity or hypothesis, which depends on determinateness of content, is not easily traceable. Yet a name is always capable of acquiring a definite content, which at once brings such a relation into prominence. The indignant denial, 'Gladstone never said that,' 'Socrates never gave immoral advice,' is obviously hovering between the sense of 'A B did not go to town to-day' and that of 'An honest man cannot say what is certain to mislead', that is to say between the assertion of fact and that of necessity And again, if, as explained in the last paragraph, the content which characterises the subject has not the nature of a complete or individual whole, then its reality must be taken as that of which alone it is capable, viz indefinite presentation in the series of space and time, and this amounts to so little (for the presentation may be as rare as is consistent with occurring at all) that the element of necessity or relativity dominates the element of unity or actuality, and the judgment appears to have as its essential content a necessary sequence or connection

[1] Cp. Sigwart, vol 1, p 69. E. Trans, I. 57

about the presence of which in reality little or nothing is affirmed. Such a judgment, and every judgment in as far as it can be thus regarded, is hypothetical, that it to say it runs wholly or partly along a line which may be formulated as 'If *a* is (or in as far as *a* is), then *b* is (or so far *b* is ').

On the other hand, even those perceptive or historical judgments, including ordinary assertions about people or places called by proper names, which betray in their content no tendency to enter a groove, i e. to exhibit a universal connection of attributes, nevertheless must be held as bound by this ideal, which is involved in the employment of contents which have a meaning, and therefore can acquire a determinate meaning If then, as in such instances we may assume to be the rule, the contents actually employed in judging embody no principle, but merely exhibit irrelevant differences as coexistent in a concrete subject, such judgments, even when true in their first meaning as mere statements of fact, are yet fundamentally false. That is to say, they are true in their categorical aspect but false in their hypothetical or relative aspect, from which, being definite judgments, they cannot escape They do not express an *a*, upon which, within some real system, the content of predication follows as *b*. But it is important to remember that we are dealing from beginning to end with aspects and not with total differences. I believe that a misapprehension on this head has of late years given rise to an erroneous conception regarding the tendency and goal of knowledge

Knowledge as both absolute and relative.

VII It is a great thing to have raised the notions of Logic to a level with the ideas of exact science. This has been done by Mill and subsequent writers,[1] and the work had become necessary, though the views to which it led were not in principle new The essence of judgment was by them declared to lie in the coexistence and rational connection of attributes, and the ideal of science to consist in the knowledge of the fewest assumptions, from which, if given, the whole course of the

[1] Mill's Logic and Lotze's earlier Logik seem to have appeared in the same year (1843) Lotze, I presume, was largely influenced by Herbart on the point in question I have in my mind also Sigwart, Wundt (whose doctrine of 'Gegenstandsbegriff' does not, however, seem to me perfectly clear), and Bradley

world might be deductively derived. As a consequence of these ideas, the universal judgment was, in part by Mill himself and more distinctly by later writers, identified with the hypothetical or abstract affirmation of necessary connections, and, further, by identifying this type of judgment with the extreme case of supposition for the sake of argument, the universal judgment as such was denuded of all affirmation having real content It was still treated as affirmed of Reality, but was held to be related thereto as a known consequent to an unknown antecedent. It was pointed out that in supposal for the sake of argument no element of the content supposed or of its consequence is affirmed either to be actual or even to be possible, and that nothing more is alleged as fact, in judgment based on supposition, than that Reality, which itself does not appear within the judgment, is such as under the supposed conditions to exhibit the inferred consequent. Truth, it might therefore be concluded, may be taken to illustrate but it cannot be taken to define, reality. The strong implication of actuality which attaches to the content of many ordinary universal judgments was dismissed as explicable on grounds of habit and confusion. Thus the categorical judgment, in the sense of a judgment which asserts any specific content to be actual, was primarily confined within the limits of affirmation about individuals in space and time, although in disjunctive judgment, and in judgments, if any, dealing with existences beyond time and space, a categorical character was admitted to reassert itself

In contrast with the conception of logical progress and with the ideal of knowledge which I have just described, it appears to me that a somewhat modified form of the views in question might yield less one-sided conclusions I should prefer to regard the normal and central evolution of judgment as categorical [a] from beginning to end, and as gaining, not

[a] The truth of this matter cannot really be grasped except by carefully considering such a catena of judgments as forms the basis of Mr Bradley's Principles of Logic or of the present work I do think it remains firm that the intention of the Judgment is categorical throughout It desires to say something true about something real But the point to master is its twofold nature, in which fact and necessity are in conflict, so that in all its phases there is an unsatisfied implication, and it never expresses either a self-contained fact, or a really perfect

losing, in this characteristic as it passes from perception and history to the more complete forms of science The implication of real existence which attaches to the content of ordinary generic and universal judgments seems to me to be of the same kind as the implication of existence—for it is no more —which accompanies the demonstrative 'this', 'here', or 'now', or its expansion by a significant idea, or a proper name, or the significant name of any actual, even if not in the full sense individual, totality, such as the English nation, or the Natural Order Rosaceae

The main function of judgment would then be identified with the exhibition of individual totalities at once in their absoluteness and in their relativity. We should thus not wholly subordinate classification, type, and individuality to the claims of explanatory theory, but endeavour to represent the two as complementary and indispensable aspects of knowledge Abstract and ideal judgments like those which embody the necessary connections of geometry we should rank as an indispensable divergence, but still as a divergence, from the natural track and tendency of reason, and as attaining their truth most fully when, returning towards that track, they are taken up into the precise determination of typical structure in space, or even of individual realities. We should refuse, in spite of identity in linguistic expression, to take supposition for the sake of argument as the type of universal judgment, and should point out that as supposition passes from selection within reality into free imagination it becomes detached from the real ground of all relations, and ceases even to exhibit a necessary relativity

It is in accordance with these views that I have treated measurement as involving both the revelation of Individuality[a] (morphological unity) through characteristic ratio, which is the same thing as proportion, and the exhibition of relativity by the reference of the unit to something outside the individual. I now proceed to speak of kindred judgments, which present the essential aspects of measurement in one-sided modifications.

connection And so in the end undoubtedly 'Judgments are conditional in this sense, that what they affirm is incomplete' Bradley, Appearance and Reality, ed 2, 361

CHAPTER IV

MEASUREMENT (*continued*)—ABSTRACT QUANTITY

One-sided forms of Measurement

2. INDIVIDUALITY as revealed in measurement may be simple or complex, and, if complex, it must involve a variety of simple factors. In a simple individuality, or the simple factor of a complex individuality, the qualitative distinctness of the parts is at a minimum; for any exceptional qualitative difference in any part would challenge measurement and constitute a complication within the unity When an individual is thus taken in its simplicity, in a single aspect, and yet considered as being a whole complete in itself, it is treated as a whole of *quantity*, that is to say, such a content as is exhibited in the predication of the comparative judgment, but taken as standing in the place of the individual Subject, now that the conception of individuality is attained

Enumeration

i In such instances we find the simple quantitative whole which is thought of as constituted by absolutely homogeneous parts—an idea which we have seen to be never strictly true, for without *some* distinctness of quality the parts would cease to be Such a whole differs from the normal individual by the lack of anything that can be called dominant, essential, or characteristic within the content itself. There is, for instance, no need to consider its unity in the light of a secondary or aesthetic quality The unity is already that of a continuous quality, and in the attempt to define it, it lapses almost wholly into relativity, for the determination of the whole depends on the equation of the parts, in an unending series, with other and independent standards Thus the purely quantitative whole is characterised by being capable of construction by ideal repetition of a unit or fixed part, and such ideal repetition is *enumeration* Enumeration may seem prior to measurement or identical with it; in measuring we enumerate units, and whenever we enumerate units we measure some totality When we count

the carriages in a railway train we are measuring the train, when we count the sheep in a flock we are measuring the flock, just as when we count the feet and inches in the length of a room we are measuring the room But usage, as we feel at once, does not bear us out in speaking of the two former instances as measurements, and the reason is plain In measurement we start from a whole which we characterise by its differences, in enumeration we start from a distinct unit, out of which we desire to construct a collective or aggregate whole—a sum total The whole in enumeration, which is a predicate, is a weakened form of the individual whole in measurement, which is a subject, and the unit in enumeration, which is a subject (generally distinguished by a natural individuality), is a strengthened form of the distinct unit, ideal part, or constant difference, which forms the predicate in measurement.

Enumeration is therefore in one sense posterior to measurement, because it presupposes, as a naturally distinct unit, the 'thing', the idea of which can only be furnished by a sense of proportion or perception of limit, but, on the other hand, enumeration is an *instrument* of *precise* measurement, which involves the notion of a scale of degrees or aggregate of homogeneous parts The two processes are constantly concurrent, and only differ in the respective values of the parts and wholes with which they deal It would be futile to distinguish them from one another, but for the consequences which result from the possibility, first exemplified in abstract enumeration, of systematising the synthesis of parts without relation to a whole The whole of enumeration is depressed into a mere aggregate, or not even a definite aggregate, and therefore the part into a mere unit, or even into the mere place in which a unit might be The process with all its corollaries, including the three unreal infinities of Number, Time, and Space, must be regarded as belonging to a form of the judgment-function in which the relation of whole and part is denuded of all structural variety, and therefore the aggregate or sum which is the outcome of that relation lacks predominant unity This principle is expressed in the saying that in a numerical system the sum of units is

the same whatever may be the order in which they are counted, i. e. any part (*qua* part of a total formed by enumeration) can be interchanged with any other part without modifying the whole in which they are parts

Corollaries. I proceed to state and illustrate some corollaries which follow from the above idea of enumeration.

Simple counting. *a. Simple counting always consists of a series of singular judgments, and is in this respect on a level with simple measurement.*

However abbreviated or abstract each step in counting may be, it can always be expanded into a singular judgment which records its own position in a coherent series. In counting the birds in a covey, or the stitches in a bit of needle-work, we often merely repeat aloud the words 'one', 'two', 'three', and the real nature of the judgment which accompanies them is open to question In fact, each of these numerals in such a case implies a separate singular judgment, though extreme abbreviation tends to conceal its structure. The essence of counting is in just such a progressive distinction as is conveyed by 'this', 'that', and 'the other'; 'alter' and ἕτερος have almost the value of numerals The exclamation 'One!' when we are beginning to count any set of objects means '*This* unit is a part in the whole which interests me', e. g. 'One—two—three—*eight* birds in that covey'. The bird is the naturally distinct unit by which in so simple a case we proceed as a matter of course, and the covey is the total up to which we wish to count. If birds get up belonging to another lot, we shall probably desire to keep them distinct, and so count them separately, beginning 'One', 'two', &c, over again. It is plain in this or any similar instance that we do not count right on as long as units can be found, but that every step of the enumeration is made with reference to a limit as well as to a unit. This limit, however disguised by our caprices and interests, is simply the common or continuous nature of the unit in so far as it interests us, and in every enumerative judgment without exception the elements of separate unit and common nature may be traced. The unit need not be externally distinct or physically separable, and the common nature,

instead of being hard to trace, may all but obliterate the differences that exist within it But it is the nature of the unit that furnishes both rule and limit of enumeration, which is a process unmeaning without a limit and impossible without a rule If we are told to count even all the 'things' in a room, we shall find ourselves obliged to ask what is to be reckoned as a 'thing' Is a bookcase with 500 volumes in it one thing or 500, or 501? 'In a room', however, is a kind of limit, and assigns a totality to be constructed by synthesis, but if we are asked simply to count, we should reject the request as pure nonsense, because it assigns no totality to be constructed by counting.

What, then, is the meaning of counting 'One'! 'Two'! 'Three'! in starting a race, or of—

> 'And still stood all who saw them fall
> While men might count a score'?

In the latter case, the idea is that of counting the names of the numbers up to a fixed limit at a rate determined by habit or by the time which is required to pronounce the words distinctly. In the former case the object is perhaps not merely to let time elapse, but to set attention in a certain rhythm, so that the tendency of rhythmical anticipation may assist in seeing or hearing the starting signal at the moment it is given For this purpose the periods should be exactly equal, and in fact at the Oxford boat-races every boat has some one beside it who counts aloud the last few *seconds* before the gun fires Of course, finally, the names of numerals may be repeated, as any words may, without a meaning But to do this is not to count.

Thus even simple counting always involves the elements of the judgment—of an identity exhibited in differences and affirmed of reality; which elements present themselves in the shape of a distinct unit within a continuous nature, its relation to which nature is indicated by number. And that the simple enumerative judgment is always singular follows from the nature of the unit, which is theoretically nothing more than the content to which we ascribe distinctness *pro hac vice*, in other words, the unit *is* the difference or part that is taken as distinct by one act of judgment,

and it may be said that what we count[a] in enumeration are primarily the acts of judgment, as acts of distinction and relation within a certain continuous quality

Does it follow from this view that Number arises essentially out of the sense of time or succession? I answer, Not 'essentially'. The connection between number and succession is a psychological and not a logical question If it is impossible for two related acts of judgment to be simultaneous, as we are naturally inclined to suppose that it is, then two enumerative judgments must always be successive, and in this sense enumeration may depend upon succession Nor do I think that this connection is necessarily disproved by such observations as that perception can take note of six balls at once being dropped into a box Such a perceptive judgment, in my opinion, is probably *one*, and applies the result of previous counting, as an attribute resting on experience, to the perceived content, just as we can judge the number of pips on a card from the mere form of the pattern which they make So if we count by twos, fours, or more, I think that this is complex counting, the unit being the mass of two or four, known to be such by previous experience[1] A *logical* order among the units, i e. an order in which the apprehension of each unit has a place in the series conditioned by the separate apprehension of other units, is of the essence of enumeration, although a *material* order among them, i e such an order that a unit changes its value by changing its place in the series, is in contradiction with the essence of enumeration Unless there is what I have called a logical order, we have no security that the unit as such is apprehended at all, and what we take for enumeration may really be an inference like that which detects from the pitch of a note the number of vibrations

[a] This view has been treated as an assertion that what we count is our own counting, which has no meaning I understand it as holding number to be the general schema of the relation of parts in any whole, taken as homogeneous so far as the distinctness of parts permits In counting, as I understand, we apply this general schema to a given whole, and are consequently put in possession, with regard to that whole, of all the relations which the schema involves Cp p 156, below

[1] See the Author's Knowledge and Reality, p 92

per second which generate it But whether a logical order of succession can only be realised in temporally successive acts of apprehension, is a purely psychological question ; the more so, that, as we have seen, succession and time do not exclude unity of judgment, and the acts of judgment which constitute an enumeration might even, like an inference, be brought under the head of a single continued state of consciousness In any case, succession in time would be a mere psychical condition of number, following from the unity of the intelligence as forbidding (if it does forbid) two judgments to be made at once There is no sort of ground for connecting enumeration with the apprehension of *equal* parts in time, unless the equality of such parts be the material purpose of the apprehension in hand.

Discrete and continuous.

β It follows from the nature of enumeration that *the distinction between discrete and continuous magnitude and the opposition of number as discrete to space, time, and other kinds of quantity as continuous, rests on a confusion* Number as mere names or mere sounds may be discrete, i e. disconnected, but then it has nothing to do with magnitude, but is a set of mere words destitute of meaning On the other hand, number considered as the vehicle of magnitude or quantity is both discrete and continuous , and the same is true of all quantity, as we saw in examining the comparative judgment, and it is the essence of quantity to be so The distinctness of natural units such as reciprocally exclusive material things does not make any difference of principle They, like all units, are numbered in virtue of a continuous quality or identity which pervades them, and every unit, though it may only be suggested by a momentary purpose, is, like them, a distinguishable part, within a whole or aggregate consisting of such parts The books on a shelf are not merely discrete, and the inches in a yard or the units of weight in a gravitating body are not merely continuous , in every case the unit is a distinct or discrete part, and the sum is a self-identical or continuous whole It is nonsense to speak of counting without saying what is to be counted ; and in specifying what is to be counted we specify at once the nature of the continuity and the rule of the discretion.

Judgments affiliated to Enumerative Judgment

ii *Judgments affiliated to the Enumerative Judgment*

The Enumerative Judgment is the root of all quantitative determination, and, as we have seen, all the matter of knowledge above the stage of pure quality is either in itself or in its conditions accessible to quantitative determination. But the judgment in question also contains, that is to say, *is*, though in a depressed form, the universal essence of judgment in the principle of identity and difference; and it is possible for this to be revived by one kind of abstraction into a different relation from that of unit and sum, as it is for it to be further specialised, by an opposite abstraction, into an idealised form of the latter relation. But before indicating the genesis of other types of judgment out of enumeration, we must glance at those which it necessarily generates, and which must be regarded as species of itself. These are, α the Plural or Particular, and β the Collective judgment.

The Plural or Particular Judgment

α The Plural Judgment, or the Particular of traditional Logic, differs in no essential respect from the singular. It is not however accurately described as a mere aggregate of singular judgments, and indeed this description does not explain itself, for is such an aggregate several judgments or only one? I cannot doubt that the plural judgment is a single act of thought, which determines a certain whole or aggregate, given at the moment, though, it may be, in process of modification, by an attribute or condition such as two, three, or some other number which expresses the reciprocal relations of its homogeneous parts. Thus the plural judgment is not an aggregate of judgments, but a judgment about an aggregate. Therefore the number is to be regarded as a predicated content or determining condition, attributed to a whole consisting of the units which have been counted up to the point at which the plural judgment is taken. That the element of continuity, or designation of the whole whose parts are *to be* counted, must quarrel with a determination by any number short of this whole,[1] and demand a continuance of the enumerative process, is only a case of what happens in every judgment of perception. Such judgments, as we

[1] See Knowledge and Reality, p. 65, and above, Introduction, pp. 51 ff.

have seen, never embody 'pure cases', i e they never fit precisely into a groove of necessity, that is, a sequence of reason and consequent There is always something relevant omitted, or something irrelevant retained

Thus the particular judgment from the very first *implies* a *ratio*; and the implication may be so strong as to take the judgment out of the category of particular judgments and place it in that of collective judgments, e g '335 members of the House of Commons are Liberals' As every one knows that the House of Commons contains altogether 670 members, this is just the same as to say 'half of the House of Commons are Liberals' It depends for its meaning on a completed enumeration, and therefore is essentially a collective judgment. It may even be regarded as an instance of simple measurement, i e. of *ratio* treated as pure historical fact, in as far as its purpose is to measure the voting power of the Liberals against that of other sections of the House The Liberals and the whole House are equated in respect of numerical strength, the whole House minus half of the whole House = the Liberals In short, as we have seen, the completed judgment of Enumeration passes into Measurement, though the idea of Measurement is prior to the act of Enumeration If on the contrary we say that 'Two Cambridge men are coming down to lecture', the number two is *prima facie* non-significant; the judgment tells us nothing of the ratio borne by two to the whole number of Cambridge men, nor even to the whole number of local lecturers Nevertheless if we think of it, we shall see that either it was wholly superfluous to mention the number, or else some context or latent allusion must imply a ratio Number has significance only by comparison with number

The Plural Judgment is equivalent to the Particular of traditional logic in the form 'Some men are mortal' There is no essential difference between 'some men are mortal' and 'four men are mortal', the two assertions, if interpreted literally, belong to the same logical class. It will appear however below that this literal interpretation does not render the true meaning of the old 'Particular Judgment'.[1]

But we must distinguish from the above a form which has

[1] See on 'Modal Conversion', chap vii, below

been employed in quasi-numerical arguments, viz the form 'Most men are mortal', or 'The majority of the House of Commons that has just been elected is Conservative' These judgments are not merely particular even in immediate appearance They present on their very surface the relation to a collective judgment which we saw to be at least latent in every particular affirmation 'Most' or 'The majority' means more than half, and when we speak of half or a quarter or any ratio we assume a completed enumeration of the whole. We must therefore now pass to the Collective Judgment or Judgment of completed Enumeration

The Collective Judgment

β The Collective Judgment has of late been rightly distinguished from the Generic and from the Hypothetical Judgments, which correspond to the real meaning of the Universal Judgment known to traditional logic It has been justly pointed out that the 'All' of mere extension or numerical totality does not really express what is intended by such an allegation as 'All men are mortal', or 'All triangles have their three angles equal to two right angles' In such cases complete Enumeration is inconceivable, and something quite different, viz the universal connection of attributes which are not results of enumeration (for number is also an attribute) is really the matter affirmed So far all is clear, and Logic has greatly gained by the distinction

But when we come to erect a difference of kind, and to treat the collective judgment as purely on a level with the singular or particular judgment, and as in fact a mere aggregate of singular judgments, and as thus separated by an impassable gulf from the universal judgment, and when we further maintain that enumeration cannot warrant its own completeness, then we fall into difficulty and confusion Even the Plural Judgment, as we saw, is not a mere aggregate of singulars

The Collective Judgment I understand to be a judgment made about a definite group or limited class of individuals, which individuals are taken to have been exhaustively enumerated or to be capable of exhaustive enumeration. It is not necessary that the individuals should all be together in space or time, it is not necessary that they should all exist or

have existed with any specific degree of reality, but it is necessary that every one of them should be brought before the mind, or should be capable of being brought before the mind, in a distinct and separate enumerative judgment Such judgments are 'All the books on that shelf are German', 'Every horse that I have bought in the last three years has gone lame', 'All the kings of England since the Conquest but three have died natural deaths'

When it is said that such judgments as these are on a level with the singular or the particular judgment, I take it that we must exclude from the meaning of singular or particular judgment all dependence, whether latent or explicit, on completed enumeration, otherwise our account of the collective judgment becomes circular But if so, then we have the 'allness' of the collective judgment staring us in the face as a distinction between it and the particular And this distinction becomes an absolute severance if we are to insist that the process of enumeration, a process which consists in singular and particular judgments, cannot furnish the warrant of its own completeness

I have maintained elsewhere,[1] and it follows from my whole conception of the judgment, that enumeration cannot be made intelligible on such a view It is impossible for enumeration to go on apart from the discriminative control exercised by the pervading nature of the totality under construction upon the successive apprehension of the units It is this control which takes the form of an inchoate perception of ratio as the counting advances, and of the warrant of exhaustiveness when it is completed Apart from such an influence of the whole there can be no purpose in enumeration No doubt therefore complete enumeration is in one sense on a level with the singular and particular judgments, because they present though in an imperfect form (as we have excluded the case of definite ratios such as 'Half A are B') the same relation to totality which the collective judgment completes

But for the same reason it is impossible to justify an absolute separation between the collective and the generic or universal judgment The collective judgment, we may say, must

[1] Knowledge and Reality, p 77

emanate from an enumeration of actual individuals, or at least of individuals actually brought before the mind, one by one. But how can we carry a genetic distinction like this into the interpretation of judgments whose actual content is precisely the same? It is the commonest thing in the world for a judgment to be taken as exhausting a group or set of distinct individuals, without resting in any way upon direct enumeration; in other words, a judgment that *might* be obtained by enumeration constantly *is* obtained in some other way. The fact is, that when we have gone beyond sets of individuals present to perception or within the power of the mind to represent at once as distinct individuals (and it would be impossible finally to limit the collective judgment to such sets of individuals), we have entered on a process which is plainly and obviously mediate and hypothetical, and the fact that many judgments thus mediated are taken to refer to a finite group of individuals is a mere instance of our general rule that every content in judgment is taken to have the reality of which it is capable

Is there then no difference between a collective judgment and a true universal? Certainly there is a difference, and it is illustrated though not constituted by the connection of the former with complete enumeration It is simply this, that a collective judgment deals with a content which can be presented to thought as possessing the character of an aggregate of exclusive units, or finite whole of enumeration This point of view would involve identification of the collective judgment with the aggregate of singulars only if all enumeration were simple enumeration. But a numerical whole may be obtained by mediate enumeration, and it is to a whole so obtained that the content of many collective judgments is equivalent It is true that a collective judgment is not a genuine universal, but it is not true that such a judgment must be equivalent to a mere aggregate of singulars. This conception is, in the first place, irreconcilable with the unity of the judgment,[1] and, in the second place, is not warranted

[1] No doubt there is a difficulty here 'George and John have gone to school,' 'The two boys have gone to school' Does the former sentence convey two judgments and the latter only one? The latter

by the plain meaning of the propositions in question. It is impossible to characterise a number of individuals by a common name as the subject of a judgment without implying a significance in the characterisation. Even if the predication is true of the different individuals for different reasons, the common interest of the judgment must give it unity of purport. An arbitrary limitation either of number or of time tends, no doubt, to interfere with this significance, and to force an extensional meaning upon the judgment, but, as we know, the purest extensional meaning is only a minimum of intensional meaning And there are collective judgments which could not possibly be taken as mere aggregates of singulars Such are reflective historical judgments 'All States of the North American Union are prohibited from interfering with the tenure of property' I need not know this by simple enumeration, I may know it mediately, as a provision of the Constitution of the United States I need not even be actually able to enumerate the States which are included in the Union But I know that they are numerable, because I know that they are an actual limited group, and so I judge it as a historical fact, though I may know it also as rooted in the nature and tendencies of the Union

The fact is, that superficial as is the view which makes 'allness' the adequate expression of logical necessity, it is if possible, more superficial to deny their connection From the first use of the characterising idea, necessity makes itself felt; and 'allness', or the aspect of a finite totality as an aggregate of exclusive units, is never without a warrant and significance however arbitrarily the totality may be taken. It is by the certainty of complete enumeration that counting becomes, as was said above, the organon of precise measurement For instance, the series of enumerative judgments, 'One, two, three, four, five, *six* ounces are in the scale balancing this packet,' is convertible, in virtue of their exhaustiveness, into the judgment of measurement, 'This packet weighs 6 oz'

We saw that the act of counting tends to assume indepen-

only one, certainly, the former conveys two at first sight, but if we bear in mind our account of the Judgment as an act in time, we shall see that these two may readily pass into one See p 70 supra

dence, as if it could have a meaning apart from any continuous nature in the units, i. e in short, apart from an identity presented as a totality This is not merely owing to the customary abbreviation of the enumerative judgment, as when we seem to count by saying 'One, two, three,' &c , rather such an abbreviation is made possible by the apparently independent reality of number. This appearance of reality depends on the fact that the numerical series does furnish a generalised scheme of the relation of whole and part when envisaged in the form of total and units[a] Such a generalised scheme, though meaningless except as applied to a positive content of thought, contains nevertheless definite and necessary relations which are imposed by it on any content to which it is applied , and the presupposition that it is taken as applied to an adequate content fades into forgetfulness that it need be applied to a content at all But need it ? We may surely investigate the numerical series for its own sake When we say that twice 50 is 100, need we mean that twice 50 of some particular kind of thing are 100 of it ? We may surely mean that in the numerical series 100 is separated from 50 by the same number of places as 50 from 0, which relation involves a variety of consequences, all true of the numerical series as such No doubt this is so, but it will be observed that we have to appeal to the idea of places in the series, and these places are the abstractions of enumerative judgments and imply relation to a content. Such places contain in themselves no reason for stopping at any point of enumeration, are applicable hypothetically to every content, but can yield, in their abstraction, no conclusion about any. It is a well-known fallacy to obtain a concrete estimate by multiplying an amount by the number of times its cause as given has to be repeated Hardly any concrete quantity is unaffected in the ratio of its increase by a great addition to its absolute magnitude To say that the stock or trading capital of England is worth so many hundred millions sterling is a graphic expression for the fact that its amount is a million times as great as an amount worth so many hundred pounds But in the economical sense of 'worth' the conclusion is

[a] Cp p 148, above

nonsense A thing is only worth what it will fetch, and who is going to buy the whole stock of England if thrown on the market at once, at the rate which is commanded by the amounts of stock which change hands in the common way of trade?

The numerical series is an ideal scheme of the relations of units within totals, but is itself unreal apart from its applications, not because it is ideal, but because it has in itself no element of limit or totality, i e its units make no choice between belonging to one total and another, and so naturally belong to an infinite series

This aspect of the enumerative judgment—the system of number—leads to the consideration of complex counting and of numerical infinity But before pursuing the enquiry in this direction I pause to indicate a reversion or convergence to which the simple enumerative judgment tends in its other aspect.

Enumeration becoming Generic

The Exhaustive Judgment

γ. Every concrete enumerative or collective judgment bears reference to an identity which controls its selection of units and fixes the limit of its enumeration This identity is the pervading nature of the units Now if this nature consists in the characteristic quality of an individual thing, then it is possible that on the one hand it suggests no conceivable limit of enumeration, while on the other hand the characteristic individuality claims completion in respect of its positive connections of content In such a case there is no true whole of repetition concerned, no whole, that is to say, which in its nature draws nearer completion by every repetition of an individual The books in this room are a true whole of repetition, the human race, to our present knowledge, is not Thus a problem which is first attacked by enumeration may transform itself unawares Meeting with a series of individuals in which we perceive some important attribute, we enumerate them as cases of it But soon their characteristic nature reacts on the function of thought, and we find our successive judgments attempting to grasp a connection of content and not to exhaust a sum total of cases Now as enumeration is on one side selective analysis, we continue to give our judgments enumerative

form, and even couch them by anticipation in the shape of exhaustive totality. Then we have what has been called Induction by simple enumeration, with its results embodied in the Judgment of Allness, which to avoid an un-English expression I shall venture to speak of as the Exhaustive Judgment.

The most varied opinions have prevailed as to the nature and value of this process, obviously because it forms a transition between two distinct lines of thought, marking the revival within mere enumeration of the sense of characteristic individuality which belongs to the judgment of proportional quality and the kindred judgments of individualising thought We meet, for instance, with the question whether the judgment 'All men are mortal' claims to represent completed enumeration or not. The answer is that its form and its meaning are at variance, in form it does make this claim and in meaning it does not Such a judgment indicates that the spirit of analogy and of characteristic quality has reawakened within the form of mere enumeration, and is sweeping the line of evolution back towards judgments which predicate individual and generic character. When we come to speak of Induction as a phase of Inference we shall see that there are good reasons for such an awakening. At present we have only to note that the exhaustive judgment 'All men are mortal' is a transitional form between a collective judgment on the one hand, and the generic judgment 'Man as such is mortal', also couched in the form 'All men are mortal', on the other hand, and arises from an incipient reaction of positive content upon ideal schematic form in the process of making number, before an external separation has been effected between the two elements. It is idle to demand the perfection of complete enumeration from the exhaustive judgment, for this latter is a popular and unstable form of thought, and must simply be recognised as such. It is better to treat the collective judgment as inevitably leading up to a connection of attributes and as therefore having its ideal in the spirit of the exhaustive judgment, than to interpret the exhaustive judgment according to the letter, as having its ideal in the collective

judgment. We shall have to recur to this subject when we return to the central development of the judging function.

We have now to trace the further abstractions which have their root in enumeration

Judgment of Mediate Counting

δ. On the side of the relations between units and total as such, abstraction being made from the positive nature of the contents submitted to enumeration, the judgment of completed enumeration is an ideal or generalised scheme of all possible constructions of such purely numerical totalities. It starts from a measurement or collective judgment of the type 'All the books in this room amount together to a thousand.' The further abstract development of such a predication may be brought about by the most various occasions, but it essentially consists in this, that the positive concrete units of enumeration which stand as subject in a judgment like the above are replaced by generalised relations of ideal units equated to a total which also becomes ideal and generalised. This substitution reveals the fact which alone makes it possible, viz that in the numerical scheme all units, being abstract, retain the same value in every part of the series; e. g the units between 50 and 60 count for as many as and no more than those between 30 and 40 or between 0 and 10 Thus a series of units may safely be named by the number of places which it occupies counting from the zero of the whole numerical series, but it is the same wherever and however often it recurs. It is on this characteristic of number that the possibility of mediate or complex counting and of equation, which is implied in these processes, depends If, for instance, we desire to recompose the sum of books in the room by equating it to component sums or to factors, we do so by conducting a number of enumerations separately from the beginning of the numerical series, and then combining their totals according to the rules of that series, which are known to us simply by experience Thus, if we count 700 English books, 200 German, and 100 Italian, we find that these sums, considered merely as numbers, are equal to the total 1000 obtained by direct enumeration. 'Considered merely as numbers,' because the rules of the numerical series cannot warrant us against any

material influence of the actual individual things upon each other. Some of them may cancel each other, or may produce more by combination, but this has nothing to do with the properties of number, except for the fact that, when known, it is capable of numerical expression The component numbers in the subject of a judgment like the above (These 700 English volumes with these 200 German and those 100 Italian make up 1000 volumes) correspond to the characterising ideas in an affirmation like 'This execrable ruffian should be hung' It is hard to say if they are general conditions, or if they are specifications of fact taken as true merely in the present instance

But it is clear that in such enumerations we are on the brink of mediate counting, that is to say of the abstract equation $700 + 200 + 100 = 1000$, just as in the individual judgment mentioned above we were on the brink of the generalisation 'Every execrable ruffian ought to be hung'. '*Mediate* counting!' it will be said, 'then we are in the region of inference, and no longer in that of judgment' It is certain that when we speak of necessary connection between attributes, of hypothetical or mediate judgment, we are in the region of inference, but it is not the case that we are therefore out of the region of judgment By 'mediate' in the present connection I only intend to indicate a judgment which has for explicit subject a generalised or abstract attribute, and being free from any demonstrative or sign of perception must be taken as conditioning its predication by that attribute Such a predication is mediated, i e is affirmed of any particular individual only through and in virtue of the attribute expressed in the subject Mediate counting forms the transition between ratio and proportion just as mediate measurement does Logically speaking every equation expresses a ratio, and a ratio becomes a proportion directly it is applied—directly its unit is taken as variable. Thus an algebraical equation, which exists with a view to a variety of applications, is *ipso facto* a proportion

Complex counting is a case of mediate counting; that case in which we count by units which are themselves sets of numerical places, i e by multiplication and division, which

may for the purpose be taken to include addition and subtraction. The only difference between multiplication and division on the one hand and addition or subtraction on the other is the equality to each other of the subordinate totals in multiplication and division, which enables them to be counted as units and their number indicated like any other number, by its place in the series counting from 0 upwards. In this sense multiplication is a mere abridgment of addition; it is only a question of form whether we say $2+2+2+2=8$ or $2 \times 4 = 8$ This latter equation represents what is essential as well in addition and subtraction as in multiplication and division, a total analysed into factors and a process. Complex counting, as a case of mediate counting, shares its abstract, hypothetical, and necessary character.

Abstract Counting and Infinite Series

6. The processes of mediate counting deal with the construction and reconstruction of any given numerical total. Even the quantitative relation of part and whole in its extremest abstraction retains thus much of structural unity, that, given a total of units, it can only be dissolved or reconstructed according to certain rules of combination or analysis. But the quantitative unit *per se*, or rather the one-sided abstraction of the quantitative unit, the mere numerical place which no positive identity links with the other places of the series, has in it no principle of totality or limitation, that is to say no reason for stopping short after one set of such places rather than after another Enumeration of units as such may be continued at pleasure and the process of so continuing it without limit is summarised in the conception of numerical infinity

We have here tracked to its genesis this paradoxical conception, in its right place so powerful for good, and in its wrong place for evil It would not, perhaps, be beyond the province of logic to comment on its use in its right place, that is to say in mathematical science, nor could a more interesting subject readily be found But lack of mathematical knowledge deters me from attempting such a comment with any degree of detail; for a logician is aware of the risk incurred by venturing beyond his knowledge, and as he preaches that there is no royal road to truth, must keep clear

of the delusion that he himself has found one But any one may offer a suggestion, and this I propose to do by saying that it seems to me most probable that the scientific use of the conception of infinity rests in every case on a justifiable neglect—justifiable, because that which is neglected has a known nature, and may be set down as insignificant either altogether or from the point of view of a specific purpose

If, to take a coarse and non-mathematical instance, we set about any task in a way which is demonstrably perverse and inadequate, a looker-on is justified in disregarding our efforts He will tell us that we shall not get it done in that way if we live to the age of Methuselah Translated into logical phrase, his comment means that our way of going to work has not the element of totality, the successive efforts which make up the series of our activity, bearing no relation to the nature of the work to be done, do not include in themselves successive parts *of it*, and therefore, *as regards it*, have no tendency to come to an end and will (unless we choose to leave off) go on to infinity We may even apply this illustration to a simple mathematical idea, say to the case of parallel straight lines We may consider as the task to be accomplished such a change of direction in either or both of a pair of parallel straight lines that they should cease to be parallel to one another And we may consider as the means adopted to bring about such a change the production of the two straight lines in their original direction Then our supposed on-looker would say, 'You might go on for ever at that game,' 'You cannot change the direction of a straight line by producing it in its original direction' Therefore it would be justifiable to neglect the production of parallel straight lines to infinity, in other words to pronounce that such production cannot alter their character as parallel straight lines, i e that even if produced to infinity (which they can never actually be) they do not meet Probably such a case as this would hardly be recognised as an instance of the mathematical use of infinity, but in as far as it introduces the conception of quantitative infinity as a term in a positive definition it would seem to be at least analogous to such a use

ζ. I may insert at this point some additional observations upon the question of an actual numerical infinity.[a] New Theory of Infinity

The previous paragraph expresses the essence of my argument It still seems to me sound, so far as it goes, and while fully admitting my own want of mathematical knowledge, I cannot see my way to cancel it, or the discussion which depends upon it.

Professor Royce,[b] I think, in his reference to this discussion, has done me less than justice by assuming that I insist, in the vein of Hegel's satirical digression,[c] on the mere tediousness of numerical infinity But my treatment contains no reference to this topic (unless it were on pp 191–2 ed 2 where I reject such a treatment), and in considering the numerical series as the abstract schema of membership in a whole I have implied and stated that it has laws and properties which give it a character and interest of its own That, taken by itself, it fails, as a consequence of its abstractness, to fulfil the conditions of an individual whole, seems to me obvious, and no less obvious that Professor Royce, in using it as a type of individuality, lays stress on possible incidents of the repetition of positive contents, rather than on the essential nature of repetition to infinity.

With regard to the present state of the question, I will venture on three remarks.

(1) I observed in the first edition of this book that 'relations may be given as actual which only an infinite series could represent on their quantitative side' (p 168 ed. 2). The same point presents itself to me with regard to the conceptions employed in the recent theory of infinity I see that certain problems and definitions are given, and that if their requirements could be carried out in actual series an actual infinity would be given, but I am unable to see that this latter condition is fulfilled. The character of givenness does not appear to me to be brought to a point beyond that illustrated by Spinoza in the case of a circle within a circle and not concentric The figure, which would need an infinite analysis, is given. But it does not follow that the series which would Problem actual.

[a] Ed 2, 1911 [b] World and Individual 1 508 ff

[c] Hegel, Wiss d Logik (ed 1841), p 2[illegible]

analyse it is given in and with it I suggest that this analogy might apply to the conception of self-representative series and of transfinite numbers I note that the latter cannot be reached by enumeration I do not know what characters they share with numbers of the natural series, but it is obvious that they can hardly be given in the same sense

Split in Whole

(2) In the definition of an infinite whole, it is obvious, and I suppose intentional, that the relation on which the infinity depends, the one-to-one correspondence of a true part with the total of parts forming the whole, expends the material of the whole, if I may speak so grossly, at an immensely more rapid rate than the whole itself progresses I am thinking of the case in which prime or square numbers are taken as a part of the series of natural numbers This answers well enough so long as you have an enumeration *ad infinitum* to draw upon But it seems to me that if you had anything approaching a given totality this spending in advance must produce a contradiction at once At any point of the consideration, the part is miles and miles outside the whole. While you appeal to infinity as incomplete, this cannot matter But the moment you seek to apply the character of totality, the arrangement seems to split in two It is one thing, I repeat, to have a definition which if fulfilled would involve a given infinity, and another thing to show that it can be and actually is fulfilled

Demand of Logic

(3) I believe Mr Russell would agree with me that fundamental principles of Logic could not be overriden by the practice of mathematics If they really conflict, he would admit, some condition has been tacitly imposed [a] in the transition to mathematical conclusions, though as conclusions in mathematics they may be perfectly sound But he holds that they do not conflict. His view is that a true Logic is one with the principles of mathematics, and carries the conclusions of the latter without reserve I cannot see my way to escape from the former conviction. We may be given something else of great interest and value, but unless it fulfils our simple requirements, we cannot accept it as an actual infinity On the other hand, we can show an actual infinity, in one

[a] On the nature of mathematical abstraction see pp 182–3 below

sense of the word, in the character of wholes with parts such as to embody and reproduce the spirit of the whole. Such parts, in some degree, are the organs of an organism or the institutions of a nation And in some degree, by what seems to me like special pleading, the character of such wholes is discovered in the mathematical concept of self-representative series [a] But I cannot think that the two conceptions can really be united, except in the sense that it is probably true that a real infinite, or self-contained individual whole, could not be numerically represented except by an infinite series, which I take to be the same as saying it could not be numerically represented at all

Infinite number in general.

η I will now continue the discussion from which this was a digression

More subtle and interesting are cases in which the continuance of the series makes a difference in the task to be performed, but the whole possible difference can be shown to fall within certain known limits These cases, which I presume to be of the nature of infinitesimals rather than of infinites (both of which must fall under the head of infinite *enumeration*), may be reduced to the same class as the former if we reflect that in the former also the difference fell within known limits, but these were limits of *kind*, whereas we are now speaking of limits of *quantity* In cases of this second type we know that a series could be continued to infinity, and that some difference would be made to the problem before us by this being done , but, on the other hand, we are aware of a limit within which the whole series must fall, and we are therefore able to pronounce that the difference which can be made by its continuance after a certain point is, at least for our immediate purpose, a negligible quantity. Such, I take it, must be the principle of any process which determines e.g the area of a circle by treating it as between the area of a polygon inscribed in the circle and that of a polygon described about it. So far as I follow the exoteric utterances of mathematicians,[1] the principle of abstraction based on a positive knowledge of

[a] A E Taylor, Metaphysics, p 116 ff

[1] I have in mind more especially Mr Spottiswoode's Presidential Address to the British Association, which unluckily I cannot refer to.

the capacities of a series must be at the root of the employment of mathematical infinity

But our immediate business is with infinite number considered logically, i e with a view to its general place in knowledge And from this point of view we have to notice—

(1) That the idea of infinite number has its genesis in a one-sided abstraction, viz in the notion of counting without having anything in particular to count,[1] which corresponds to the idea of difference without identity, and of parts without a whole By such an abstraction the enumerative judgment is destroyed, the essence of judgment as such—the exhibition of identity in difference or of the whole in its parts—being withdrawn from it; and the names of the numbers are turned into a meaningless repetition, the purpose of enumeration having disappeared We are no longer saying 'One tree, two trees, three trees', &c, but we are merely saying 'One, two, three', and it is for this reason that we need never stop

(2) Being one-sided, the idea of infinite number is self-contradictory The essence of number is to construct a finite whole out of homogeneous units The idea of numerical infinity arises from neglecting the continuous nature of the unit, and therefore omitting the element which alone arrests computation at one number rather than at another Thus an infinite number would be a number which is no particular number, for every particular number is finite

(3) It follows from this that infinite number is unreal This does not mean that there may not actually be more units of one kind or another than we can count in the time at our disposal or in any finite lapse of time The statement deals with the nature of number, not with the extent of the sensuous universe If, to put a common idea coarsely, we are asked, 'Supposing that you *could* travel through space for ever, and never come to an end of it, must not space contain an infinite number of units? or even if you can go on subdividing a given portion of matter for ever, must it not contain an infinite number of parts?' to such questions we could

[1] A series which is *ex hypothesi* infinite comes under the head even if it seems to have a positive nature For its nature *ex hypothesi* does not determine the number of its units

only reply, 'Things or the parts of things may quite conceivably transcend our power to count. But except in view of a finite goal, number does not help us, does not tell us anything, grounds no ratio of parts to whole. We should in fact never give up counting any units that had interest for us, and should in doing so always be at some finite number But if it could be miraculously revealed to us that there was no end, then I think we should stop counting, unless the units in question entered into subordinate or graduated totals which had an interest for us Thus we go on counting the stars for definite reasons Their relation to us is graduated, and several subordinate totals within their number have already been completed by enumeration, e g stars of the first eight magnitudes have been identified and counted. In counting them we have always in view some definable total to be constructed or to be corrected Who counts the waves of the sea? The hope of complete enumeration is the justification of counting'

Then why do we count the years and centuries? Do we pretend to know that they will have a numerable sum? and when they have reached it, do we imagine that the race will survive to take an interest in the completed enumeration? In the first place, for each of us time seems to have an end, and in the second place, all history is parcelled out in overlapping epochs which we have an interest in measuring We do not in fact ordinarily know or compute the whole numerable series of years that has elapsed since the first events of ascertainable date in history, we adopt this or that era according to some overpowering historical interest which makes it seem to mark a fresh beginning We do not count the years to ascertain their total quantity, but to give them names by which we can fix events; and as a means to fixing the relations of events we no doubt desire to note the quantitative relations within the total of historical time which has elapsed down to any given present If we are pressed further, and told, 'But, after all, the years may go on for ever and the human race may go on counting them for ever', we can only reply that the faculties with which we are endowed refuse to express this 'ever', that at any point taken in the series we should

be at some finite number, and that if a conviction of the endlessness of the series could be miraculously impressed upon our minds we could only conclude that, except as a record of the past, counting the years was an unmeaning form, seeing that the nature of the series could not be represented in number. A very simple case of enumeration *ad infinitum* would be that in which, by persistent errors of identification, we should count the same objects over again, round and round, without being aware that we were doing so In such a case it is obvious that the conception of number would be destroyed so far as these objects themselves were concerned, though if they were at known intervals of space we might none the less use them as a measure for other things When we measure with a foot-rule we do in fact count the inches marked on it over and over again in this way, not for their own sake or the process would be infinite, but only for the sake of some other quantity which we characterise by them In this respect the inch-marks on the foot-rule correspond to the physical changes which indicate the day and year, and which serve as a measure for occurrences other than themselves

An infinite series, then, is not anything which we can represent in the form of number, and therefore cannot be, *qua* infinite series, a fact in our world Relations may indeed be given as actual which only an infinite series could represent on their quantitative side, such as the ratio of the diameter to the circumference of a circle But for this very reason they never are adequately represented on that side, although we may know and argue from the positive character of the series, which *ex hypothesi* its prolongation to infinity is not to change Our constructive judgment requires parts *and* a whole to give it meaning Parts unrelated to any whole cannot be judged real by our thought Their significance is gone, and they are parts of nothing

Thus it is nonsense to speak of any definite number, say 100, as a portion of number, in the sense in which a foot is a portion of a yard, or a minute of an hour. The question 'what portion'? at once disposes of any such relation Number as such cannot be identified with any particular total such that a given number is a definite fraction of it.

Closely allied to infinite number, and in a great measure depending upon it, are the conceptions of abstract or ideal time, and of abstract or ideal space, tending respectively to generate ideas of infinite time and of infinite space.

θ. In speaking of Comparison we saw that every 'now' tends to become a part within 'now' and 'then', and every 'then' again within 'then' and 'then'. This analysis is very gradually brought about, speaking historically, in the evolution of the tense-system. The sense of Time is in the first instance the mere consciousness of continuity in succession, that is, the mere perception of *a* succession or process of change This sense however being only possible through setting off the succession against a comparatively permanent background of consciousness, is in embryo the comparison of successions, with the development of which comparison measurement of Time, and with this the abstract idea of Time, are brought into existence The measurement of Time consists in the equation of one set of perceptible changes identified by a common nature, to another set of perceptible changes, in the sense that the beginning and end of a numbered series of the one coincide with the beginning and end of a numbered series of the other The enumeration of phases of one series that coincide with one or more phases of the other series might conceivably be undertaken apart from any belief that either series has a constant duration in time, i e if repeated, would occupy the same amount of duration as before; but in enquiring whether such a belief actually exists we must distinguish between the reasonable doubt whether any portion of any series ever can or will be repeated absolutely without physical modifications which may affect its duration, and the unmeaning doubt whether a series assumed to be repeated without physical or causal variation may not nevertheless have varied in the absolute amount of duration which it occupied. The former kind of doubt will only lead to a demand for criticism and reciprocal adjustment of our time-measures, together with the temperate scepticism which our lack of exhaustive knowledge must produce about our acquaintance with even those natural conditions which we have most fully investigated. The latter kind of doubt, if pressed to its con-

Abstract and Infinite Time.

clusion, would reduce the enumeration of successive changes to a statement of mere numerical fact wholly devoid of significance Such enumeration would not be impossible, but would scarcely fulfil the requirements of judgment It would be on a level with the mere repetition of the names of numbers It is hardly necessary to give instances; every one can see at once that if we entertain the idea of variation in the measures of time occurring without any reason, all equation of successions becomes futile, and ceases to afford any ground of expectation or of inference.

But though a doubt of this nature has been mentioned by great writers, yet it has never been extended to affect the only element of our time-perception which is essential to its utility, viz the constant ratio obtaining between the successions employed as measures of duration This limitation of the doubt arises from the confusion in which it originates, the confusion which consists in treating the whole as if it were a part Time, for us, is a relation, a ratio, and its constancy is the constancy of this ratio If all processes in time maintain a constant ratio to one another so far as they are unaffected by physical modifications, then there is no meaning in suggesting that tried by some unknown or impossible standard[1] they may be variable But yet this suggestion is a natural suggestion arising out of a natural confusion. We naturally frame an idea of duration in itself, as that which has successive parts really and absolutely equal, because equality of successive parts *in terms of one of our time-measures* is what we are always endeavouring to ascertain about some other time-measure And we forget that equality, if we exclude from it the idea of measurement, ceases to be an intelligible conception This ideal of duration then, whose successive parts, though *ex hypothesi* not measurable, are assumed to be equal, we actually set up as an imaginary measure against the totality of consenting measures and processes the ratio of which to one another forms our world of time, and is merely represented in an abstract and ideal form by 'duration in itself'. 'Duration in itself', says

[1] E g by measurement of successive phases against each other, which is *ex hypothesi* impossible

Locke,[1] 'is to be considered as going on in one constant, equal, uniform course. But none of the measures which we make use of can be known to do so, nor can it be assumed that their assigned parts or periods are equal in duration one to another'.

But 'duration in itself' not being a relation of successions, could not be perceived as time, nor could any being that perceives as we perceive be aware of equality or the opposite in its successive parts, for equation is necessary to equality. If, on the other hand, we set it against our time-measures, then it becomes one measure among many, and in case of a discrepancy that should remain unaccountable it is only caprice that could choose which we should regard as right. The question in fact would be unmeaning, for the whole discussion obviously originates in the attempt to transfer an attribute which depends on a comparison to a set of terms considered in themselves and apart from comparison, and then to suggest a comparison between them (the supposed equal parts of ideal duration) and the totality of our time-measures of whose reciprocal relations those parts are an idealised form.

But this confusion does not naturally originate a doubt of the constant ratio, physical disturbances being allowed for, of our measures of time *compared with one another*. For it is this constant ratio from which the idea of duration as such, hypostasised by abstraction into duration in itself with equal successive parts, is derived, and to doubt the constancy of this ratio would be to deprive ourselves of that idea of duration on which the confusion itself depends. If oscillations of a pendulum of fixed length, such as are normally equal when measured by the rotation of the earth, may vary without a physical cause affecting either of the compared motions, the conception of uniform duration is destroyed, and no equation of successions is more than an isolated fact. On such a hypothesis there would be nothing to generate the idea of uniform duration, and no measure of time with which to compare that idea.

The antithesis between duration in itself and our measures of it is merely a case of the antithesis between the thing-in-itself and our knowledge of it. It is possible, though un-

[1] Essay, Book II, 14, 21.

meaning, to doubt whether our knowledge as a whole is real knowledge—i e corresponds to some test or condition which we may imagine as imposed upon it from without The possibility arises from our possession of an ideal of knowledge, which by an act of abstraction can be set in antithesis to the actual whole of knowledge from which it is abstracted. But it is impossible when operating in detail upon the matter of experience to doubt the existence of rational connection in any one particular group of appearances; for by so doing we paralyse the understanding, which can only act in the search for causes, and leave neither knowledge to condemn nor an ideal by which to condemn it. We shall have to return to this subject in a later chapter, when we speak of the postulates of knowledge.

We have just seen that it is idle to treat the whole as if it were a part—our reciprocally adjusted measures of time as if they were one measure among many It is also worth while to observe in the same instance the impossibility of making a part do duty for the whole, which is strikingly exemplified in the ultimate data of time No process of perceptible change is a trustworthy measure of time except in as far as it is equated with other processes of known constancy, and observed to be in itself free from physical causes of variation That is to say, the part can only be taken as a definite standard when it has been criticised in the light of the whole This is true of all premises of knowledge

Amount of Time, like all quantity, is measured by the enumeration of units which have a known value But, unlike any other kind of quantity with the exception of space, it follows number not only into mediate counting (all quantity does this), but into enumeration *ad infinitum*

Simple enumeration in the case of Time gives such judgments as 'He died three years ago', 'It is seven days since I saw him',—which expand by reference to a standard of succession the mere indications—probably of various nature and origin [1]—conveyed by the tenses

[1] Tense need not have originated in the idea of succession at all, and may often have arisen out of the expression of emotions or anticipations or out of the mere negation of presence as a perception of a certain kind

Mediate or complex counting in Time gives such judgments as the equation '365 days = one year' Such judgments deal with our real and ideal time as the result gained by comparison and equation of actual processes in experience But the abstraction which stands as subject tends to assert itself as a something apart from the actual processes whose relation it is, and thus as we have seen generates the conception of 'duration in itself' or 'absolute time', which again having lost the element of totality precipitates us into the idea of infinite time.

Enumeration to infinity, when applied to the parts of time, has characteristics analogous to those of infinite number, but more striking, inasmuch as time is closely bound up with the attributes of actual existence Infinite time is, like infinite number, an *unending whole*, which is a contradiction in terms That is to say, we are prevented by the nature of our minds, if by no other cause, from attaching any meaning to infinite time as a quantitative expression indicating an aggregate expressible by enumerative judgments. Whether the problem (for so we must consider it) which is put before us in this form is capable of becoming, not a problem, but a fact, in some other form, is a question which does not concern us here The instances which are alleged to show that an infinite series may be *given* point somewhat in this direction, I allude to such an instance as that of the relation between diameter and circumference However this may be, it remains true that infinite time, *as a mere quantity of duration*, is a phantom generated by a meaningless abstraction.

But, as we asked whether there may not *be* endless numerable units, so we may be challenged to ask, May there not or must there not have been an endless chain of actual occurrences in succession? And if we are prepared to deny reality to every endless series, must we not first of all deny it to the actual world as in time, and we may add by anticipation of the next section, as in space? There can be no doubt that the relativity of events and appearances in time and space does involve for our minds an infinite progression in the way of referring one thing or appearance to another as its cause or explanation, or at least as in some way its determinant.

We can hardly conceive that we are really counting in a circle, but our position is just as hopeless as if we were Our ideas of reference, determination, causation, do not allow us to fancy a first event, a beginning of time, or a limit of space On the other hand, so far as we can understand, it is impossible for such a succession as we postulate to be actual in the sense in which a known section of history is actual It is not merely something more than we do construct as a whole ; it is something the essence of which is to be incapable of construction as a whole We may say if we like that we are bound to think of such a succession as actual in the sense that it is a problem inseparable from the relativity of our world , but we cannot take the endless series *qua* endless as a positive element in our organised experience

In dealing however with the succession of actual events having positive character and content we are in spite of their serial appearance on different ground from that of abstract time or equable succession as such Actual events—the history of our world—have more in them than a mere series. We may say either that the world in time and space is not mere succession and externality, or that the real world which intelligent perception presents to us is not merely a world in time and space The human mind and will are always busy in turning a series into a coherent and almost individual whole, projected more or less definitely on a scene of time and space, but not exhausted in its meaning by the nature of that background Greece for instance or England are not mere 'geographical expressions' ; and if they were they would still have a physical unity of a deeper kind than the juxtaposition of extended units or the sequence of a series They are historical realities, but their coherence lies in their meaning Therefore in denying that for us, in our way of understanding, an endless progress can be a real and controlling factor of organised experience, we do not deny the reality of the phenomenal world *as presented to intelligent perception*

Abstract and Infinite Space

c. The nature of Space is in many respects analogous to that of time, and bears on the whole a similar relation to the system of number, with its degrees of simple or categorical

counting, mediate or complex counting, and counting *ad infinitum* The corresponding grades of abstraction in dealing with space may be identified as—

a The measurement of actual distances

b The theoretical relations of spatial qualities, including the whole of the mathematical sciences, excepting what may be included in the mere system of number, viz all forms of simple and generalised (algebraical) arithmetic, and

c The conception of infinite space

In defence of the subdivision here adopted, I venture with a good deal of diffidence to suggest that the idea of infinite space is not an idea belonging to geometrical science. The employment of the idea of infinity in geometrical reasoning belongs, if the account above given of it is correct, to the second of the above heads, being really an abstract mode of describing a geometrical whole If enough space is given to make manifest the positive nature of the whole before us, it would seem that no addition can really affect the case. Quantitative infinity may be a roundabout description of a direction or a quality That a certain straight line is infinite may only mean that its direction is such that it can never meet a certain other straight line.

Measurement of actual distances.

a The measurement of actual distances is *prima facie* a case of simple measurement, and must obviously arise at the point where positions distinguished by the comparative judgment are discovered to have relations of distance and direction reducible to degree Degrees of distance from assigned points, and proportions of such degrees, considered as properties of objects, pass into the determination of concrete individuals and of their characteristic attributes In this respect we have considered them above. But also, receiving their significance from a system of equations by which all spatial magnitudes are brought to a common denomination, they contribute to reflection upon space in the abstract, a reflection which is developed by the process of enumeration applied to the parts of space when thus idealised and considered for their own sake The system of measures, as we have seen above, is a connecting link between simple and complex measurement It is *prima facie* a system

of ratios, and requires an arbitrary starting-point to give it meaning But when taken as general and typical it passes into a system of proportions, as we see in such a judgment as 'This map is on the scale of twelve inches to the mile' There need not be twelve inches in any direction in the actual map before us, the phrase expresses a proportion, not a simple fact of enumeration It is not necessary to know, for the purpose of the scale, how much absolutely (i e in the mass of other relations) an inch or a mile is; we can interpret the scale if we know how many inches there are in a mile. The abstract numerical expression 1 63360, which is a *proportion* as governing the relation between every part of the map and the corresponding part of reality, represents what the formula 'one inch to the mile' really means to say We may compare this case with the financial expression 'five per cent'; some newspapers *will* print this as £5 per cent, which is a confusion between singular or merely actual, and generalised or proportional ratio They do not really mean that £5 need be concerned, their predication is as true if the interest in question only amounts to five shillings The expression five per cent is simply a fractional or proportional expression

Now, when, e. g in the two instances just quoted, the reference to an arbitrary magnitude is dropped out, and when moreover the generalised equation is taken as expressing the relations between distances in space combined in a certain way, then we have ideal or mediate enumeration as it exists in geometrical science 'Mediate' because the proportion contained in the equation is affirmed of reality as qualified by definite spatial attributes, which therefore enter into the content of the equation as interrelated conditions The equation of an ellipse is a hypothetical judgment asserting that the axes of an ellipse (or other quantities involved in an ellipse) being treated in a certain way will be always equal to themselves treated in a certain other way. It is obvious that being mediate this geometrical enumeration or computation is also complex,[1] for the conditions by which it is mediated may involve units of any degree of numerical com-

[1] See pp 160–1 above

plexity. Thus judgments dealing with squares and cubes, still more with conic sections, presuppose, *on their numerical side*, more or less elaborate enumerative processes as involved in the formation or apprehension of the spatial unit

b The generalised relations of spatial attributes form a systematic science with a distinctive object-matter Geometry

(1) 'Any two sides of a triangle are together greater than the third side' 'Triangles upon equal bases and between the same parallels are equal to each other' 'The square of the hypotenuse is equal to the squares of the containing sides' 'The angles at the base of an isosceles triangle are equal to one another' Individuality of Spatial Figures

Such judgments as these are among the simplest results of mediate enumeration [1] as applied to space, and they present an obvious peculiarity shared with them in a high degree by complex judgments of mere number, and in a less degree by judgments that compute time. Although geometrical science proper consists, as a science, exclusively of the equation with each other of variously described spatial units, which, if we further consider the sciences of motion, we must take as referred to units of time, yet under this 'variously described' there lurks a whole classificatory science of forms possessed of structural unity and quasi-organic relations between part and whole Thus in the definitions and definitory judgments of geometry, as in the inductive judgment of enumeration, there is a revival of that structural subordination of part to whole which, though dormant, yet is never dead so long as judgment has a meaning The triangle, the square, the circle, the ellipse, though each of them capable of being exhaustively defined by generalised enumerative processes dealing with homogeneous units, have also an aspect of structural unity and subordination of parts to the dominant quality of the whole Both clauses of this statement are however subject to reservation.

In the first clause, the expression 'homogeneous units' is not entirely adequate to the facts. Distances in the three spatial dimensions are indeed as distances homogeneous,

[1] For the further distinction between arithmetical and geometrical reasoning, see II, chap. II.

but it can hardly be said that an angle is homogeneous with a distance or direction, although a proportion of distances is an exhaustive measure of an angle Therefore, though an angle may be represented in terms of distance, yet in considering the elements in the spatial structure of a square or triangle we must assume the angle as well as the straight line The apprehension of converging direction is hardly given in the mere apprehension of direction.

In the same clause we must qualify the assertion that *all* geometrical shapes can be exhaustively defined by equation of quantities. I presume that the impossibility of squaring the circle implies a difference of kind between circle and straight line which is disguised but not destroyed by the efficient methods employed to express the one in terms of the other.[1] Here again, in the adequacy, as distinguished from exhaustiveness, with which quantitative equations represent characteristics of kind not wholly reducible to quantity, we find an analogy with the various systems of necessity which are superimposed upon one another in the organic and moral worlds

And the truth of our second clause would be a good deal interfered with, if not annihilated, by enlarging our list of geometrical figures, and by regarding every figure in all its possible variations As to the first point, every spatial figure is a geometrical figure, and those which we mentioned, together with others that occur to the mind at once as commonly considered in plane and in solid geometry, have no real prerogative of existence to the exclusion of trapeziums or any other irregular figures, even if partly curved and partly rectilinear in outline. The world would be easier to explain than it is (or so it seems to a superficial glance) if irregularity, ugliness, and disease did not, as they do, exist by law and necessity[2] just as much as symmetry, beauty, and health It may be said in defence of treating the regular figures as

[1] The quadrature of the parabola shows that the impossibility of squaring the circle does not arise from the mere difference between curve and right line

[2] I do not say by the same necessity, I only mean that they result mechanically from natural conditions Any limitation of which this statement may be capable must refer to a distinction of kinds of necessity.

if there were no others, that all others are reducible to them, —all rectilineal figures to triangles, and so on But this is a mere ideal reduction by measurement, and in no sense a deduction of the existence of the one from that of the other

And again, most 'figures' of which we speak are really classes of figures, even if we take all 'similar' figures as the same, i e disregard size and only look to characteristic proportion I do not know why we should not take the conic section as 'a figure' and treat the circle simply as a case of it. Anyhow the ellipse comprises in itself a whole class of figures which are qualitatively quite different from each other, and pass by imperceptible gradations into figures which are not elliptical By insisting on facts like these we might melt away the individuality of typical figures, and exhibit every group of geometrical shapes as destitute of common characteristics beyond those which flow from the mere genetic relation itself Thus, for instance, an ellipse when just passing into a circle or into a straight line has none of the characteristics which we associate with elliptical form, although the analytic relation by which its nature is theoretically determined may be within the limits which must as a matter of geometrical classification be assigned to that figure. Such a treatment would be the triumph of explanatory theory and necessary connection over individuality and characteristic quality. But it is a treatment of which, in geometry, the facts admit, and which even in the organic world is rendered indispensable by the idea of continuous evolution

It is right that attention should be drawn to the above reservation No habit is more pernicious than that of assuming what is obvious and familiar in a certain sphere to exhaust the contents of that sphere. Every figure is geometrical; and even if all irregular figures can be reduced for purposes of reckoning to the more regular types, this does not justify us in speaking theoretically as though there were no figures in space besides those which have been selected as typical by geometricians.

We must meet such conceptions now as we shall have to meet them again, perhaps more than once, in the evolution of the judgment, by the reflection that one positive existence

cannot, by the mere fact of its existence, cancel another There may be non-elliptical ellipses, figures which fulfil one set of conditions that mark the ellipse, but lack attributes without which we hardly recognise the figure , and there may in the same sense be or have been (time makes no difference) non-human human beings. And explanatory theory may be able in both cases to trace step by step without *saltus* or miracle the transition from the one phase into the other by development of elements fundamental in both But this will not obscure our perception of the elliptical shape of a characteristic ellipse, or of the humanity of a typical human being. The typical human being is made typical, as we shall see, by real teleology. The typical ellipse is only typical through a subjective quasi-teleology This is the difference between the two cases

Thus, in spite of the above reservations, it remains true that in complex enumeration as applied to Space the antithesis of individuality and necessity is strikingly illustrated. A generalised relation of distances, not obviously differing in kind from any other generalised enumerative relation, when interpreted into an actual figure, *may* at least produce a structural totality complete in itself and of a marked individual character. It is enough to mention such simple instances as the equilateral triangle, the square, and the circle. These totalities do not refer outside themselves for definition in the obvious sense in which simple magnitudes are forced to do so They are primarily cases of internal proportion, of proportion, that is, between one and another element of a single totality. Relativity appears in them in another shape than that of simple equation with absolute (in the sense of arbitrary) standards of magnitude. A particular triangle or ellipse is relative and finite chiefly in the sense of being derived from arbitrary conditions, no one value of which has any prerogative of existence as against any other, and an extreme modification of which will always destroy the essence of the figure. There is also a further sense in which all natural existences are finite as compared with mind, because they cannot refer to themselves, but only *are referred* whether to themselves or to external conditions

The subordinate totalities of Time, such as the hour, day, and year, are not really cases of the same principle, for they are mere aggregates of units without a limiting totality, and are simply formed by arresting enumeration at intervals prescribed by external interests A better parallel may be found in the numbers of the numerical series which are distinguished by any peculiarity, e g prime numbers, squares, cubes, &c, if we consider these peculiar cases in their relation to the one unvarying process of adding together homogeneous units This simple synthesis of number may be regarded as analogous to the mechanical or necessary aspect of an individual thing or figure, and its peculiarity as square or cube to structural totality.

Existence of Figures in Space.

(2) Figures in space then, and numerical relations, although abstract and ideal, and arbitrarily selected out of continuous sets of value of which no one has existence by preference to the other, yet seem capable in a sense of possessing characteristic quality and self-sufficing totality. What is to be thought of their *existence*? Do such judgments as are mentioned on p 177 involve the assertion that the qualifications of Reality which form their subjects are actual, and if so, in what sense are they actual? The difficulty of the question is only displayed in its true extent if we add to the above instances some ideas for which actual existence, as diagrams in books or as thoughts in the minds of individual students, cannot with probability be claimed; e. g the idea of a polygon with a thousand and one equal sides, of any trapezium chosen at will, or of any irregular figure or high number.

It is clear that the square or triangle *qua* spatial figure has no actuality which does not equally belong to all such less familiar shapes, and therefore mere presence in the individual mind is not the existence in question And indeed to speak of it as such would be to enter upon a vicious circle which would stultify the judgment, for it is essential to the judgment to affirm a reality outside itself, and it would be too ridiculous that your judgment should refer merely to the content of mine as the reality asserted, and mine in turn to that of yours It is obvious that the two manifestations of

the thought-function are on a level, and if each refers to the other, each might just as well refer to itself, i e find its truth in the simple fact that it is made [1] A triangle must be just as real [a] when no one is thinking about it as when many students are engaged upon the conception of one This is of course not the same as to say that spatial figures do not depend on the spatial consciousness, or on consciousness at all. We are only saying that they cannot depend for actuality on one particular reflective consciousness of those particular figures. The world as it is for perception and intelligence is the object-matter of our whole enquiry, and we have no occasion to raise a question that assumes the destruction of the object which we are considering

The absence of material existence and also of any mode whatever of particularisation in determinate forms (the selection e g of a typical ellipse not being justifiable on purely geometrical grounds) must however make a distinction between the actuality of the contents under discussion and that of material things or their sensible properties Admitting, as we must admit for logical purposes, that Space is to be reckoned with as having a peculiar actuality of its own, still it is not clear in what relation geometrical figures, apart from the shapes of actual objects, stand to actual space Geometrical figures as such, the subjects of judgment in geometrical science, are not the shapes of actual objects, they are not identical with any perceptible figure; they are not distributed through space nor present as special characteristics in any portion of it I do not know how to describe them better than as a peculiar class of *laws* or *attributes* of the spatial relation as such, which are concrete in the nature of their content, though abstract in their medium of presentation. And in the same way the characteristic totalities of number must be taken as laws—embodied laws—of the enumerative relation of part and whole

[1] Reference to the world of meanings or objective reference (Introduction, sect 7) is not merely reference to judgments in fact made by others It is an inadequately conditioned reference to reality

[a] This is overstated Every thought, we must suppose, contributes something, however practically inappreciable, to the reality of what it thinks of

We may illustrate the nature of such attributes by comparing them with any purely imaginary ideal content which bears (so far as an imaginary content can, for it is always, in my opinion, found wanting somewhere) the character of self-sufficing totality. Such an ideal content for instance is Shakespeare's Hamlet,[1] or the material spheres of ancient astronomy, or, to come nearer to our present subject, the conception of 'Flatland' as space in two dimensions only, with sentient beings confined to it. These conceptions, though doubtless based on elements of fact and illustrative of real conditions, yet exist only in the minds of those who read and think about them, or more strictly in the identical reference which these minds are stimulated to make to a world of meanings, but a world of meanings *explicitly* discontinuous with and detached from the actual world of fact, or what we call in short an imaginary world Such a world is indeed maintained by judgment, but it is judgment of a peculiar kind and under peculiar conditions—subject, not like the common world of meanings to a wholly indeterminate, but to a *conscious* and explicit, abstraction from reality, which becomes semi-conscious in artistic fiction, and utterly lost and obliterated in mere error and superstition

The figures which represent the properties of actual space are not imaginary in this extreme sense of the word. It might however be a question whether the difference between them and such ideas as have been mentioned is one of kind or one of degree. Both kinds of ideas it might be said involve abstraction from concrete perceived reality, both kinds are therefore hypothetical and not actual existences, and how far the abstraction is carried cannot be a question of principle Nor, I must add, do I mean to insist on the manifest *contradiction* with experience, or self-contradiction when viewed in the light of experience, that some of my instances of imaginary ideas may be held to present. The distinction which I desire to draw is simply between abstract but real, and purely imaginary contents, when employed as subjects in judgment.

[1] Omitting the considerations which arise out of the artistic truth of the conception, and taking it merely as an illustration of an imaginary idea

Perhaps the distinction might be found as above suggested to consist formally in the *nature* of the abstraction to which the two kinds of contents are severally subject A merely abstract content is subject only to the abstraction which its ostensible nature implies The name of a figure in space is the name of a figure in space and not the name of a man or a mineral or of any material object But the name of Hamlet for instance is and yet is not the name of a man The name of a knot tied on an endless string is and yet is not the name of a reality in space These imaginations are subject not merely to the abstraction which separates every content from all that is not included in it, but to a further abstraction which says, 'This is *a* meaning, but not *the* meaning you would take it for ' in other words, it is conditional within a world which itself can only be predicated conditionally and not directly of the reality with which we are in contact by means of perception.

Thus we may say, if we choose, that our ideas of actual space have for their meaning only possibilities, but these are at least *real* possibilities, that is to say, their fundamental generating relations actually exist in the world which centres in present perception They are therefore as real as colour in the dark or as sweetness which we do not taste And if we pronounce these attributes unreal outside the moment of perception, we have laid the axe to the root of the perceptible world We might as well say that the wall in front of me is actual and that behind me is not But what corresponds with these ideas to the actuality which colour and taste do partake of when perceived? Here we find a real difference Such actuality in sensuous presentation they cannot have But we do not of course admit that Reality is restricted to sensuous presentation All we can say is that in all relations of actual things these spatial attributes make themselves evident as controlling conditions, and are introduced as conceptions without which the mind fails to construe the phenomena. They are abstract characteristics of the actual spatial relations of things, and are as much a fact for logic as any secondary quality, *qua* general quality, and they are not on a level with mere imaginations and fictions, even if

these are consistent, or not notoriously inconsistent, with reality.

(3) The judgments mentioned on p 177 are thus found to display in their own line of abstract evolution which we are now pursuing a character analogous *mutatis mutandis* to that which the Generic Judgment will be found to display in the concrete evolution of thought The subject of each such judgment is Reality qualified as a structural whole which embodies properties rooted in an *actual* relation and controlling the consequences of that relation at every turn The figures in question do not claim sensuous particularity and are not capable of it The judgment therefore is a degree less generic and more hypothetical than classificatory judgments which retain much of the meaning of the collective judgment The quasi-generic judgment

Yet the judgment in question *is* generic In the first place, the particular figures which arise if particular conditions are assumed are individual totalities, not indeed having sensuous singleness, but self-identical as laws of space And in the second place, although no doubt any series of such figures (triangles, ellipses, &c) is in one point of view an infinite series[a] (the transition from value to value of the generating conditions being absolutely continuous), yet the whole falls within known limits, and is bound together by a characteristic quality which might probably be found to vary with the variations of the generating factors[1]

Therefore a judgment like this presupposes, *not* as do the judgment of zoology a limited even if very large number of actual individuals forming a real historical unity though spoken of mediately and by help of an abstract qualification of Reality, but a series of laws regulative of form, or rather

[a] 'Then there are infinite series,' a critic may say The case illustrates my point If you could make actual all the cases of a continuous series, it would be a given infinite series But I presume you cannot

[1] It seems obvious that as one generating factor, e g one axis of an ellipse, approaches disappearance, we should expect the characteristic quality to diminish But any such conception is not easy to carry out. E g if we take equality of the axes as the characteristic point in a figure of the conic section class, we get the circle as the characteristic type to the exclusion of the ellipse The question has perhaps only an aesthetic interest

a law expressed in a series of forms, having positive common characteristics and bounded by definable limits within which the whole series must fall. 'All triangles' is hindered from meaning every particular triangle (i e variety of triangle—I abstract from particular sensuous presentations all through this discussion) not only as 'all men' is hindered from meaning every particular man, by the *practical* impossibility of dealing with such a meaning, but also by a theoretical absurdity, for all particular triangles would be an infinite series, which need not theoretically be the case with all particular men. But, as we saw above, a series which has a known positive character or falls within known limits may be treated as an actual unity in spite of its infinity 'All triangles have their angles equal to two right angles' is a judgment about what is really a single continuous relation, but embodied to the mind's eye in certain salient types resulting from geometrical classification It resembles the generic judgment more strikingly than appears at first sight, for the generic judgment too deals with a section of evolution in which a vast though not infinite array of transitional types has really bridged the gulf between the marked species which are familiar.

Infinite space

c. Infinite enumeration applied to the parts of space is the last result of abstraction in this region. Here, as in time and number, we have the idea of the absolutely homogeneous part, i e the part whose repetition has no tendency to generate a whole The idea of infinite space is the idea of the endless synthesis of such parts, which must always present to us the appearance of an unsolved problem If the problem has a solution, it must consist in changing the point of view from which we regard it, as if, to repeat an illustration which I employed above, we were suddenly to awake to the fact that we had been counting parts extended in a circle and not in a straight line

Will not the doctrine known as the subjectivity of space and time help us to explain the nature of this contradictory reality? Up to a certain point it has undoubtedly done good service by showing that the difficulties which attach to sensuous reality are rooted in the nature of the percipient intelligence itself, and must be reckoned with as inherent in

sensuous experience But I am unable to see that the 'subjectivity' of these forms of apprehension can carry us further, unless or until we are enabled to put something better in their place. At present we seem only to have learnt that the difficulties of knowledge are not external to it, but are inbred and inevitable, at least so far as concerns the series of sensuous phenomena But we gain nothing, so far as I can understand, by attempting to erect a world beyond as a non-sensuous counterpart of the sensuous series If a counterpart, then it would seem to share the difficulties attaching to this series, while as non-sensuous it lacks the compulsory reality of sense-perception[1] Our present knowledge rather points to the conclusion that *if* we are to attain something less contradictory, more capable of self-sufficing reality, or if we like to use the phrase, more above sense, we must look for it in facts and purposes which deepen the significance of life, not in a shadowy counterpart which repeats the world of sense without enhancing its value Mere series, mere space and time, we must always remark are mere abstractions; and though no human knowledge is free from relativity, i e from the reference to what falls outside it, yet on the other hand no actual human knowledge is, like the abstract infinities, mere relativity and nothing more There would be some justification for saying that, as *contrasted with the concrete structure of individual things*, a 'subjective', i e artificial and unreal, character might be attributed to number, space and time as infinite wholes, on the ground that they conflict with the nature of actual fact however comprehensive, and that the extension to infinity deprives them of relation to the phenomena in which they are known to us But the distinction is hardly sound, for it is at best one of degree The difficulties of relativity do not wholly cease as soon as we turn away from the abstract infinity of mere number, time, and space; in common ideas of causation they affect the actual content of phenomena From the very beginning of knowledge, as I have tried to point out, Absoluteness co-exists with Rela-

[1] Lotze has well brought out the difficulties attaching to the conception of an 'intelligible' counterpart to the world of sense The strange thing is that they do not appear in any way to make him discontented with that conception

tivity, but it is impossible to form categorical judgments of a comprehensive type until the idea of causation has been freed from its primary implication of an endless series.

The question might be asked, why are we exclusively tempted to demand the reality of infinite time and infinite space when there are other abstract conceptions of homogeneous parts not subordinate to any whole, which might in the same way be pushed to infinity? In the first place, it may be replied that abstractions of this class are not so common as might appear. The essence of them is that progressive enumeration shall not tend to modify their character Thus intensive quantities, such as infinite force, infinite velocity, are conceptions of a heightening the later stages of which would modify the earlier and not remain indifferently beside them Therefore although the phenomena of velocity or of force do suggest the idea of quantitative infinities of those kinds, yet they do not impel us to judge those infinities to be real, because the perceived forces do not in their nature refer to and presuppose infinite degrees of themselves, but rather each manifestation *per se* excludes the infinity which would involve a qualitative change in itself Infinite force or velocity is as contradictory an idea as infinite space, but is not in the same sense a problem or a paradox, because it does not in the same sense claim reality

And secondly, Space and Time may be called the Categories of sense. That is to say, they are the only principles according to which the world of sense-perception, both of our own immediate feelings and of external objects, appear to us to be possible The question is not merely whether we can imagine the absence of either or both. I take it that experience would reply to this by saying that we cannot *seriously* imagine (i e conceive with full consideration) the absence of either *in a world of sense-perception*[1]

[1] Psychologically speaking, I should suppose that we may lose consciousness of either, perhaps more readily of space, e g when listening to music Time is perhaps the more importunate of the two ideas because it extends to our inner feelings, &c, and I suspect this to be the reason of Lotze's notion that time is more 'objective' than space Yet we may, of course, in a fit of absorption lose consciousness of time I do

But the real point is not merely psychological, although of course in dealing with it we must appeal to facts of the mind. The point is that the very character and essence of sense is isolation, and therefore in apprehending variety, series. So extreme is this character of isolation that the presentation of sensuous contents even as a series is due to an intelligence that goes beyond sense Space and Time are *for us* the first work of knowledge, as the conquest of them is the second But all we are concerned with here is that, assuming the impulse to construct out of our sensuous perception a whole of the same nature as itself, we cannot but attempt to erect space and time also into wholes, an attempt which is frustrated as we have seen

Mechanical view of Universe

iii But, lastly, reflective science in pursuing an analogous attempt does meet with analogous difficulties It assumes as further characteristics of the sensible world the abstractions of matter and motion Matter and motion are the abstractions in which the sensuous world is reduced to homogeneity in order to be susceptible of quantitative treatment, and in this treatment they are able to a large extent to represent genuine and actual relations of that world. In this respect they correspond to the structural classifications of geometry, and form the content of mechanical science It would hardly be true I suppose to say that the infinity of matter and motion in space is an inevitable paradox to the scientific consciousness It appears possible to conceive of the universe as a coexistent finite mechanical whole, demanding no determination from without. But this is only because the determination from without is thrown back in time by the doctrine of the eternity of motion, which, with a similar doctrine applied to matter, introduce the infinite series under the guarantee of the law of causation. Here again we have the insoluble problem which arises from the relativity of the sensuous world and presses upon us in its naked form as soon as, by reduction to homogeneity, the element of absoluteness or totality which helped to balance it is destroyed It is to the latter element that we really look

not suppose that these half-illusory states are at all perfect in their neglect of the non-obtruding element

for a solution in the degree in which it is possible The infinite series cannot itself be reality, but so far as we can transmute the series into an articulated whole, so far we can gain a reality out of it.

This distinction suffices to justify the well-grounded conclusions of science respecting the past and future of the material universe. Such conclusions are contributions to the projection that forms the actual world in which as percipient and intelligent beings we live But with eternity in the shape of infinite regress and progress such a projection can have nothing to do

We have thus traced to its climax in mechanical science that form of identity and difference in which an identity is regarded as the sum of the differences in which it is presented This one-sided aspect of identity and difference is what takes the shape of whole and part in the strict or quantitative meaning of those words, the meaning in which the whole is taken as equated to a relation, whether particular or generalised, of homogeneous parts

It naturally occurs to us to ask at this point, how, if quantity is homogeneous, and if *proportion* is, as we have reckoned it to be, generically a quantitative relation, the concrete individual (see p 127) whose characteristic quality takes the shape of proportion differs *in content* from a relation in number, figure in space, or system of motions which, though purely quantitative, is also, as we have seen, characteristic and self-contained In the first place, we have spoken of the quasi-individuality which does attach to the structures that embody geometrical and, we may now add, mechanical laws And we must remember that their quasi-individuality is only made possible by a certain revival of the qualitative element within the whole of quantity, even if the quality so present throughout the parts is, like the curvature of a circle, constant in all of them. And in the second place, we have to point out that in a *true* concrete individual its individuality exists in the form of a conscious purpose, a real teleology, and is the cause of its homogeneousness, the proof of this being that if the elements are isolated and removed from the individual they fall back into disparateness. The proportion in

which its parts are held together is as we saw[1] secondary and not primary, it is a proportion between proportions And though it is true that in the simplest forms of comparison, such as matching a colour, judgment and equation are hardly to be distinguished, being in fact as yet in their common germ, yet the peculiar secondary unity of a complex whole characterised by internal proportions is not fully expressed by reciprocal equation of its elements Thus the concrete individual is from the first characterised by rather than equated to pure quantitative relations of parts With abstract totalities just the reverse is the case Their elements, homogeneous to begin with, are placed arbitrarily in any whole (in as far as the elements of various curves resist such construction they are not pure quantity), and only acquire the semblance of a relation to a whole by their non-resistance in such a construction *ab extra* In the former case the *differences* involve the character of *parts* as the concrete involves the abstract; thus head, arms, legs in a man have quantitative relations to each other and to his whole figure, which vary only within narrow limits, and which sculpture or painting must not violate, or again, his whole life is only possible subject to definite quantitative relations of energy supplied to work done In the latter case the parts, indifferent in themselves, are forced by construction into the function of differences. The reservations to which this last assertion is subject have been explained above (see pp 177–8).

Thus equation and judgment are no doubt closely related in their origin, and this is further illustrated by the facility with which judgment drops back or crosses into the equational form, which demands altogether less effort and insight than the attempt to grasp the differentiated structure of things I cannot refrain from quoting in illustration a paragraph from a powerful and sensible writer whose one fault is the love of moving in the lower categories and avoiding the effort to grasp entire realities as they are. 'As to the general result[2] [of human progress] what is it? Say, roughly, three hundred million Chinese, two hundred million natives

[1] See p 131, above

[2] James Fitzjames Stephen, Liberty, Equality, and Fraternity, p 177

of India, two hundred million Europeans and North Americans, and a miscellaneous hundred million or so,—Central Asians, Malays, Borneans, Javanese, South Sea Islanders, and all sorts and conditions of blacks, and over and above all the rest, the library at the British Museum This is the net result of an indefinitely long struggle between the forces of men and the weights of various kinds in the attempt to move which these forces display themselves. Enthusiasts for progress are to me strange enough "Glory, glory. the time is coming when there will be six hundred million Chinese, five hundred million Hindoos, four hundred million Europeans, and Heaven only knows how many hundred million blacks of various shades, and when there will be two British Museums, each with a library"' The numbers here are not merely descriptive, they are essential, otherwise the element of progress could not consist in their augmentation Of all instances that show in what thin abstractions a writer who prides himself on contact with realities may live and move, I know none more grotesquely striking than this, and it cannot be defended by suggesting that its absurdities are in some degree imputed rather than adopted For they can only be so imputed because they are adopted The faith in progress *need* involve no assumption of numerical increase of population Had the writer ever heard of virtue or knowledge?[1] That his main thesis in the passage is somewhat of a truism does not justify so gross a misconception in supporting it. The more we examine the more we shall find that it is indolence which makes us drop into the equation when our subject-matter demands the judgment

I have finished the account of the equation before proceeding with the judgment proper chiefly with a view to coherence in treatment, and not with an intention of representing the former as inferior to or less ultimate than the latter, though in a sense which may appear from p 85 above, such a representation might be held justifiable. But the

[1] Contrast Dante's lines —

'Considerate la vostra semenza,
Fatti non foste a viver come bruti
Ma per seguir virtude e conoscenza'
Inferno, xxvi. 118.

conception of divergence, of a co-ordinate evolution generated by abstraction, seems more appropriate to the matter before us than that of linear development [1] The equation is, as we saw, hardly even a momentary phase in the growth of genuine judgment; the two functions part company almost as soon as their significance reveals itself

And quantity is more than one among many categories; as the simplest point of view which admits of difference and system it aspires to be, and in one sense is, *the sole* category, or ultimate ideal of knowledge. It may be treated as sole category falsely or truly. It may be sole in the sense that though abstract, yet, subject to the reservations involved in its abstractness, it has universal applicability. Every science, as we read in Aristotle, assumes its subject-matter, and does not give an account of it. The schematic world of space, time, and mass is in this sense, as an object of science, beyond question, it has only to serve as an abstract postulate in working with perceptible facts, and from this point of view is *a* truth, if not *the* truth about the universe as a whole. As enabling a coherent reflective view to be obtained of perceptible phenomena as a quasi-totality (always encumbered indeed by the infinite series), it is of immense scientific value and co-extensive with definite existence For these reasons, again, the equation—the judgment of quantity—is rather co-ordinate with judgment than a phase in its development And still more is there reason for so considering it if we take account of the false employment of quantity as sole category. This false employment arises, or would arise, supposing the category of quantity to be considered not merely as co-extensive with determinate existence, but as, in its abstraction, the ultimate reality of all determinate existence, and consequently as furnishing the final ideal of science. It is obvious that the true use of this as of every category slides easily into the false one. Every science is occupied with its own abstractions. Every individual mind tends to magnify that with which it is occupied. The category of quantity, for reasons mentioned above, lends itself to universal application It seems a short step from universal application to sole application, but it is

[1] Above, p 85.

the step from truth to falsehood It is not made exclusively by votaries of physical science, nor perhaps by them chiefly. It meets us in theology and in philosophy under the form of the quantitative infinite as a sublime attribute of the Deity, or of soul life, or of the universe as contrasted with the 'finite' mind of man We find it again in barbaric or vulgar art, in as far as this relies for effect on mere magnitude, mere evidence of expended labour, or mere costliness of material And we do also find it no doubt in a formulated shape wherever matter and motion are invested with the dignity of real existence in a sense and to a degree that degrades the individual and concrete realities of life into something secondary and fictitious But it is plain that no such tendency is necessarily involved in the treatment of these abstractions as real characteristics of the perceptible world There is no special virtue in non-atomic continuous extension, nor any especial iniquity in the resolution of material objects into systems of vortex-rings, if such resolution either is a good working hypothesis or represents a real fact The only error is in taking either a hypothesis for a fact, or *a* fact for *the* sole fact—in confusion, not in mere abstraction.

The category of Quantity is, as we have seen, in its nature wholly relative It is therefore incapable of furnishing an absolute and ultimate account of things It not only cannot escape from the reference *ad infinitum* from term to term and condition to condition, but is forced to make this contradictory conception the very basis and postulate of its scheme We have seen that the moment characteristic quantity or proportion makes its appearance in the judgment, as in any quantitative judgment it may, the whole between parts of which the characteristic proportions obtain is tending to exhibit itself as an individual synthesis of true differences, not as a mere aggregate of indifferent parts. The pure quantitative judgment or mere equation [1] is possible only by abstraction from one aspect of the essential judgment-function

[1] An equation that embodies a characteristic proportion is not purely quantitative It involves in its interpretation the material differences between the parts which are in the assigned ratios, e g between angle and arc See pp 177–8, above

It is not easy to find a parallel to so comprehensive and systematic an employment of a single class of abstractions, except in any attempt which may have been made to regard the world as simply a congeries of qualities, say of pleasures and pains No such system indeed exists—the point of view excludes system, but one may conjecture of some such state of feeling as forming the consciousness of children and childish adults who have no judgment to pass on things, persons, or events beyond the expression of their likes and dislikes.

Thus I have thought it desirable to treat as in some degree co-ordinate developments the two series of judgments which diverge from the simplest measurement or equation such as a colour-match On the one side we have the full evolution of concrete thought, as it builds up the actual and individual world within the series of relativity, on the other side we have the truncated evolution which embodies relativity almost pure and simple, but, as the abstraction is never quite complete (for then it would annihilate itself), may in particular matter revive its relation to totality, as we see in the exhaustive judgment of enumeration, and in the quasi-generic judgments of geometrical classification And in the same way the more concrete judgment may in particular phases and under particular stimuli borrow determinations from or generate approximations to the abstract series This happens when the life of a nation is subjected to statistical treatment; when the disjunctive judgment is taken in the weakened form of enumerated alternatives, and so gives rise to the calculus of probabilities; or when any one of the grooves or threads of relativity which compose the perceptible world is taken as a problem *per se* and tracked to its consequences by means of a pure hypothetical judgment.

CHAPTER V

SINGULAR AND UNIVERSAL JUDGMENT

Singular Judgment.

1. THE transition from the singular to the universal judgment is the transition from the affirmation of particular fact to that of general fact This transition may be expressed by distinguishing two forms of the singular judgment, which may be called respectively the Individual and the Corporate Judgment

Individual Judgment

1 In approaching the Individual Judgment we are returning from the one-sided offshoots of measurement to the normal and concrete evolution of the judgment. We saw in chap. iii that it is the judgment of proportion which first reveals individual quality, that is, quality which, although particular and characteristic, yet does not refuse to admit diversity into itself and itself to enter into various contexts This quality, however, we found, if merely indicated as the content of an abstract idea, stood in antagonism to the demonstrative indication of present perception which alone could attach it to actual reality We found ourselves entangled in such judgments as 'This oak-tree has a leaf-spiral of $\frac{2}{5}$', 'This teasel has the bracts longer than the head', 'This tower diminishes in width from story to story'. Such judgments as these must rank as singular, for it is of their essence to qualify present perception by the meaning of ideas, yet their content is really ambiguous, for, as we saw, the characterising idea which ekes out the demonstrative reference to the concrete subject tends to grow into a condition and to make the judgment abstract, and in that sense universal

Judgment with Proper Name.

a. The ambiguity which tends to split in two the imperfect singular judgment of the above type, which we have ranked among the judgments of measurement, is apparently removed in the class of singular judgments which we now proceed to consider, and which are avowedly based on the fact of recognisable individuality Such are judgments whose

subjects are designated by Proper Names Enough has been said in the Introduction of the essential nature of Proper Names. We have now only to consider the logical value of the judgments which are made by their means

The determinate idea, present in the judgment of proportion, is omitted in the judgment made by means of a proper name. On the other hand, the demonstrative particle, which by itself is helpless, being only an indefinite reference to presentation, is replaced in the proper name by the indication, not to be effected without some kind of meaning, of a particular individual. Thus it might be said that the two elements of the subject in such a judgment as 'That young soldier is the victor of Actium' are fused together in the subject of 'Caius Julius Caesar Octavianus is the victor of Actium' But the union is effected at the cost of a mutilation of the significance; although as in the present example the diminution of determinate content may be more than compensated by the accession of suggested ideas. Identification no doubt involves ideas, but with the proper name, as we saw, identification is the end and ideas are only the means. In unfamiliar matter, say in a chronicle of remote date, we might conceivably identify the unknown possessor of some name as figuring in several scenes or incidents without being sure what he, she or it might be, whether a man or a woman, or a favourite horse. In this sense the judgment that deals with a proper name is merely particular. It has no meaning that can carry its application beyond the unique individual to whom it is taken to refer

But, though subject to this imperfection, yet Singular Judgments of the class now before us form a real advance on the Singular Judgment of Proportion. They rest upon the fact of characteristic individuality capable of change and persistence without sacrifice of identity. Instead of an abstraction limited by pointing as if with the finger they refer us to a unique[a] concrete thing in its continuous permanence. In this sense, because attached to a continuous element

[a] I say 'unique' because that is the purpose of the proper name. Of course it has a general nature of its own, and cannot really guarantee uniqueness.

of reality not shut up within a particular time or a given perception, the Individual Judgment is universal, and as we shall see leads up to a transition which takes us in one respect beyond the Singular Judgment But universality—concrete universality—is not, we must understand, antagonistic to individuality 'Caesar crossed the Rubicon' is an Individual Judgment, yet in it we are not[1] confining the reference of Caesar to the moment in which he was engaged in fording the river If we thus refused to refer the predicated content to the whole extended identity of Caesar the significance of the judgment would be destroyed, and an eristic error committed by reducing an assertion to a tautology. Where is the significance of crossing the Rubicon if we do not affirm it of the conqueror of Gaul, the rival of Pompey, and the true founder of the Roman monarchy? Thus the judgment regarding a person, place or other object that bears a proper name introduces a reference that is determinate without being abstract, and particular without being confined to present perception It should be observed that not all classes of objects are suited to be distinguished by proper names. This fact is akin to the inapplicability of significant names indicating *a thing* to many objects endowed with material existence The range of proper names fall within that of significant names of things, and the spheres of application of these two kinds of symbols compared with each other and with the sphere in which neither applies have a curious bearing on the subject of individuality (see above, p 128 ff)

Judgment with Name and Ideas

β With these Individual Judgments must be classed all predications dealing with particular events, individuals, or objects, in which the demonstrative particle is dispensed with and replaced by a symbol referring to the individual It will be found that these particular events, persons or objects have ultimately to be designated by reference to a proper name, or to some symbol which nearly approaches the nature of a proper name. As such e g may be ranked all chronological indications—'Christmas Day, 1885 A D'

It is obvious that such a symbol, or a proper name, as a fixed point in history, may be supplemented by any amount

[1] Contrast Lotze, Logik, sect 58.

of definitely significant ideas, and as the proper name or the date is often understood or presupposed, e. g. in a continuous narrative, we are apt greatly to under-estimate the part played in judgment by the content of such symbols Page after page of discussion about political or social tendencies may chance to be found in a reflective history, say in Lanfrey's History of Napoleon, without the mention of a proper name. But in so far as these discussions are to be taken as significant of the actual conditions of an actual epoch, they are understood as ideal content predicated of the nation, age and persons with whom the narrative is concerned Chronological symbols exhibit the transition from the demonstrative to the proper name in a peculiarly clear light. 'To-day,' 'yesterday,' 'last year,' are just on the line between demonstratives and proper names. 'To-day' seems naturally to = 'This day', a demonstrative indicating mere relation to the percipient subject [1] 'Last year,' 'twenty years ago,' show the relation to the percipient subject growing into an objective system And when we come to the employment of an era, A U C,, B C., A D, we have the system transferred from the accidental percipient subject, and attached to the content of a proper name Wherever we have 'I' or 'my' &c, as points of reference in narration, we are dealing with something between a demonstrative and a proper name And every narrative judgment which goes beyond a mere impersonal or demonstrative reference to present reality or to my own perception may *prima facie* be said to involve a reference to some proper name. No *mere* abstract idea can form the subject of historical predication How far this *prima facie* conception must be corrected by allowing for judgments which may be capable of uniting without mutilation the powers of unique reference and of determinate notions is a question which will occupy us in the sequel

ii. Closely allied to the Individual Judgment, and perhaps in rigid technicality not distinguishable from it, is what for Corporate Judgment

[1] I take what I believe to be the actual meaning in use Philology may or may not support it as the original meaning We are bound to take philology into account as evidence of evolution and as a guide to observation, but it cannot override present usage.

want of a better name I call the corporate judgment. This title is meant to include all such affirmations as deal with comprehensive totalities or aggregates which we *bona fide* take in their corporate or singular aspect, and do not consider either as in the collective judgment, in the light of sums of enumerated particulars, or as in the hypothetical judgment, in the light of mere abstractions whose very existence is not absolutely postulated Such a judgment may be expressed indifferently by a singular or by a plural enunciation, so long as the name used in the plural is a *bona fide* designation of a known or knowable unity in respect of its characteristic features e g. 'The ancient Greeks were at once a most scientifically and a most imaginatively minded race' This judgment obviously = 'The Greek race was', &c This is not a collective judgment in the sense which has above been given to that title. It is not capable of being obtained by successive synthesis of the component units—by enumeration —and retains therefore no special extensional reference to the individuals who as an aggregate constituted the ancient Greek race; it starts from the idea of a common stock as a historical entity with peculiar endowments and with its own rise, decline and fall Other instances are such as 'Europe has acted harshly to the modern Greek nation'. Europe is here not a geographical expression, nor even a mere body of nameable states, it is an organisation acting upon definite resolutions and through a known combination and proportion of forces Several states in geographical Europe probably do not count in political or concerted Europe Again, we may say 'The House of Commons detests a bore'. This might be transformed into 'All members of the House of Commons' &c, but the two affirmations are only equivalent if the latter is understood of the members *qua* members, i e as engaged in debate in the House; otherwise this latter becomes a mere collective judgment, dealing with the members as a collection of individuals who share two attributes, viz. belonging to the House of Commons and hating bores, but wholly neglecting any reference to the House in itself as a single body with its own functions and peculiarities, among which is the one predicated in the judgment. The glacial

period, The French Revolution, The Italian renaissance, The solar system, are 'corporate' realities and subjects of singular judgments of the species of which I am now speaking.

The distinction between such individualities and those of really single objects or persons such as form the subjects of individual judgments is not a matter of principle, for in both the existence of the subject is affirmed, or, as I prefer to say, absolutely postulated Nevertheless, the fact that such individualities as we have now before us do on one side consist of immense aggregates of particulars, and are therefore capable of being regarded at any moment from points of view antagonistic to that of their individuality, either as abstract ideas or again as series of numerable units, makes it desirable to mark by a specific distinction the fact that *in their unity* they can be regarded as individuals

On the other hand, the subjects in question are again not easy to distinguish absolutely from the content of scientific class conceptions whose individual unity and actuality are disputable The line which I have drawn is intended to correspond with the distinction between history on the one hand and truly physical science on the other. History deals on the whole with contents into the essence of which time enters, and which therefore, however comprehensive, are ultimately particular within the phenomenal series. Abstract science as a rule deals with timeless connections and systems of attributes, though it will be necessary in exhibiting this tendency to make allowances for a considerable admixture of and recurrence to the historical attitude, for in the light of evolution, time, or at least amount of change, enters into the essence of most things And besides Abstract Science we must not leave out of account Classificatory Science on the one hand and Philosophy on the other, both of which, though in different senses, may claim to deal with actual realities

The Corporate Judgment marks the fullest development of the affirmation of particular fact, and at the same time a near approach to the affirmation of general fact. In other words, the affirmation with which we have so far been dealing is beyond possibility of dispute Categorical affirmation, treating of contents which the judgments affirm, or at least

absolutely postulate to be features that have or have had actuality in the world continuous with present perception But from this point onwards the Categorical character of judgments becomes a matter of theory and of analysis There is indeed, according to the scheme here adopted, no absolute separation between the Singular and the Universal Judgment Still the distinction, unknown to Formal Logic, which is the ground of that separation as made by recent writers, is the basis of our scheme also The Historical or Singular Judgment, and *a fortiori* the perceptive or imperfect singular judgments that precede it in our scheme, are not on all fours with the judgments of science, whether classificatory or analytical, or of philosophy Before attempting to trace in these latter types of judgment the elements which we have observed in the former, though in other proportions and combinations, it will be well to examine more particularly the one leading difference between the two chief stages of affirmation

Time and existence in Singular Judgment.

111 The Judgment that asserts particular fact coincides with the Categorical Judgment in the primary sense of the latter title According to the standpoint which for the present we have adopted, a judgment is Categorical when it asserts some thing or event to belong to the actual world in which we live This assertion is made both by the Individual and by the Corporate Judgment For the subject in these judgments is something that can exist as a particular reality, and is therefore taken or presupposed to be a real particular, while at the same time its reality is so far determinate that it can intelligibly be denied, which we saw not to be the case with the 'this', 'here', or 'now' of the Imperfect Singular Judgment In these judgments, therefore, though not necessarily in them alone, we have existential assertion And the simplest test of the presence of such an assertion is to ask whether the non-existence in reality of the content which enters into the judgment renders that judgment false To such a question the primary answer is that *at least* in all judgments where time enters into the content — i e which assert facts in time—the non-existence of this content renders the judgment false And these judgments will be found to be coextensive with the Historical

Judgment, affirming as it does not necessary connection, but particular reality within the phenomenal series And thus, as said above, the Historical or Singular Judgment is equivalent to the Categorical Judgment in the sense here under consideration

This answer, however, demands explanation and limitation. If non-existence of the content or subject of a judgment renders the judgment false, we have further to ask, Non-existence *when*?

In every judgment we must distinguish between the time *of* predication and the time *in* predication The time *of* predication, i e the time at which some thinking being makes the judgment, is relatively to the content of the judgment a mere accident, and alters actually while we are occupied in judging, and *a fortiori* when the judgment is rethought after a lapse of days or years The time *in* predication is the relation of the predicated content to the total content of the temporal series of events which we construct and contemplate as objective The time *in* predication, if any, is affirmed as an attribute by the judgment, the time *of* predication is not affirmed as an attribute by the judgment, which therefore is not made false by any relations whether negative or positive between its content and that time Hence it follows that the non-existence of the individual subject or content which falsifies a Singular Judgment is non-existence at the time *in* the predication, not non-existence at the time *of* the predication 'Thucydides is among the greatest historians of the world' is not false, although Thucydides is in fact, when we speak, not a living man[1] 'The House of Commons is an integral part of the British Constitution' does not depend for its truth on Parliament not being in dissolution at the moment of predication, but only on the co-existence of a House of Commons with the British Constitution in the sense and to the extent demanded by the

[1] It may be objected that it is false to say 'Thucydides *is* a general in the Peloponnesian war', simply because Thucydides does not exist at the time of predication Where we have true tense this is so, we have then not got rid of the personal era See p 199 It must be remembered that in 'Thucydides *was*' &c, Thucydides *is* non-existent, and yet the judgment is true

import of the judgment. It appears to be a corollary from this principle that if no time in particular is involved in the import of the judgment, which is the case at any rate with geometrical truth, it becomes all but impossible to convict a judgment of falsity on the score of non-existence of its content; although the demonstration of utter non-existence, i e. I suppose of impossibility, would so convict it. This bears on the categorical character of the generic and hypothetical judgment.

A subtlety is introduced into the problem by the phenomena of *tense* which include the time *of* predication, or a relation to that time or personal era, within the content of the judgment, and so within the allegation of time *in* the predication. The content so superadded is of the most fugitive and relative nature, and is constantly neglected even by historical narrative. Picturesque history neglects it by the use of the historical present, and philosophical history by the use of the logical present Nevertheless where a past, future, or true present tense (neither historical nor logical) is intentionally employed, its relation to the personal era or date at which the narrative is drawn up beyond a doubt enters into the judgment and makes an assertion which demands a particular limited existence on the part of certain objects and events, and is false if this existence is not as affirmed In the case of the present tense there is a peculiar subtlety arising from the confusion between the atomic or nearly atomic and the continuous or ultimately non-temporal present The former is the import of the present as a true tense, the latter of the present as it approaches to a mere vehicle of affirmation Compare the following examples 'The Derby is being run at this very moment', 'The horses are leaving the paddock for the Derby of 1861' (from a narrative written in 1883); 'The Derby is run once a year', 'The Derby race in England is an instance of those customs which owe nothing to government but yet amount to national institutions' The first of these judgments if true now must be false after the lapse of five minutes The second is only false if there was no paddock or no Derby in 1861 The third is only false if at the time *of* its predication the race has altogether ceased to be an annual event, but its present is well able to compre-

hend within itself the intervals of time which the import of the judgments admits and requires, and is not falsified by reason of the non-existence of the race during these annual intervals. And the fourth is false only if there never was nor will be such a thing as a Derby race having the importance alleged in English life. I am discussing of course only how far the respective judgments become false if we assume non-existence of the content, I am not raising the question of the material truth of the attributes enunciated. It is only in the first of these four examples that we see the present operative as a true tense In the case of the past and future the difficulty of tense does not arise in its full extent, the tenses which indicate them must of course introduce the personal era, but for that very reason cannot be confused with a mere form of predication, for which they have not the appropriateness that the present possesses. Therefore they do not risk the reduction of every possible judgment to a statement about a momentary date or epoch, which would be the result of neglecting the above distinctions in the use of the present tense. And moreover, these very tenses ' was ' and ' will be ' prove that it is at least not necessary to the truth of a judgment that its content should exist in the moment *of* predication It is however necessary for judgments dealing with past and future that their content should have the alleged *relation*, however fugitive and accidental, *to* the time of predication But at any rate in none of these cases, neither in present, past nor future, need the content be shut up *within* the time of predication or the time related to that personal era Past and future do not pretend to be momentary, and the present cannot possibly succeed in being so. It must be credited either with duration in itself or with a continuity that shades by degrees into the past and future.

Thus, though the time of predication has the slightest possible relation to the content of judgments, yet no doubt, when predicated as an attribute by help of true tense, a relation to the time of predication or subjective era enters into predication And in so far as history as a whole falls in the past, the use of the past tense in narrative bears true witness to the essential particularity and limitation of the

existences with which history deals But after all, the past is not a point but a line, and so for precise temporal import even of narrative propositions in the past tense we must go, not to the tense, but to the import of the judgment. And *a fortiori* this is the case with the present, where the true tense is difficult to distinguish from the same tense used as the mere form of predication

We have thus seen in what sense and to what extent Singular and Perceptive Judgments are identifiable with Categorical affirmation The Judgment whose content involves a limitation of time is plainly false if in the time-relation prescribed by such a limitation its content is non-existent This test shows that all such Judgments are assertions of particular fact Even the example (on p 204) which goes beyond common historical usage nevertheless asserts fact which is essentially in time, although the truth of the assertion is not relative to the date of predication If the Derby race never existed the judgment would be false, and *its* existence involves a multitude of temporal relations which are necessary to its being what it is It is essentially a fact in time

When we get away from the proper name and the relations of events in the temporal series, we find it much less easy to say what non-existence of particulars, if any, would imply the falsity of the judgment, and in what sense therefore, if in any, the judgment alleges actual fact belonging to the real world We must bear in mind that we have up to this point been dealing with unanalysed perceptions and with proper names and their expansions We have therefore not got rid of the element of irrational concreteness which attends all judgment whose subject is given as if by simple pointing with the finger, for *qua* a mere symbol of identification the proper name itself partakes of the character of simple pointing; and until we reached the proper name we were encumbered with the demonstrative particle

Therefore the judgments thus far considered, omitting the one-sided forms that arise out of measurement, possess certain peculiarities They assert, to begin with, the existence of things or events in time In doing so, they restrict and burden with irrelevant matter the application of determinate ideas,

which, as the only symbols of meanings, they cannot avoid employing Taken therefore as rational connections of attributes, a point of view which determinate ideas challenge, and towards which, as we saw, the import even of proper names tends to develope, the judgments which we have been considering are *false*, being burdened with irrelevant and deficient in relevant matter This is the same thing as to say that they present an aspect of necessity, in which aspect they are defective and so false

The Universal Judgment.

2. By the title of 'Universal Judgments' I mean to designate, in all their phases of import, the assertions usually typified by such well-worn examples as 'All men are mortal', 'All fire burns', and 'All triangles have their three angles equal to two right angles' It has been shown above that perceptive and singular judgments, and more especially those which employ proper names, possess an element of universality as predicating identities into which differences enter or which persist through differences But in spite of this fundamental unity of the judging function, the distinction between the 'Singular' and the 'Universal' Judgment has an importance for us which it has not for traditional Logic; which indeed treats the Singular and the Universal Judgment as on the same level in a strictly logical point of view, and both, consequently, together with the particular judgment, as species of Categorical enunciation The attitude which we have to adopt towards this arrangement was defined in an early part of the present book,[1] where we saw that it was impossible for us to retain these species of enunciation in their traditional relations It is beyond a doubt that the Universal assertion must at some point of its development assume in some aspect or in some degree a hypothetical character, while if the genus Categorical, which certainly includes the Singular Judgment, extends into or over the domain of the Universal, it can only do so subject to reservations which are unnecessary so long as it is confined to the judgments of present perception or of simple narrative

The Universal Judgment, then, is not distinguished from the Singular Judgment by the mere feature of Universality,

[1] See *supra*, chap i, p 87 ff

but by a special phase of Universality, that is to say, by the predication of a *universal law of connection*. It has, indeed, been obvious to us throughout the analysis of judgment that universal connection was everywhere at work in the background, exploiting any qualification expressed or implied in the Subject, for the benefit of systematic connection or necessity, and at the expense of the simple perceptive or narrative conjunction of contents; until, surrendering for the moment the task of intelligible qualification of reality, we fell back, in the proper name, on an attempt at unique designation [1] But here again we fail to escape from universal connection or necessity, and in the names of illustrious individuals, as in those of nations, epochs, or movements, we have found that every significance tends to break down the mere conjunction of *data*, and to exhibit itself as a connection of reason and consequent. We attempted to draw the line at events and individuals into whose content time entered, and to show that judgments dealing with such matters as these were inevitably allegations of fact, and not of abstract connection. Now we adhere to this distinction, and it is perfectly true that any fact which is especially involved in one portion, however extended, of the temporal series of phenomena, must have existence within that portion if an affirmative judgment about it is to be true The assertion of such a fact in its accidental concreteness is therefore radically different from the assertion of a mere law or relation, and if taken in this latter sense would necessarily be false But we found in the corporate judgment that it was not perfectly easy to distinguish the facts of history from the truths of science; for sets of events greatly extended in time appear to pass by a sort of *sorites* into sets of events which, though in respect of existence precisely on all fours with actualities limited in time, are nevertheless either not treated as relative to time or really are not held to be so relative In order, while marking the

[1] I have said above (Introduction) that I do not give this as a historical account of the genesis of proper names, which must no doubt have been applied and recognised by a gradual differentiation. But none the less, wherever language has fixed them as a class of words, they perform the function and are subject to the modifications indicated in the text

distinction demanded by the absence of limited particularity, to give full weight to the continuity of import between these types of judgment, I shall not follow Lotze and other modern writers in identifying the universal *ab initio* with the hypothetical judgment I shall prefer to distinguish within it two species, of which the first, the *generic* judgment, alone belongs to the main evolution of thought, and the second, the *pure hypothetical*, is regarded as an abstraction of a quasi-mechanical character, and consequently as a divergence in the direction of the arithmetical and geometrical judgment.

The Generic Judgment.

1 The generic judgment is the qualification of reality under the aspect of a Natural Kind by attributes or relations incident to that Kind A Natural Kind is for our purpose a Kind accepted and treated as such by any science We have already seen that the geometrical sciences are in this respect in a peculiar position, and we have discussed the limits under which their quasi-generic judgments may be taken to embody truth about actual reality The question of alternative classification, which arises on account of the different points of view introduced by different sciences, was treated in the Introduction, and should cause us no difficulty *if* we are once able to understand the nature of the truth embodied in *any* science. For every science employs some abstraction and idealisation, though there is an all-important difference of degree between contents which are merely abstract as not sensuous, and contents which are abstract as not concrete

The restriction 'incident to that kind' is not intended to exclude relations which one kind shares with other cognate kinds A purist logic, following a suggestion to be found in Aristotle, might indeed require that every generic judgment should be 'commensurate' or characteristic It would then have to deny that the possession of breathing apparatus was incident to man, because he shares the respiratory function with the whole organic world Such a view would be illustrated by the fact which was commented on above, that e g the animal properties of man are, in man, modified by their relation to his humanity, so that in order to represent them as features of *man* they ought ideally to have certain modifications assigned to them, while in their abstraction

they can only be set down as characters correlative to the no less abstract idea of animality as such But this principle seems needlessly purist The attributes which man shares with the animal world are elements of identity, however partial, between him and it, and there can be no reason against characterising him by these identities which would not tell equally against any knowledge which falls short of perfection. The rule to be borne in mind about such cases is that imperfect knowledge only becomes *false* when mistaken for perfect knowledge In as far therefore as the form of the judgment implies a truth completely adequate to its subject, something may be said for the view which has just been stated. But we shall find if we press the matter home that this ideal is to be regarded as the vital principle active in knowledge, but not as hostile to any genuine fact that is free from confusion

Under 'incident to the kind' then are included the attributes and relations which lie at the root of the individuals' being, although shared by them with individuals of other kinds, or even with mechanical or geometrical wholes It is only to be borne in mind that such relations, e.g the characters of vertebrate organisms, are not thought as mere abstract properties, when applied to specific kinds, but are regarded as concrete schemes presenting both a general and a specific aspect. It is in this characteristic of graduated identity that the intelligible order of the world reveals itself to us

The generic judgment has always been the battle-field of conflicting logical tendencies, corresponding to actual needs and features exhibited in various points of view by the judging activity. I shall endeavour, on the same plan which I have hitherto pursued, in some measure to satisfy these tendencies by distinguishing as different, in phase and line of evolution, intellectual acts which are often reckoned as one and the same. But it follows from the nature of thought that all such aspects have a real connection, and are in fact rather distinguishable than separable Thus in distinguishing the generic judgment from other acts of thought we shall also be analysing this many-sided judgment itself and justifying to some extent the views of those who have recognised in it only one or other of its many sides.

a. The Generic Judgment—the Universal Judgment of common life and of classificatory science—was regarded by Aristotle, as is well known, under two connected aspects, as a judgment of Allness and as a judgment of Necessity. The former of these aspects has been seized on by formal logic, and the doctrine of logical universality has been adapted mainly to the consideration of subject and predicate as names or ideas applicable to groups of individuals We examined the judgment from this point of view, as dealing with an aggregate arising out of enumeration, in a former chapter. We there saw its actual goal in the Collective Judgment and its reversion towards a more concrete and natural mode of thought in the Exhaustive Judgment, which amounts to nothing but a Generic Judgment very strictly taken in extension. It must however be remembered that we did not think it possible for any judgment, however closely confined to an aggregate resulting from enumeration, to avoid characterising the individuals by a common attribute, and predicating an attribute of them Attributes are enunciated by extensional no less than by intensional judgments, but in the latter they are connected with attributes, in the former they are centred in the identity of individuals Therefore it is possible to consider the Generic Judgment as differing from the Collective Judgment simply in degree, viz. by predicating attributes of an unknown or unlimited and not of a known or determinate aggregate, the actual means of predication being in both cases alike a general attribute, though in the latter case attached to reality by a proper name or its equivalent. But an unknown or unlimited aggregate of individuals is a contradiction,[1] a numerical problem which proclaims itself insoluble by enumeration, and therefore the judgment which is couched in this shape, and which in fact conveys a perfectly intelligible meaning, must derive this meaning from some other source than from such an enumeration as that on which the collective judgment rests. The Exhaustive judgment must be interpreted by the Generic and not by the Collective

The Quasi-collective Judgment

Examples of the Quasi-collective judgment of which we are speaking are—'All men are mortal,' 'All organisms both

[1] See p. 166, supra.

breathe and assimilate,' 'All unstriped muscle in the human body is inaccessible to the control of the will' This last example, however, is suggestive. The Exhaustive judgment, i e the Generic judgment in its aspect of 'allness', is helpless in the face of the most trivial exception. Thus '*Nearly* all striped muscle is under the control of the will', but the muscles of the heart form an exceptional case, and, though striped, are normally inaccessible to volition. Nevertheless there can hardly be a doubt that the coincidence expressed by the judgment must indicate some sort of connection, however circuitous, between the appearance of the muscle and the degree in which it is under control, and that the exceptional case must be accounted for by special conditions. But on its purely enumerative side the judgment has nothing to say to this, it only knows that the sum-total of Enumerative judgments cannot be made, and the judgment of allness is therefore unwarranted It is obvious that the affirmation of universal connection which in such an instance we feel to be all but warranted is not approached from the side of the individual units, but from the side of the common or continuous nature which binds them into a whole

True Generic Judgment

β As dealing with a common or continuous nature the Generic judgment may be more properly expressed in the form, 'Man is mortal,' 'Water boils, under one atmosphere, at 212° Fahrenheit,' 'A society organised on a purely commercial basis treats the working classes as little better than slaves' These propositions are accepted as practically equivalent to 'All men are' &c, 'All water' &c, 'All societies' &c. Here however it is plainly the connection of attributes that warrants the affirmation concerning individuals and not *vice versa*. When thus regarded, the Generic judgment challenges comparison with the Singular judgment in both its forms, both as Individual and as Corporate

Ordinary Generic, or Analogical Judgment.

α 'Man is an animal capable of social life,' 'The bacillus is a septic organism,' 'Throughout the vast Orchidean order, including 433 genera and probably about 6,000 species, the act of fertilisation is almost invariably left to insects' These are affirmations that unquestionably refer to something real, but yet employ neither perception nor a demonstrative nor

a proper name Therefore, *prima facie*, it seems as if the determinate idea had come to its rights, and were no longer obscured by any irrelevant elements of the phenomenal concrete. The act of thought ought, it would appear, to fall at once into the groove of abstract necessity '*If* man, then social,' '*If* Orchid, *then* insect-fertilised' But ideas such as those now before us offer a resistance to such treatment. The determinate idea is abstract, indeed, as all thought is abstract, but nevertheless it may have a content which is concrete, and in the example before us we have such concrete contents. These, therefore, bear the morphological character of individuality, by which alone even the unique object named by a proper name is made recognisable, persistent, and so universal Compared with such an individual subject the Generic subject has lost unique reference, but it has gained abstract significance, with which the proper name was incompatible And it is in virtue of this significance, the significance of individual self-completeness, that the Generic subject persists as an identity through the differences which form its attributes Now the individuality when reduced to a content is not single, but exists in instances Thus, in attaching differences to the individually characteristic content as such the judgment goes altogether beyond the synthesis of differences in an actual individual subject, and affirms such a synthesis mediately of a number of subjects, which may be taken as endless seeing that its limit is at this stage not held essential and not enquired after Such a judgment, which treats a concrete individuality as an abstract universal, and extends its incidents to all individual instances, may be described as an *analogical judgment* And this is the fundamental nature of the ordinary Generic assertion.

The introduction of the term analogy into the theory of judgment may indeed be objected to on the ground that analogy is a kind of inference But the fact is, that apart from any *general* question of a connection between judgment and inference, we are now at any rate on the threshold of an activity of judgment in which inference is unmistakably present All that we can do in order to avoid a premature discussion of inference is to approach the analogical judgment

rather as a conclusion the content of which is open to analysis than as a complete inference whose process lies before us It may be added, that logic is quite familiar with the idea of 'necessary' judgments Yet necessity involves inference far more explicitly than does analogy

In the analogical or *ordinary generic* judgment, then, we have neither implied reference to perception as in the impersonal judgment-form, nor the demonstrative 'this' or 'that', 'here' or 'there', nor a conventional implication of unique reality by means of a proper name The subject—I speak of the immediate subject or subject within the judgment—is an idea, and *qua* idea, is abstract But we must distinguish between abstractness as incident to thought in contrast with sense-perception, and abstractness as a character attaching to contents present in thought In the former way of speaking all thought is abstract In the latter, some is abstract and some is not The ideas which form the subject of the generic judgment in the phase now before us are not abstract in the latter sense They are ideas of totalities, existences complete in themselves, to which we cannot indeed venture to apply the conceptions of teleology proper except in so far as the wholes in question are products due to the human mind, but which must be regarded from the standpoint of that secondary finality which may be described as morphological unity or quasi-teleology We have thus a character or complex of attributes which is at once general and individual, abstract in thought and concrete in content As abstract, it defies enumeration of instances, and implies necessary sequence or connection of attributes. As individual and concrete, on the other hand, it refuses to be taken as a mere ideal antecedent in a relation of necessity, i. e. of reason and consequent The conciliation demanded by these antagonistic elements of import is found in the judgment of analogy The essence of this judgment is that it is neither purely subsumptive, as expressing a *de facto conjunction* of attributes in a single subject, nor purely constructive as expressing a *de jure connection* of attributes independent of the immediate subject in which they may exist, but is something intermediate, as expressing a perception or

presumption that the content enunciated in the judgment is bound up with the characteristic individuality which forms the immediate subject

The ultimate foundation of any such insight must be the final cause or teleological idea of the individual, which however, when considered as an immanent or embodied final cause, is most prudently treated on the level of morphological character. We may indeed safely say that the purpose or final cause for which we make a microscopic lens is to combine magnifying power with light and definition, and from this purpose, by help of a number of further judgments dealing with optical and mechanical truths, the physical attributes of a good lens may be constructed But in dealing with things not made for a known purpose we cannot apply any such abstract rule, and must fall back on the idea that the thing discharges an actual function, or at least looks as if it had a function, which must be taken as immanent and identified with the thing in its concreteness. The judgment that pronounces what is involved in this content and what is not rests on the presumption of the individual unity of the content, and on the capacity of discerning from the structure of this unity aided by empirical knowledge of instances what is essential to it and in what degree. The insight in question has undoubtedly some kinship to aesthetic judgment, for both depend on the power of seizing the concretely presented import or principle of unity of a concrete whole. To judge the structure of a fossil creature from a vertebra, or to detect the affinity between two zoological species which are externally much modified, is a synthetic apprehension of the same nature as that which realises the construction of a picture or of a drama. Such judgment, however, is merely the condition precedent, and not the essence, of the true relation between the mind and fine art

The analogical judgment, like the aesthetic judgment, is essentially outside relativity and necessity, and incapable of being resolved into them It is true that judgments of abstract relation, drawn from the mechanical or geometrical sciences, are perpetually coming in aid of analogical truth, by indicating that this or that *de facto* service within the concrete individual

can only be performed by its parts under this or that condition To support a certain weight the plant stem or spinal column must have adequate strength To impel a certain bulk and mass through the air at a certain velocity the bird's wings must have a certain area and striking rate, the arrangements necessary to which of course react on the whole muscular circulatory and respiratory apparatus But even this merely rhetorical selection of an abstract final cause is really unjustifiable Which comes first ? why such a weight on the stem ? why should the bird's body have such a bulk or mass ? There is nothing to fix any one of these elements as a given final cause to which the others must be adapted Adaptation to the bird's prey or the like is again simply *de facto* An animal might have to change its prey by reason of a change in their relative powers, just as probably as it might develope new powers to keep pace with those of its prey. And further, in the background we may see such a law as that of the Conservation of Energy dominating the entire system and operations of everything that moves In all these relations we observe the ultimate character of necessity, viz the reference of a subject to a whole other than itself, e g the treatment of an animal as a part in the whole of moving matter, or as a figure having properties in space, so that in each of these relations it appears as determined by the character of a totality other than its concrete self The nature of space for instance is *per se* a *datum* or fact ; but when it determines the results entailed, e g by the shape of a leaf, it is exhibited as a whole prescribing the relation of its parts, which relation as regarding something that is not merely a part in space is external but constraining and so necessary

Yet even if the entire construction of an individual content were laid before us in terms of mechanical analysis, still the analogical judgment would force itself upon us, as the aesthetic judgment would in a parallel case Analogy would then indeed no longer be the chief instrument in discovery, or at least in presumption of universal connections, because these would be capable of constructive apprehension of a more direct and relevant kind Such a state of knowledge may already in some degree be illustrated by

the Darwinian analysis, say, of an orchid-blossom, in as far as it succeeds in tracing the mechanical modifications, which, each of them representing a definite physical adaptation to some external circumstance, have generated the present structure of the flower. The same observation might even be applied to the identification of types and their affinities. The mechanical history of any organic structure would, if ideally complete, include the nature, degree and physical causes of its deviation from kindred structures.

But all this would not interfere with the import of the generic or analogical judgment For this import consists in the identification of individuals with a concrete content, and such an identification involves connections which differ in kind from the identification of abstract relations which are not Things They take the content not in its external relativity, but in its relation to self or to an immanent final cause—a final cause identical with itself We have examined this self-relation in the more difficult case of geometrical figures which are absolutely and adequately reducible to examples of general conditions, and seem merely to mimic the self-contained relation of the concrete thing This relation is an element of import which does not wholly disappear even in those kinds of existence which are hardly ranked by common language in the category of individual things—as we saw to be the case e g with the elements, and in short with all unorganised substances I do not restrict the meaning of the term unorganised to = 'inorganic' in the technical sense; but I employ it to designate any portion of matter, organic or inorganic, which is not shaped into a whole by human activity, or regarded in respect of its natural subordination of parts with interest due to its unity for our intelligence. Every element has no doubt its peculiar minute structure, and every fragment or portion of matter has no doubt its spatial or other relations which unite it into a whole A pebble or a bit of rhomboidal spar or a nugget of gold has a self-relation, a characteristic peculiarity which makes it single, and distinguishes it as a persistent universal from things external to it Much more has any organism a typical individuality which introduces the distinction of inner and outer,

essential and relative, into what as a mere example of general laws has no self-relation [1] and no inner or outer.

And in these last two sentences I have omitted the strongest case, because it is so strong as to dispense with the reservation which we were trying to illustrate But when we come to reflect on the conception of a *thing*, we must be struck with the fact that by far the greater amount of what we most readily recognise under that title are objects made by man for purposes which he consciously embodies in their structure. I cannot think that, apart from our familiarity with such objects, the conception of a thing would seem so simple as it does. A mountain, a waterfall, a wave of the sea, are *things* chiefly to the aesthetic perception ; and if we left this perception out of account, it would not be easy to assign the boundaries of their individuality, or to single out its essence. Complaint has been made [2] that those who lay stress on the progressive interpretation of the idea in nature do not find room in their theories for the achievements of the screw and the lever, *and* for the laws of the equilibrium of fluids, of pressure, and of tension These examples may be regarded in two aspects The screw and the lever are best known to us as *tools*, in which capacity they belong to the sphere of *mind*, as objects endowed by human foresight with an immanent significance depending on their adaptation to a determinate purpose But as mere characteristics of matter mechanically considered they rank with any of its general attributes,—rigidity, gravity, inertia,—attributes which are the basis of all material organisation, but do not by themselves suffice to give individual interest to any fragment in which they are embodied , and among such attributes must be ranked equilibrium of fluids, and the effects of tension or of pressure.

But what are we to say of the shell that screws itself into the sand, of the screw propulsion exerted by the porpoise's tail, or of the levers which form the limbs of animals ? We

[1] Of course such a point of view is unreal in its abstraction. If the thing were absolutely regarded without self-relation, its external relation would be gone too, for what would there be to determine it ? The point is that the centre of interest, in relativity, falls outside the self

[2] Lotze, Mikrokosmus, English Translation, vol i, p 17.

dare only speak, in relation to such phenomena, of a *de facto* purpose or actual function In virtue of this function, this contribution to an obvious and real end, the total life and motion of the animal in which they are found, we claim for these arrangements a morphological unity which forces us to grant them the character of elements in things that have concrete individuality We dare not ascribe to them the unity of an ideal purpose, as we safely may to the screw of a micrometer or to the lever of a balance, but we treat them as elements in a unity *analogous to* that with which we are familiar in objects that represent the purposes of our mind The rudest mechanical contrivance is in this respect on a level with the products of fine art and superior to those of nature, that it unites the abstractness of thought with the concreteness of sensuous existence; i e while in one aspect a mere material object, yet in another it embodies an idea, and does so determinately and without irrelevancy.

Thus the conception of machinery has a double and not a single import for logic. If on the one hand it accents the fact that matter is indifferent to our purposes and simply acts and reacts according to its own nature—and it is this of which we are constantly being reminded as the mechanical or uniform aspect of the world—yet on the other hand it is the most obvious example of a concrete embodiment of mind in matter, and corroborates if it does not awaken the reflections of the understanding on the rationality of things. Of this rationality the existence of individual types as concrete universals recognisable by the analogical judgment is a higher phase, a phase more akin to individual intelligence, than matter in its abstract and general modes And therefore the generic judgment resting on analogy, i e on the perception of concrete identity of content, is not capable of being superseded by the abstract judgment of pure relativity. The latter, if ideally complete, gives a true account of what occurs in terms of mass and motion, but necessarily omits the teleological or quasi-teleological import which gives the content of the judgment its interest and significance for knowledge

I will analyse a single example Exogenous trees display 'annual rings' in the wood, which are due to the augmented

pressure of the bark as each year's new wood expands the stem, resulting in the flatter formation of the outer and later cells in every year[1] As this stands I call it a generic or analogical judgment It is indeed based on one of the above-mentioned simple mechanical relations, the effect of pressure, but it predicates this relation within a concrete individuality which gives it an import that as a mere mechanical problem it would not possess Let us reduce its essential points, however roughly, into the latter shape. We shall obtain some such residuum as this 'A fabric gradually constructed under increasing pressure out of a material which hardens after a time will show increasing effects of pressure in its later-formed portion' Here we have, in part at least, 'freed the direction,' to borrow an expressive phrase from Bacon, i e stripped off circumstances which are irrelevant to the production of the effect in question But with these irrelevancies we have lost in the case before us not merely confused concomitants of perception, but the interest which gave the example its place in knowledge The fabric is no longer wood, the gradation no longer that displayed in the annual rings of timber, the subject of the judgment is no longer reality embodied in the characteristic individuality of the exogenous trees The import of the judgment is gone 'But its content is subsumed under the simple mechanical relation, if this is rightly understood' Perhaps; but what does this mean? If the import of the concrete thought is to be saved, it must mean that the analogical judgment is re-thought in its full depth, but with the explicit knowledge that it includes the abstract mechanical relation. The typical character of exogenous trees, though we must not call it a final cause, yet prescribes the extension and gives a definite reference to the content of the judgment. And I must here put the reader in mind, that, wishing to gain nothing from what may be called accidental ignorance, I have laid no stress on the present impossibility of constructing any living thing on purely mechanical principles Individuality rests on a difference, not on a confusion, of categories. I am

[1] Probably other causes concur in this process I have purposely simplified it The rings are annual only if the period of growth in the year is single A second hot season may cause a second ring.

convinced that no organic nor spiritual movement accessible to human intelligence is without a mechanical aspect. I have therefore treated the present subject from a point of view which admits such an aspect to be knowable in all vital and spiritual processes This point of view may seem absurdly fictitious when we consider the present state of exact explanation in the sphere of biology, or again of social science. The generic or analogical judgment now has, and seems likely long to retain, what we may describe as a secondary function; a function not merely of interpreting but of predicting—not merely of resuming sensible facts under higher categories, but of anticipating their actual occurrence In all science that deals with subjects beyond our power to construct, we draw our conclusions by means of analogical judgment in this secondary sense When we judge a particular plant of deadly nightshade to be poisonous, or a particular red stag to be dangerous at a certain time of year, we are judging on analogy, on an anticipation based upon a concrete character whose particulars we cannot construct The precise nature of these inferences will occupy us in the theory of inference; it is plain that the larger part of inexact science consists of them

We must not confuse analogy in that secondary sense, as a *mere* anticipation of nature,[1] with the true generic or analogical judgment which is compatible with complete analytic perception of mechanical cause and effect within the subject considered. We have perfect examples of these latter judgments in the case of things made by man for a purpose, in which a complete and accurate perception of their structure, interior causal nexus, and inevitable course of movement in no way supersedes the summary of their import which a knowledge of their purpose enables us to embody in a generic judgment. When we wind up a watch of which we know the construction, we do not merely anticipate that it will go because we have seen other watches go; we can point to the specific causal connections by which it must (excluding accidents) result that the main-spring, unwinding itself, will draw round the wheels, that the motion passed through the wheels will at

[1] Anticipatio naturae.—Bacon.

one point be regulated by the escapement, &c, &c. If we knew nothing of the use of a uniform measure of succession, but had some experience in mechanics, we should be quite certain that the watch must go, but we should have no notion of its generic content—we should not know 'what a watch is,' i e. what its purpose is. We could not therefore make the pure generic judgment, 'A watch is a motion regulated by an escapement so as to maintain a uniform rate' In order to this judgment we must know the purpose of the instrument, viz. to maintain a uniform rate. With this knowledge however we are in a position which, in strict theory, we can never attain with regard to any natural product. We can dictate the generic import of the watch, we can say that if any watch possesses this import imperfectly it is a bad watch; if it possesses it not at all, it is not a watch at all.

Judgments such as this form the ideal to which the universal judgment in the form now under consideration always aspires The properties expressed in such judgments are not merely anticipated or presumed, they are, or at least may be without altering the nature of the judgment, deducible with the utmost rigour. Yet, again, they are not mere causal sequences; it is possible to have before us all the causal sequences concerned in the object, and yet not to make the true generic judgment which unites them into a coherent system. In this class of objects we may fearlessly say that it is the purpose[a] which is the essence, and that generic judgment rests on the knowledge of essence In all other classes of objects such a view has degrees of precariousness, and can only be applied to the purpose as immanent, and therefore as not determinate, and as uncertain in its boundaries Nevertheless, when we predicate in the organic world 'growth', 'development,' 'self-preservation,' 'irritability,' we are really referring mechanical processes to an idea of life—an idea of self-relation, of 'inner' and 'outer', which is a higher result, though it *is* a result, of their purely mechanical nature.

[a] Such a purpose or essence, however, is limited, and e g. excludes the means, and therefore part of the whole which embodies it Teleology in this current sense, therefore, cannot be used as a principle in explaining any whole, e. g the universe.

The above is the best account that I can give of the normal generic judgment, which represents the really central phenomena that were designated under the title of Universal Judgment by Formal Logic The point of view which emerged in comparing this Universal with the Collective Judgment, and which was suggested by the quasi-collective form of the plural subject with 'All', can never have been really felt to include what the judgment intended to affirm. But, as was said above, 'allness' is undoubtedly *an aspect* of universality.

Existential meaning

b In order to bring to a focus the nature of this judgment we have now to consider how its affirmation is to be classed —whether as asserting the existence of fact, or the connection of attributes.

To determine this question, we must recur to the distinction laid down above[1] between abstractness as a character of thought in contrast with sense-perception and abstractness as belonging to a kind of thought as distinguished from concrete thought. Abstractness in the former sense is compatible with individuality, while in the latter sense it is not; and it is in the former sense that we apply the term abstract to the ideas which are subjects in the generic judgment. Thus though we have no longer a proper name as in the singular judgment, yet we have a concrete idea, which being as a whole capable of reality, presupposes such a reality. We saw that in the case of the demonstratives 'This', 'Here,' &c. the reality which is the immediate subject cannot be intelligibly taken as affirmed to exist, but only as presupposed. The ideal qualification which sometimes accompanies such demonstratives showed us the point at which presupposition of existence tends to pass into affirmation, simply because a *significant* presupposition can be intelligibly denied. But this tendency is never absolutely fulfilled. The union of actual and ideal qualifications in the demonstrative judgment—e g. '*This* bad man,' for though he were a good man he would still be *this*—always leaves the ideal qualification the option of being read as a condition The proper name, again, in its primary function, being void of determinate meaning, presupposes rather than affirms the existence of its content. It only tells

[1] p. 214, supra.

you that *some* individual is in question; and you cannot deny that an individual may be in question. But then as the proper name becomes more charged with import, which may even be made explicit in ideal contents as it is in the Corporate Judgment, it also becomes as we saw capable of conditional meaning. Now as the primary function of the proper name can never be cancelled while it remains 'proper', there arises within the singular judgment a parallel ambiguity to that which arises within the demonstrative judgment. The presupposition that reference is in any case made to actual existence is at war with the determinate qualification which can and will only refer to some determinate existence that may or may not be forthcoming. A determinate, i.e. significant, ideal qualification standing as or in the subject of a judgment is never unambiguously affirmed to be an actual existence. It may always take refuge in a conditional meaning. The reference to actual existence is presupposed in the sense possible for the subject, in perceptive judgments because we are never without perception, and in singular judgments because the form of the subject-idea suggests an individual, and reality consists of individuals.

The independence of these two kinds of qualification, ideal and existential, and their consequent liability to contradict one another, is the very root both of existential and of conditional affirmation. In existential affirmation the two qualifications are taken as meant to coincide, though it is a purely material assumption or assertion that they do so, in conditional affirmation the two are allowed to fall apart, i.e. the ideal qualification is not read as implying an individual reality that possesses it.

Returning now to the concrete idea which stands as subject in the generic judgment, 'society,' 'man,' 'art,' 'the bird,' 'the rose,' 'a time-piece,' 'a telescope,' we find the same elements of meaning, but in reversed proportions. The subject here consists technically and primarily of ideal qualification and nothing more. It has reference neither to an unnamed perception nor to a unique individual undetermined by abstract significance. Hence the presupposition that the subject is an actual reality is less prominent, while the abstract con-

ditional import of the ideal content is more so In the earlier types of judgment we feel that we are referring to reality, and we assent with reluctance to the analysis which shows us that our reference is conditional In the more abstract forms at any rate of the generic judgment we feel that we are affirming conditionally, and we at once acknowledge our reference to actual reality to be merely implied or presupposed. It will be noticed that I do not admit this side of the antithesis to be completely developed in the simpler generic judgments. I do not think that in 'The rose has pinnate leaves and perigynous flowers' the existence of the subject is merely 'implied' and not 'asserted' It is not uncommon to find in a manual of botany such and such a variety 'is no doubt a mistake of the observer', which shows that the observer's description asserts existence so far as existence is asserted by any judgment [1] All this however is a mere question of degree. What I am here concerned to show is that the mere *implication* or presupposition of real existence, to which in one way or another we do undoubtedly come in the Universal Judgment, is not extraneous to the affirmation and dependent on a mere fancy or habit of ours, but is the lineal descendant, *mutatis mutandis*, of that so-called existential affirmation which we have traced in perception and in narrative And the strength of this implication depends on the concreteness of the idea which here forms the immediate subject in judgment. We have then here as before two elements in the content, or rather a content regarded in two lights. We have self-relation, existence, or a categorical aspect, and external relation, necessity, or a hypothetical aspect But the nature of the generic affirmation, as analysed above, shows for the first time a trace of reconciliation between these two points of view The concrete self-relation is no longer void of meaning and purely designative, it is a system of assignable import, and the analogy of which we have spoken is the anticipation or the insight based on this import But again, this analogy introduces relativity and necessity, and as we saw interprets

[1] It may be said that in such a case the observer has alleged the plant to have been found in a given spot at a given time But this is not essential, he may simply send in the description

relations that unite the individuality in question with other totalities which prescribe to it either conditions or purposes. Thus the generic judgment is categorical in respect of its concrete self-relation, and hypothetical or necessary in respect of the analogical or constructive nexus to which the import of that self-relation gives rise

But, it will be objected, this might be all very well if we were speaking of individuals, whose nature is to be unique, like the present King of England, or the centre of the material universe, but here we are speaking of an indefinite set or series of individuals whose common nature is nothing and nowhere but in them This is, it may be urged, Scholastic Realism over again. What existence does a generic judgment presuppose, when and where? In reply to such an objection, I insist in the first place that we cannot treat any imperfection of knowledge as incident to knowledge unless we can prove that it necessarily is so, and that to treat a natural kind as an indefinite set or series of individuals is an imperfection of knowledge which can be shown not to be necessary This consideration however belongs to the subsequent section

But in the second place I reply that even without treating a kind as an actual unity, and though in fact we do not treat it so in judgments which are true of each individual singly (as common generic judgments are), yet still the individuality of the content dictates its own time, place and measure of existence. And it is this time, place and measure, wholly without reference to subjective era,[1] place or fancy, that is affirmed[2] in generic judgment It is characteristic of the rose to exist in a certain epoch of evolution and within certain limits on the earth's surface Existence within this time and place, subject to such variation as the nature of the content allows, is what the generic judgment affirms (or implies) of the rose; that is to say, in affirming that 'the rose has

[1] See above, p 204, discussion on tense in judgment

[2] I should not object to replacing 'affirmed' by 'implied', in order to mark the unquestioned line between the singular and the universal judgment, if it were admitted that the implication is an integral part of the judgment, and not a fancy of our own The point is that existence is implied or affirmed in these judgments, just as Necessary connection is implied or affirmed in Perceptive and Singular judgments

perigynous flowers' we mean that individual actual roses, found within these limits, have the attribute in question. If there are none such, then the rose is like any genus or species that has been imagined to exist by a mistake of identification, the kind in question would in that case not exist, and the judgment would beyond question be false Of course in every-day subjective judging the place and time &c of existence is but roughly indicated by what we happen to know or believe about the actual subject of judgment, but it is never referred to the time or place in which we judge, unless *per accidens* our knowledge is limited to, or the content especially concerns, that time and place Rose in the abstract does not exist But it is a concrete universal which has power, in the context of the real world to which we refer it, to dictate the epoch, place and quantity of its individual embodiment

I need hardly guard myself against the misapprehension that I am alleging that anything and everything exists which we choose to fancy. I am maintaining just the opposite, viz that if we attempt to embody fancied realities in judgment, such judgment is false, for all judgment is a definition of real reality. We can only escape this result if the fancied content is such as is in its logical nature debarred from being real, i.e a mere abstraction, and is therefore incapable of claiming to stand for a reality

c. The reality involved in a concrete universal will be made plain by insisting on a third aspect of the generic judgment, viz. that in which it challenges comparison with the Corporate Judgment. For this purpose we must think of generic judgments which are not merely analogical, but which for want of a better term I may designate as Individual. These judgments are characterised by not being true of any and every individual singly, but only of the kind taken as an individual. Such judgments are, 'The animal world represents an evolution coordinate with that of the plant world,' 'The Orchidean order includes 433 genera and probably about 6000 species,' 'Space has three dimensions,' 'Humanity is the object of worship to Positivists,' 'Monarchy disappears with the advance of civilisation' Individual Generic Judgment

In comparing such judgments as the above with that form of the Singular Judgment which we called the Corporate Judgment, we find at first sight little distinction between them beyond the fact that these Generic Judgments do not employ a proper name in the subject, whereas the Corporate Judgments do And even this distinction is in some degree bridged over when we call to mind that such determinations as 'now,' 'last year,' 'this,' and 'mine' appeared to us essentially to rank either with proper names or with demonstratives, and also that there is a tendency on the part of proper names themselves to assume abstract significance, so that a proper name is not always easily distinguished from a generic name The Greek race, Europe, are proper names; but it is more doubtful how we should class 'the Aryan languages,' 'the North Pole,' and 'the Mahometan religion' Again, 'the earth,' 'the solar system,' seem free from all arbitrary reference, but in speaking of them we really imply 'our earth,' 'our solar system,' and so fall back into some form of the singular judgment.

The difficulty is worth noticing, but it is simply one of those which must arise from the Sorites-like character of any continuous evolution It is hard to say whether 'the Mahometan religion', as we mean to employ the term, involves a reference strictly of the order of a proper name,[1] or on the other hand is simply an ideal content, and, so far as ideal, abstract The reason of this is that the conception is in fact on the border-land between the proper name and the mere determinate content, and in all probability it is sometimes employed in the one sense and sometimes in the other. The question is whether the general meaning or the individual identification comes first in the mind As we saw above, it is not improbable that in primitive careless and unscientific thought the significant word is not made distinct from the proper name—in other words, the intension which is the

[1] In every case we must keep etymology out of the question The reference to Mahomet as a historical individual is certainly not the chief element, and perhaps hardly an essential element at all, in the direct significance of 'Mahometan' at the present day Compare the rhetorical antithesis that has been drawn between 'Christianity' and 'the religion of Jesus'.

mere means to identification is the only intension signified, but for this very reason the purpose of identification is not distinguished, so as to be considered primary, from the purpose of definition In fact, when we now speak colloquially of 'the bay mare,' 'the low pasture-field,' we are using 'bay' and 'low' merely as signs of identification, though of course by help of their meaning Such is the type of usage which we may imagine to have been the common root of the significant and the proper name The terms of the Linnaean arrangement of plants, in as far as they are subservient to mere recognition, are a somewhat similar case in point.

But when all deductions are made, there remains a clear distinction of principle between judgments which use proper names, and judgments which do not. From this point of view the generic judgments now before us agree with those last discussed and contrast with the singular judgment. They are able to convey their reference to reality by means of a determinate ideal content.

On the other hand, in the *nature* of their reference to reality they agree with the singular judgment and differ from the common generic judgment. They do not rest on analogy. The individual to which they refer is a real and a single individual, and not a mere individuality. So far from being mediated predications about a number of particulars they are not even true of the particulars that enter into their content. When we said, in the former section, 'The rose has perigynous flowers,' we were treating the individuality of all roses as one by analogy.[1] But when we say, 'The rose family is a descendant of x, a divergence from y, and a transition towards z,' such a judgment is not made about each particular plant

[1] Subtlety of transition must have an end somewhere *in writing*, but *in fact* it has none Thus the reader may object that if it is nonsense to say that the rose family as an actual individual has perigynous flowers, yet we may always safely say in such a case that it prescribes that the particular rose shall have perigynous flowers I can only admit the objection, the fact is so *if* we are *bona fide* regarding the genus as a whole of evolution, whose actual individuality expresses itself in this and in other common predicates But if, as is most probable, we realise nothing of the kind, but are merely going by analogy to a common property of roses, then we are treating the kind as a mere individuality, and must not pretend to be treating it as an individual.

within the tribe, nor even about each particular species. If true, it is true of the whole section of plant-life in which every particular rose-plant is a distinct and separate progressive or divergent phase There can be no doubt, I think, that from an ideal point of view every natural kind in the concrete sensible world must be thus regarded, and of course when we consider existences in which intelligence is more definitely active—society, mankind or at least civilised man, fine art, or morality—in these phenomena the totality is more real and concrete, and the reciprocal relations of its parts exist not merely for the microscope of analysis, but as patent every-day facts

Although universal, the Generic judgment in the aspect now before us is fully Categorical It is in this respect wholly on a level with the Singular judgment, being in fact related to the judgment of Analogy with its dual nature much as the Singular judgment is related to the proportional or comparative judgments that are introduced by a demonstrative The Singular judgment may be regarded as a *premature* attempt to concentrate individuality—the 'characteristic quality' which the proportional judgment had revealed—into an individual, resulting as we saw in the omission of determinate quality from the individual content by the use of Proper Names. The generic judgment raises (in its Analogical form) and meets (in its Individual form) the same problem in a more adequate way, concentrating individuality into an individual by completion and not by omission It is as a system of such individuals, united perhaps in a yet more concrete individual reality, that we must conceive of the world known to us through space and time, if we are to assign it any existence beyond the present of presentation *For us*, it is plain, such individuals are intellectual constructions, and only attached to, not shut up within, the actual present perception The distinction between concrete realities and abstract truths is not, for us at any rate, that the latter are intellectually initiated and the former are not, it is not a question of origin, but a question of nature, i e of the degree in which a content is capable of being regarded as something that exists as a whole and can be considered in relation to

itself, or on the other hand is incapable of being given as a whole and affords no matter for consideration in relation to itself. All contents must theoretically be regarded as combining these two characters; and as an important application of this idea I may instance the answer to a question which arises when we make the categorical nature of assertion depend upon the degree of concrete self-relation.

Is it possible, we shall be asked, to lay down a hard and fast line, by which abstract shall be divided from concrete contents? And if not, does not our view surrender the self-dependence of reality and make it purely relative to fancies and notions in the individual mind? Does it not enable us to treat as actual any content however abstract or trivial, and any however concrete or significant as a mere element in hypothesis, simply by varying the point of view from which we regard them? And the answer is, that as reality unites these two characteristics, we can always emphasise either at will; and further, we commit no error in so doing, unless we assume and assert the relation which we happen to be considering to be the only relation that there is. Our knowledge always falls short of reality, and apart from false identification of relations—with which false antithesis is at bottom the same—we have a right to see all that we can either of absoluteness or of relativity in any content whatever. Reality is such that any element or feature of it, however slight or superficial, can be raised by our intellectual gaze to the position of a self-related significant whole. The nature of mind is present in everything, the only difficulty is to see it there. And such an elevation is not false, except in as far as it is exceptional, in as far, that is, as we fail to view the remaining contents of reality with the same constructive insight. Not merely a fragment of stone or metal, but a colour, a curve, a relation of size or weight, is ideally capable of being passed through the stages of generic judgment, of being regarded first as an individuality, and then as an individual What is false or forced in such a mode of contemplation depends on the want of proportion between it and our ordinary careless vision of organisms and fine art, of men and of society. All contents are relative except the absolute;

but the import and degree of their relativity is not the same.

A further corollary may be worth drawing in a few words from the above considerations. Our present treatment of logic starts from the individual mind, as that within which we have the actual facts of intelligence which we are attempting to interpret into a system. But our consequent preoccupation with the phenomena of the individual mind, with its imperfect grasp of reality and the varying aims and tendencies of its thought, brings with it a double danger which haunts every phrase and every idea in a logical treatise. Either one may speak as if reality were simply relative to the individual mind, a ridiculous idea, but one which the very caution required of a modern writer is apt to encourage, for he hardly dares to allude to Mind as such or in itself, or one may become interested in tracing the germination and growth of ideas in the individual mind as typical facts indeed, but only as one animal's habits are typical of those of others, and so we may slur over the primary basis of logic, which is its relation to reality. For mental facts unrelated to Reality are not knowledge, and therefore have no place in Logic. The difficulty is, in other words, simply that modern Logic has a hard task to hold its own between Metaphysics and Psychology. I entertain no doubt that in content Logic is one with Metaphysics, and differs if at all simply in mode of treatment—in tracing the evolution of knowledge in the light of its value and import, instead of attempting to summarise its value and import apart from the details of its evolution. My object however in mentioning the difficulty at this point is merely to protest that though I assume reality as the norm of the mind, in constructing which it is reconstructing and not creating *de novo*[a] out of itself, yet I can entertain no doubt that intelligence is essential to the being of Reality, and that an abstraction which tries to regard the one apart from the other is a hopeless and helpless self-contradiction. As a working conception

[a] See II, p. 319. Reconstructing, for me, never meant representing or copying. And thought is as creative as anything in the universe can be. But creation *de novo* is a contradiction in terms, because it forbids the conception of a creative nature, and so of any interest or continuity throughout reality.

in Logic we are forced to adopt some such idea as that of a normal intelligence operative in all human minds, but subject to the accidental limitations of each The evolution of knowledge is, as Plato long ago portrayed it, the emancipation of individual minds from their accidental limitations, and their education into the knowledge of the one real and intelligible world [a] But the duty of modern science is to preserve the continuity of this evolution, and to admit no *saltus* at any point between the world in which we live and the world which is real and intelligible And in this continuity we have a standpoint which Plato, although he reached it, did not consistently maintain Objective Intelligence presents itself in Logic as the mere postulate required by such a continuity, and, starting as we have done from the individual consciousness in time, it is merely as a postulate that we propose to treat it To say that the real world is the intelligible world is only to repeat what we found ourselves obliged to suggest as an elucidation of the earlier stages of judgment, that reality is something at which we arrive by a constructive process [1]

We are now to consider the consequence of emphasising the abstract or relative aspect of the Analogical judgment. We are thus led to a form of thought which is antithetical to the Individual Generic judgment of which we have just been speaking, and consequently must be regarded as a divergence from the concrete evolution of thought towards the mechanical or analytic judgments which begin with enumeration.

[a] It does not seem necessary in Logic to develop this point further. A 'first philosophy' would have more to say about the sense in which the one Real world is intelligible, and would not admit that the limitations of individual minds ought to be called, in principle, accidental, little as we may be able to explain them

[1] See above, Introduction, p 38 ff.

CHAPTER VI

UNIVERSAL JUDGMENT (*continued*)

Pure Hypothetical Judgment

11 THE Universal Judgment, when pushed to the extreme point of abstraction, becomes the Hypothetical Judgment.[a]

Its relation to previous forms

a The Hypothetical Judgment is distinguished from all which have thus far been spoken of, by its essentially abstract character, abstract not merely as thought is said to be abstract when compared with sense-perception, but as the

[a] A word must be said on the distinction between Conditional and Hypothetical propositions, to which Mr Keynes, following Mr W. E. Johnson (Keynes, p 249), attaches much importance

Conditionals, as I understand, affirm a connection between phenomena, and their elements are events or combinations of properties, the connection of which in a subject or in time or place is affirmed Hypotheticals have for their elements independent judgments, propositions of independent import, a relation between the truth or falsehood of which is affirmed The characteristic phrases of Conditionals are 'Whenever,' 'In all cases in which,' 'If any—then that' The unambiguous form of a Hypothetical is 'If — is true, then — is true'

I think this distinction tends to obscure the meaning of the pure Hypothetical I note some points

(1) I do not see why the elements of a pure Hypothetical should be taken as propositions or judgments Its characteristic, I hold, is to trace a thread of logical necessity, and I see no gain in throwing the sequence into the form of two or more propositions And this opinion encourages

(2) the idea that true Hypotheticals coincide with what I have called 'broken-backed sequences' (p 236 below), the propositions retaining their full meaning when separated from one another (Keynes, p 251) For the criticism of this view I may refer to my discussion on p 236

(3) It would have made a neat series if we could have said that Categoricals (in shape) are only assertorical and existential, Conditionals (adding 'Because,' 'Since,' to the above-mentioned phrases) unite modal, assertorical, and existential meaning, and true Hypotheticals are modal only But Mr Keynes rejects any such distinctions, which are indeed incompatible with usage Therefore I see little or no *raison d'être* for the class of Conditionals It is hard enough to distinguish Categoricals (in shape) from pure Hypotheticals

(4) I am glad to agree strongly that it is not the differentia of propositions of the type If A is B, C is D, to imply *doubt*

(5) The student will distinguish from the doctrine I am discussing

thought of an ideally isolated attribute is abstract compared with the thought of a self-dependent and self-related individual. It represents the fourth of the elements or aspects which have been confounded, or at any rate have not been duly distinguished, by traditional logic within the so-called Universal Judgment Its differentia is that it does not refer to a concrete subject, not even to what we called an individuality or the concrete self-related content in its aspect of self-relatedness, and that consequently we do not consider whether its subject is given in actuality or not For it is essentially the judgment of necessity or relativity, in which the subject is taken, not given, and taken not for its own sake nor with reference to its individuality, but for the sake of that which is to follow from it, that is, for the sake of its relativity. It is a judgment which follows out the single thread of a nexus of attributes, and does not heed the import of the pattern into which it enters If a gravitating body is set free to fall, it falls with an acceleration proportional to the squares of the times, whether it is a drop of rain, or a tortoise with the head of Aeschylus below it. Here we have, in an explicit shape, the relativity of knowledge which has haunted us throughout the evolution of judgment, forbidding us to feel satisfied in connecting together any data which we might merely chance to light upon in conjunction, and requiring that every idea should always be limited and controlled by its reference to something else, and not simply taken as we find it in perception or in ordinary life And just because this principle has so haunted us, the judgment that embodies it cannot be sharply severed in meaning from the earlier forms of the universal judgment, and even the quasi-collective 'All gravitating bodies &c' may, and most frequently in this case does, contain what is really meant as a hypothetical affirmation 'In this case,' for the distinction really goes, as we have maintained all through, not by the shape of the proposition, but by the content of the judgment The connection however between all the types of universal judgment is intimate and essential,

Mr Bradley's view that all judgments are conditional, i e that there is always a supplement, which is never completely known, necessary to make the predicate fully true of the subject Appearance, ed. 2, p 361

so that in popular usage one easily slides into the other, or even combines the other with itself as its ground or consequence When I say 'All animals need food' I am probably expressing a *quasi-collective* conclusion about a property shared by all species of animals, taking its significance from an *analogical* perception of the generic function and immanent purpose of animal life, but ultimately resting on the *hypothetical judgment*, expressing a *necessary or relative* principle, 'If force is to be expended it must be supplied' It will be observed that the second type of generic judgment, which for want of a better name I have called Individual Generic, is omitted from this combination of aspects It represents a tendency divergent from that of the Hypothetical assertion, while the Analogical judgment is undecided between the two If the Individual Generic judgment is capable of combination with the Hypothetical, we must look for the result in the Disjunctive and not in the Universal affirmation

External Form.

β The type of the Hypothetical Judgment in traditional logic, so far as it is recognised at all, is stated in one of three forms 'If A is B is,' 'If A is B, then C is D;' and 'If A is B, then it is C' The third of these forms is that which guides us to the true import of the judgment, though conformably to the habitual irrelevancy of popular thought the second is that most commonly in use But this second is obviously a broken-backed sequence, in which no point of unity is formally recognised between the antecedent and the consequent When, indeed, significant words are substituted for letters, the unity would generally be obvious, supposing the sequence to have scientific value at all; but in such a case the expression is not essentially distinguishable from that of the third form The first form, 'If A is then B is,' has been said to be an abbreviation in which letters stand for clauses; in that sense of course it must be reducible to either the second or the third form We get the same result if we try to take it as a combination of single-word or impersonal predications Contents may undoubtedly be ascribed in judgment to an unanalysed present, but an unanalysed present can form no bond of union for a necessary sequence 'If guilty, then death' is a mere linguistic abbre-

viation for 'If he is guilty then he will be put to death'. And even 'Where there is smoke, there is fire' superadds to the impersonal 'There is' a true local particle in the 'where' of the antecedent, and this reacts by a curious equivocation on the impersonal 'there' of the consequent No doubt, but for the unpleasantness of the sound, we should say, '*Where* there is smoke, *there* there is fire' Here again, then, we have in essential meaning the third type of the Hypothetical judgment, 'If A is B, it (A) is C' I will next illustrate the transformation of type II into type III 'If the barometer (A) falls (B), the weather (C) becomes stormy (D)' 'If the atmosphere (A) decreases its pressure locally (B), it (A) must leave a gradient for wind (C)' But now if we take the lines,

> ' when in Salamanca's cave
> Him listed his magic wand to wave,
> The bells would ring in Notre Dame,'

we find that the *saltus* from antecedent to consequent is all but essential to the judgment; the point of the mystery is that we cannot get at the underlying unity Thus we see the extreme case of type II. in a judgment which has for its object to assert magical, i e irrational, connection Of course the general scheme of reduction would have to be, 'When his magical power (A) was exerted (B), it (A) could act at any distance (C)' It is in this sense that Schopenhauer calls some of Euclid's demonstrations conjuring tricks, because, although in a demonstration some unity of course must be shown between antecedent and consequent, yet the unity shown is often not central or fundamental,[1] and is therefore a *causa cognoscendi*, and not a *causa essendi* In the pure type, 'If A is B, then C is D,' we have no indication even of a *causa cognoscendi*

Much more might be said about the forms of conditional sentence; but the subject is really grammatical rather than logical, for the hypothetical judgment can be expressed without a conditional sentence at all Hypothetical and Categorical Judgment, as we understand the terms, are a question of content, not of grammatical form, and the hypothetical judgment is found wherever we frame asser-

[1] Schopenhauer, Werke, I 136 ff

tions about an abstract content, in the above sense of abstractness, although there is a difference of adequacy in different grammatical expressions for any kind of judgment, and the conditional sentence resists any attempt to embody in it a purely categorical meaning 'If this man dies, our cause is lost,' takes 'this' as a sign of unanalysed content and not as a point of attachment in reality We know that the reason is somewhere in the unanalysed content, and so take it as an antecedent in the lump [1]

Assertion made by Hypothetical Judgment

γ. What is the precise nature of the assertion conveyed by a Hypothetical Judgment? In answer to this question I shall speak first of the idea of Ground or nexus in general, secondly of Ground as compared with Cause, and thirdly illustrate our view by the attitude of the individual mind in hypothesis or supposition

The idea of Ground

α. The contrast which we have anticipated throughout the above account of judgment determines the central attribute which we are now to consider The content of a true hypothetical judgment is abstract; abstract in itself and not merely by the absence of sensuous perception. In other words, the subject of a hypothetical judgment is not an individual, not a whole, nor anything considered as a whole, i e as a self-related system On the contrary, the content of a hypothetical judgment is composed of ground and consequent, each referring to something other than itself, and hence essentially a part For a system as a whole, such as space, or the totality of gravitating matter, or the British Constitution, is a mere fact, complete in itself, and neither a ground for nor a conclusion from anything else [2] It is only as parts within a system that elements can be so relative to one another that '*if* this is *so*, then that must be *so*'

It is only a question of detail how far the system in and by which the nexus subsists, is itself made explicit as a content within the hypothetical judgment We may say, if we like,

[1] I take this analysis, which appears to me exceedingly felicitous, from Bradley's Principles of Logic, pp 89–90

[2] This applies to the examples given only when considered with reference to their internal nature It may be said that space implies an intelligence but this is as within a further whole There is, of course, no ultimately Absolute whole except *the* Absolute whole

that the ideal of logical sequence demands that the system as a totality should be so made explicit, because the system is the real ground of the nexus, and if the system does *not* appear in the content, the real ground does not appear in the content But this argument, from the comprehensiveness of the real ground, does not overcome the essential principle which is involved in there being a ground, as ground, at all Ground implies a consequent other than, though fundamentally one with, itself This transition or otherness ceases to exist if the content does not formally present itself as part to part For say that the totality of the system is explicit in the ground, still this totality is depressed into the relation of a part by the fact that a part is selected to appear over again as consequent, and so as formally at least external to the ground Thus it remains true that the elements of content in a hypothetical judgment are related as other to other within an identity which determines the one on the basis of the other Such an identity, as far as exhibited in the one term that, in virtue of it, determines the other, is what we mean by Ground. It is obviously capable of all degrees of completeness up to the ultimate fact or whole which embraces in itself as parts both ground and consequent commonly so called The various degrees of imperfection or broken-backedness in Hypothetical judgments, such for instance as were illustrated in the last section, are simply the degrees in which the system that determines the nexus fails to manifest itself within the content connected. It must not be forgotten however that we have refused to treat the grammatical form of propositions as decisive of the character of judgments Where no rational nexus is traceable, but only a coincidence in fact, however general, we cannot admit that the essentials of hypothetical judgment are present But then if we are impelled to make a judgment in hypothetical form, there always is some presumption of a rational nexus We shall consider in the following section what attributes of true hypothetical judgments are shared by analogical assertions such as 'If he is a negro, he has woolly hair' It is not worth while to insist more fully on these degrees of imperfection, except in so far as they will

come under our notice in dealing with the doctrine of causation and with the kindred subject of the negative in hypothetical judgment.

Let us attempt to make perfectly clear, before we go further, the nature of the relativity within a system which we ascribe to the contents before us The simplest cases of such relativity are drawn from the field of numerical or geometrical construction A Chinese puzzle or dissected map may give us a first idea Any selected piece out of such an arrangement determines nothing by itself, but when a second piece is given some relation between them emerges, though perhaps only a negative one It is further possible for a piece entirely enclosed by others to have its place relatively to them determined long before the whole arrangement is completed; but this determination will really be partial, for the place of the whole group of pieces cannot be determined till the whole puzzle is put together Now the arrangement *as a whole* is a mere matter of fact, it is only within it and by reason of it that each piece has a prescribed place in virtue of its own shape combined with the shapes of all the other pieces All the pieces being given, of course the arrangement is given too, but if nothing is given, of course all is in the air, and one arrangement and set of shapes is as likely as another.

Or again in the region of number, we may take as equivalent to Hypothetical judgments those which in treating of Enumeration we called Mediate $50 \times 3 = 25 \times 6$ would run in conditional language, 'If 50 is multiplied by 3, the product is equal to,' &c Here we have one form of the numerical whole 150 presented as a term from which another term, viz another form of the same whole, may be inferred. The system within which the relation exists is of course not the whole 150, but the system of number as such; or we may say if we prefer, the whole 150 as involving and exemplifying the nature of the whole of number as such This system may be brought to bear either by simple counting, the process which is so to speak the medium in which number exists, or by developing any of the relations which are embodied in the several places of the series with their individual names

Thus the above statement may at once be reduced to a tautology by taking 50 as 25×2 and 6 as 2×3. The combinations which might be made with the same result might be pursued as long as we cared to continue the enumeration of places in the series, and from whatever point we started in the system we should obtain the same result so long as both sides of the equation were subjected to treatment that was equivalent according to the rules of the system. On the other hand, if we were to assume the invalidity of the equation, we should at once make the whole numerical system inconceivable—a unit, say, would have to be taken as varying in value without being subjected to any arithmetical process, and such variation is incompatible with the fundamental principle of number

The same may be shown in space, treating it not *qua* extended whole of parts outside one another, but in respect of the connection of its attributes 'If two straight lines have the same direction, they can never meet.' This is a consequence drawn from the conception of direction in homogeneous space. If we destroy the idea of homogeneous space, the relation, which only holds within a totality having that attribute, is annihilated If there can be a change of direction which yet is not a change of direction—I do not know how else to express the notion of direction in space which is itself curved—then, I presume, the judgment from which we started is no longer true

The same characteristic might be pointed out in relation to gravity, inertia, or any property which is the basis of exact inference The consequences of gravity hold only within the totality of gravitating matter, of inertia in the combination of motions, and so forth Every sphere of this kind, every set of relations within which certain nexus of attributes hold good, is itself ultimately a *fact* or datum, relative no doubt within some further totality, but absolute relatively to the inferences drawn within it Hence we are brought to a conclusion of the last importance All hypothetical judgment rests on a categorical basis That is to say, all relativity rests on an absolute datum and all necessity on fact Why then is mechanism, necessity or relativity,

opposed to individuality, fact or absoluteness, if all mechanical relations are themselves characteristics given in some individual whole?[1] The answer seems to be dictated by what has been said Individuality is in self-relation, Necessity is in external relation But as all relation is within some whole, it follows that wherever we have necessity or relativity we are concerned with more than one whole or individual content, that is to say, we have a whole or content with its own import and significance taken as a part within a wider or completer totality[2] The ground or necessity which forms the affirmed nexus of attributes lies then in the systematic nature of the wider whole, that within which the terms of the nexus are capable of being opposed as part to part

We will examine two or three instances of ground and consequent in the light of the above doctrine 'A picture 6 ft. × 7 ft cannot be hung in a space 5 ft × 6 ft' Here we are taking the picture as a whole in itself, but as a part within space, and as therefore having external relations determined by the spatial system as including other objects. The arrangement of other spatial objects so as to leave only the area 5 ft × 6 ft is incompatible with the occupation of the area 6 ft. × 7 ft by the picture But again, we may judge that 'If the boat in the right foreground of the picture were erased, the arrangement of the distances would become confused' In this judgment we still call attention to spatial determinations, but only as involved in the concrete individuality of the work of art, which assigns them their meaning and value We are thus taking not space as such, but the individual picture before us, as the totality or determining system, and contemplating the necessary relations which this fact or whole, from its nature and structure, imposes on its several parts Of course, apart from the effect of the whole picture, there would be no such necessity or relation between the parts This is an illustration of the ultimate

[1] Space and Time are, as we saw, imperfect individualities But it is their individuality and not their imperfection that makes them sources of general relations in things

[2] See on Measurement, chap iii, supra

nature of logical necessity or relativity and its relation to fact, which is, if not specially felicitous, at least true in every detail In a true work of art we have the bearing of every part on every other, the innumerable details, none of which could be altered without necessitating the alteration of others, all concentrated in a unity which is itself constituted by all these parts, and yet, as a whole, prescribes the relations existing between them And yet the whole is itself a comprehensive *fact*, and apart from it or outside it all these prescribed relations lose their necessity and disappear.

Again, take such a judgment as 'The sound of the violin is of peculiarly piercing quality' If we describe the sound merely in terms of the mechanical system of vibratory movements, it is governed by the necessary relation, 'If a string be dragged by a bow, slipping from it at intervals, its vibration is of highly angular form,' i e in terms of its effect on the air, produces sharp and not gradual transitions from increase to decrease of condensation Omitting the effect on our hearing, this is a nexus of attributes grounded in the properties of vibrating bodies, and in the laws of friction and of undulatory transmission But here again the whole system of physical properties, though comprehensive, is a datum, and except in it no necessity could be shown why motion communicated by a bow must have this particular form, or why this particular form should find a correlative in a peculiar type of impulse communicated to the air. It is this system which as an identity in differences appears first in the effect of the bow on the string, and then in the peculiar impulses communicated to the air by the sounding-board. It is only as having such a unity behind both of them that the one of these phenomena can condition the other. And here again we may obtain a relative absolute by considering the compound tone of the violin string as perceived by the mind through the ear, simply on its own merits. It then becomes an ultimate fact, embodying certain relations between musical sounds And within this fact we may distinguish the necessary relation, 'When a tone is piercing in quality, the higher overtones are strongly marked in it' And finally, we may bring the physical and the musical system together

under the complex fact of the correspondence between the shape of oscillations and the character of tones, and say on the faith of this complex fact that where the oscillations are angular, the higher overtones are audible, and proceed to the whole system of deductions made possible by Fourier's analysis of all vibrations into combined single oscillations.

The idea on which we have been insisting—that of a system or unity which prescribes the relation between its parts or differences—is the idea of Ground, which includes the sphere of the Hypothetical Judgment, and indeed wherever it appears may be said to involve a Hypothetical element It is difficult to express the essence of this conception otherwise than by saying that the system is the same in the one difference or aspect as it is in the other We thus appeal to the notion of identity in difference, which we have taken throughout to be the content of judgment Only, as Ground, it is not *mere* identity, but *systematic* Identity,[a] a notion easier to illustrate than to define, but apparently equivalent to 'identity such that the differences in which it is manifested have definite relations to one another' Of course any such definition only repeats the characteristic which the account of the Hypothetical Judgment presupposes

Assuming however these characteristics as summarised in the above definition, we can draw from them two consequences that affect the idea of Ground First, it is plain that when once a Ground is rightly stated, in conformity

[a] The question has been raised (Keynes, p 264) whether the consequent in a true Hypothetical should be called an inference from the antecedent My answer is given by the doctrine of Ground I certainly agree that the consequent in a typical Hypothetical does not follow from the antecedent alone, if that means that it would follow if the two stood, without further implications, alone in a universe On the other hand, I hold that the warrant of the sequence lies in the relevance and adaptation of the clauses to the systematic world which they presuppose (see p 245 below), and assuming this world as a basis, I should say that the consequent is always a necessary inference from the antecedent It appears to me off the track of the distinctive Hypothetical type to treat the consequent as inferrible only when it is a sort of converse of the antecedent, or when the antecedent has the full premisses of a syllogism put into it (Keynes, p 264) Even here, after all, you have to presuppose some law of identity or Contradiction, so that you do not get rid of the assumption of a systematic world

with the true nature of the system which it presupposes, and with which it is in fact identical, such a Ground is unalterable except by alteration of this system itself. With what justification, theoretically, we refuse to contemplate such alteration of the universe as a whole, or how far practically we permit ourselves to contemplate it in respect of subordinate systems, e g man's moral nature or the type of disease, are questions that must be reserved for a general discussion of the postulates of knowledge. Formally, we may say, the whole cannot alter, because any alteration must be included in the whole But we shall see that so purely formal a postulate would not satisfy the purposes for which a postulate is required

And secondly, it is plain that a ground is not rightly stated unless it either embodies the whole essence of the system which constitutes the ground, or at least is exactly relevant to or compatible with that system, and to the particular bearing which a given interest in any context imposes upon it In the former case it is clear from what has been said that the hypothetical judgment must tend to expand itself into a categorical one. When we go to the root e. g. of geometrical truths we find ourselves affirming facts regarding the nature of space We shall thus at a later stage have to face the conception of judgments at once categorical and necessary, we have indeed anticipated something of the kind in speaking of the individual generic judgments The latter case is that which gives rise to hypothetical judgments having strict reference to a systematic ground, which they therefore imply, but do not need to express. Such are the ordinary statements of 'pure cases' in exact science, or geometrical truths as commonly treated without raising fundamental questions of the nature of space A 'pure case' is a nexus of differences reduced to their expression as the differences of the system in which they have their nexus Without knowledge of such a system we may analyse *ad infinitum* and yet never be sure that we have obtained a 'pure case' This has, as we shall see, an important bearing on the theory of Induction I gave a rough instance of a pure case on p 220 in reference to the annual rings of exogenous trees Such,

again, are the mechanical and chemical elements in the vital processes of man, e g the pumping action of the heart, the oxidation of the carbon in the blood and so forth, in stating any one of which as a necessary sequence of ground and consequent it must be treated as belonging to its own mechanical or chemical world, and not as an element in human life All such judgments are abstract in the fullest sense, and analytic, their very point is that they disregard the import which constitutes the individual. On the other hand, a system which is the combination of individuals in their full import, i.e the state in relation to moral beings within it, is most naturally dealt with not in hypothetical but in categorical judgments For the subject is either the concrete system itself, or an individuality subordinate to it, taken in its full concreteness It would be sheer pedantry to speak in hypothetical language of man's moral being, its ground, and its necessary relations

It is a corollary from the idea of Ground as a relation purely relevant to a positive determinate system that the hypothetical judgment, when ideally complete, must be a reciprocal judgment 'If A is B, it is C' must justify the inference 'If A is C, it is B' We are of course in the habit of dealing with hypothetical judgments which will not admit of any such conversion, and the rules of logic accept this limitation as they accept the custom of ordinary speech as to the comparative range of subject and predication Some cases of non-reciprocal sequence and their justification will be considered in the next section But here we are only concerned to explain the principle upon which necessary sequence must ultimately rest, and according to that principle, the unity of a system in its determinations, it follows that if A B necessitates A C, then A C must also necessitate A B We are not now speaking of causation, but simply of coherence in principle, and it is obvious that the idea of coherence in a system is reciprocal. A cannot cohere with B unless B coheres with A If in actual fact this is found not to hold good, and A B is found to involve A C while A C does not involve A B, it is plain that what was relevant to A C was not really A B but some element $\alpha\ \beta$ within it 'But may not the irrelevant element be just

the element which made A B into A B as distinct from A C, so that by abstracting from it A B is reduced to A C, and the judgment is made a tautology, i e destroyed?' 'If he is drowned, he is dead,' e g is not reciprocal But it may be objected that the judgment I desiderate is really 'If he is dead, he is dead' The suggestion is tempting, because it aims at cutting up by the root a troublesome scientific problem, viz the statement of connected attributes as purely relevant to one another and yet as distinct. We constantly tend either to insert irrelevancies by way of distinction, or to let both attributes fall back into the undistinguished abstract relation which connects them. To grasp a distinction in unity is an effort, and we dislike effort. Nevertheless, if it were impossible, the idea of system, of the one in the many, would be gone. In the above instance, the reciprocal judgment required would be something like 'If his heart has stopped for good, he is dead', or 'If he is drowned, he is dead through suffocation by water'. The former shape obtains reciprocity through the expansion of the antecedent, the latter through the limitation of the consequent [a] A systematic relation is always within an individual whole, and the priority or antecedence of its elements belongs to the imperfection of knowledge, and not to the relation in itself I am not saying that every individual reality exists endlessly in time, but I am saying that every whole in as far as its parts form a system has a nature which is independent of time, or (what really comes

[a] Mr Keynes's remark (p 271) called my attention to my omission of the case in which reciprocity is secured by dealing with the consequent (see however p. 255 below) My use of the term expansion is not his Applying it to the consequent, which *ex hypothesi* in a non-reciprocal Hypothetical is wider than the antecedent, he ought to mean expansion of *the expression*, but limitation of *the case*. But from his use of the phrase 'a more complete statement of the consequence' as equivalent to 'expansion', I think he has in mind his cases in which something like a middle term is introduced in the antecedent and dropped in the natural statement of the consequent, and that he hardly notes the point that as a rule the consequent, as a case, has to be cut down and not expanded in order to secure reciprocity My reason for insisting on the ideal of reciprocity has to do with my general attitude to Logic and to the Hypothetical Judgment (Preface, and p 236 above) I do, as Mr Keynes says I ought, maintain that the proposition A is imperfect See Knowledge and Reality, pp 190–2

to the same thing) continues positively and actively through the fugitive moments of time

But apart from time on the one hand and irrelevant elements on the other, I cannot see how the relation of conditioning differs from that of being conditioned Every B that is conditioned by A is the condition of A being such as to condition B, i e of A being what A is, and if the being of A were wholly relevant to B, this would be equivalent to saying that the existence of B involves the existence of A. In other words, if there is nothing in A beyond what is necessary to B, then B involves A just as much as A involves B [a] But if A contains irrelevant elements, then of course the relation becomes one-sided, as if we were to say that a plane section of a sphere has its radii equal The mention of the sphere makes the relation of coherence one-sided, the circle need not be regarded as a section of a sphere But, always assuming the homogeneous nature of Space, the relation between equidistance from a central point and uniformity of curve is inseparable, and it is impossible to see that either of these essential differences of the circle is prior to the other It may however be questioned whether in an ultimate sense any incomplete case can be pure, i e whether irrelevancy can be wholly avoided except by including the whole fact to which the judgment belongs What, e g has distance to do with curvature? The only answer is in the nature of space This amounts to a doubt whether in the end any Hypothetical Judgment can be true, and points us again to a further type of judgment in which such deficiencies may be made good

The relation of Ground is thus essentially reciprocal, and it is only because the 'grounds' alleged in every-day life are burdened with irrelevant matter or confused with causation

[a] Mr Bradley's remark should be noted 'I certainly agree here that if the judgments are pure, the relation holds both ways [ref to this place] But if in the end they remain impure, and must be qualified always by an unspecified background, that circumstance must be taken into consideration' Appearance and Reality, ed 2, 362 note I presume that the effect would be to demand great caution in asserting a reciprocal, e g not to assert it unless we knew by experiment or other special consideration that *ad hoc* the background might be neglected

in time, that we consider the Hypothetical Judgment to be in its nature not reversible The habit of thought is to proceed by determining an undetermined datum, and this habit is never wholly laid aside even in the Hypothetical Judgment which is theoretically its negation But a given condition, though interpreted in a single aspect by the judgment which draws its consequence, may have other consequences just for the same reason for which its consequent may have other (alternative, not merely co-operative) conditions The 'other' conditions arise through a variation of the irrelevancy present in the given condition, as, if a circle has been said to arise through cutting a sphere in a plane, this condition may be varied by altering the superfluous relation in which a plane figure bounded by a line equidistant from the centre can be regarded, e g it may be taken as a section of a cone, or as an ellipse with equal axes And just as each of these irrelevancies would present the antecedent of circular curvature in different garb, so the presence of an irrelevancy which is thus capable of variation involves all the independent consequences that follow from the irrelevant idea—in this case that of a sphere-section—which has been included in the condition If we restrict ourselves to the relation of equidistance in a plane, we can get no result beyond that of a circular figure with the properties which belong to it in the geometrical system

A ground that admits of such variation is not only partial or abstract, i e one which leaves the true ground in a measure to be understood, but is actually in part 'impure', i e burdened with matter which gives rise to diverging consequences, and makes the ground itself one among many converging grounds.

We have thus seen the idea of Ground in three aspects; as an actual system, interpreted in its bearing upon its parts; as a 'pure case', i e a factor within a system stated in terms precisely relevant to the system and entering into a nexus in virtue of that system, and as an 'impure case', i e a condition weighted with irrelevant matter and so failing to express the real nexus which is aimed at The first of these three is necessarily categorical in import, and may perhaps be identified with Schopenhauer's 'Seyns-Grund', or Ground of Being, the two others are primarily hypothetical and only

imply reality behind them, and correspond together to his *Erkenntnissgrund*, or Ground of Knowledge It need hardly be remarked that the ground of Being is also the only genuine and complete ground of knowledge. In respect of reciprocal character they must be divided differently; the first two, the complete and the abstract ground, being necessarily reciprocal with their consequents, and the third being as obviously not so.

The idea of cause

b Cause may correspond either to the complete form or to the incomplete forms of Ground In the former sense it can scarcely be taken to differ from ground at all In the latter sense it is a distinct species which is included in a common genus with the incomplete forms of Ground

Cause as complete Ground

(1) Cause[1] as corresponding in meaning to complete Ground would consist in the exhibition of some selected attribute or event—the effect—in the totality of systematic relations which constitute its necessity And in such meaning it can scarcely be taken to differ from Ground, because the temporal succession, which seems the natural differentia of Causation, disappears in the reference of the effect to a positive and continuous system Mere temporal relation is negative, is nothing It is only the unity behind the temporal relation that can bind cause to its effect, and in the real or complete Ground this unity is made explicit The cause of the earth's being where it is at this moment may indeed be popularly indicated by saying that it was, wherever it was, at the previous moment; but strictly of course the relation of the present position to the last position when fixed before the mind as discrete and successive in time is simply that the one is not the other, which is so far the same relation that subsists between the earth's present position and the sun's or moon's last position, and amounts to nothing at all The cause of the earth's present position is the persistent velocity, together with the persistent influences regulating the direction, of its passage through space This meaning of Cause is the ideal logical import of the term, and is what Mill meant to indicate when he defined Cause as the 'sum of the conditions' The word 'sum' is unfortunate, because it indicates a special

[1] On the conception of Cause, see some very acute remarks of Professor Clifford, Lectures and Essays, 1 150

way which may be inappropriate of combining the factors. The totality of the relations would be a better phrase than the sum of the conditions

The only difference between Cause in this sense and Ground would be that Cause, though not a sensible event, still retains an import relative to the explanation of sensible events or of attributes entering into events (character, health, &c.), and is therefore not coextensive with Ground, which includes e g geometrical relations where the phenomena of process in time are wholly wanting There would be no sense in saying that the attributes of a triangle are the cause of those of parallel straight lines, or *vice versa*[1] The distinction, however, is more one of usage than of theory On the one hand, the effect, subsequent in time, which is exhibited as one relation or difference within a necessary nexus, is necessary to the persistence of the whole system and to the evolution of its significance, so that the parts of the unity or system are reciprocally necessary in *complete* Cause as in complete Ground, and on the other hand if in investigating a ground, say in geometrical matter, we go back to the whole system of fact which is at the root of the necessary connection, we shall be justified in treating this fact as a Cause We could hardly be censured for saying that the nature of space is not only the ground, but the cause, of the attributes of triangles and of parallels alike

Cause then, in its largest sense is a real ground, and ultimately there is no complete ground which is not a real ground Ground and Cause are thus not identical but convergent conceptions, i e as they are completed they tend to coincide, and the striking differences between them depend on a comparison of their imperfect and ultimately self-contradictory forms

Complete Cause, like complete Ground, corresponds to a Hypothetical Judgment whose condition and consequent are reciprocal If, as is perhaps the case in Mill, the phrase 'sum of the conditions' is not limited to relevant conditions, and the

[1] Schopenhauer's 'Seynsgrund' describes the relation of such cases, supposing the rational connection to be central and fundamental to the contents connected

hypothetical judgment which expresses the nexus of such a sum with its effect is consequently not reciprocal,[1] the notion of sum of conditions loses the only merit which it appeared to possess But if it means, as it seems to mean, a persistent and systematic fact, then it agrees with other indications in suggesting that for the complete Ground *or* the complete Cause we must go beyond the Hypothetical Judgment

Cause as an event, distinct from Ground

(2) Cause as an event in time is thus an imperfect conception Indeed it is hardly possible to formulate the idea of one event in time as the cause of another that falls, in time, wholly outside the first Cause is always taken to be more or less of a complication of relations and circumstances; and these, as acknowledged to bear on one another, are not mere events in time It may indeed be retorted that *mere* time is an unreality and that no one ever said that causation was in mere time, i e in succession taken as discrete, but that real time involves continuity as well as discreteness, and in such real time causation really is Such a retort might be ill-founded as a statement of common logical opinion, but would in substance express the principle which I am endeavouring to explain *Mere* time is mere succession, but real time involves something that is not in succession, though it remains through succession The consciousness for which there is time has begun a process which tends to abolish time To say that in this sense Causation is in real time is to say that Cause corresponds to an incomplete ground, i e. a partially known unity including the factors which are in question as Cause and Effect But when we come to speak of an incomplete Ground, the difference between thought and reality emerges, for it is only the complete Ground that is the real ground When the ground in thought is distinguished from the ground in fact, then the cause is one with the ground in fact and is separable from the ground in thought, which latter is sometimes called by analogy the *causa cognoscendi*

[1] Cp Essays in Philosophical Criticism, p 96 note Of course if Cause as = sum of conditions is compatible with Mill's plurality of causes, the Cause as sum of conditions cannot = Ground But it ought to be incompatible, for in any concrete circumstance that may be named as condition, what is not relevant is not condition So the sum of conditions ought to be restricted to the relevant or minimum conditions

Of course the *causa essendi*[1] must be a *causa cognoscendi*, but a *causa cognoscendi* need not be a *causa essendi*. As a matter of fact the ground in thought or *causa cognoscendi* often belongs to the effect in time,[2] but may be any element whatever related to the real ground, whether cause, effect, or abstract principle

The root of these distinctions is that the nexus of ground and consequence is at this stage still charged with irrelevancy The cause and effect, ground and consequent, are all of them at this stage concrete events, or groups of incomplete relations, among which the special aspects belonging to any *nexus* that may be in question have not yet been *freed* by analysis As a result of this state of things the hypothetical judgment which embodies such a connection follows the analogy of singular judgments or of imperfect universals, and has an antecedent which is not affirmed by affirming the consequent, just as the subject of a singular or generic judgment is not affirmed by affirming its predication And in so far as the hypothetical judgment is taken to be the natural vehicle in which to assert causation, this characteristic of it agrees with the popular view that the same cause always has the same effect, but the same effect need not always be due to the same cause This doctrine, formulated by Mill under the name of the plurality of causes, and wholly incompatible with his view which treats Cause as 'the sum of the conditions', is a mere translation into analytic science of the notion of subject and attribute, here quite out of place The degree of truth which the view possesses depends solely on an imperfection of knowledge and not in any way on the nature of causation

[1] Schopenhauer's distinction between the *Causa essendi* (Seynsgrund) and the *Causa fiendi* (Ursache, cause proper) is reduced by the view taken in the text to a distinction of degree Effect cannot be in succession to cause in the sense of falling outside it, there must be a real whole which includes both

[2] It is worth remarking as a matter of usage that 'antecedent' in the conditional sentence has absolutely no allusion to the temporal relation of the events connected Its name may have come from the usual grammatical place of the condition, or from some profounder idea of priority But this would be a mere historical connection Mill's application of the term to succession in time creates quite an unfounded idea of correspondence between Causation and the Hypothetical Judgment

It is an old story that if, having said that 'All men are mortal', you then further say that 'A B is a man', you are committed to the assertion that he is also mortal, but if you prefer to make the more cautious assertion that he is mortal, you do not thereby pledge yourself that he is a man The same maxim in relation to the Hypothetical Judgment is summarised in the formula 'Assert the antecedent or deny the consequent'. It makes no sort of difference in the application of this formula whether cause is antecedent and effect consequent or *vice versa.*

Thus the reason why the law of Causation has been stated in the form 'The same cause always has the same effect' rather than in the complementary form 'The same effect always has the same cause' is that popular philosophy tends to start from the event which comes first in time, as logical antecedent, because the primary source of knowledge is simply to observe processes in time and so the further determination of any datum or circumstance *when effected by this elementary method* corresponds to the succession of events in time, and that event which comes first is taken as the datum to be further determined, and that which comes after is regarded as its determination Of course then the same datum always has the same determination, for every content—and a datum is a content—is an identity, and having attended to an identity in respect of one of its differences we are quite safe in saying that this identity—this datum—will always have the same difference. For if it seems not to have, we may say either that the difference is disguised, or that the datum is not the same And so we come first to the principle that the same cause always has the same effect, and sometimes, to make quite clear that we are simply regarding a real content in respect of a difference which we have selected out of its concrete nature, we add 'the same cause under the same conditions', or 'in the same relation'; thereby showing that we know very well that the concrete cause has all sorts of different effects under different conditions and in different relations

It is usually presupposing the truth of the first principle that we go on to consider whether the same effect always

has the same cause; and neglecting in the effect, which we take at first as a goal in which thought can rest, the idea of a limiting reference to a particular antecedent, we are impressed by the variety of relations and conditions compatible with the undetermined result, as contrasted with the single aspect in which we watch the operation of a cause; and we forget that each set of these generates the effect in a slightly different form It seems so common-sense to say 'If a man is drowned he is dead, but if he is dead he need not therefore have been drowned', that we forget that, if he is dead *in the particular way produced by drowning*, then he has been drowned We might, from the very first consideration of an effect, draw a parallel to the popular form of the Law of Causation, 'Same cause in same relation, same effect,' by a Law of Effect which should affirm, 'Same effect, in same form, same cause.' But, as our first impression in starting from Cause is of the Identity of Effects, so our first impression in starting from Effect, because there is no simple guide to further determination, is of the Plurality of Causes Really however we have to supplement these ideas by those of the Plurality of Effects and of the Identity of Causes It is, technically speaking, an accident which of these four points of view attracts our attention first The knowledge that the same effect has the same Cause is not necessarily later than or dependent on that of the converse maxim 'If a man is dead his heart has stopped' does not involve a knowledge whether stoppage of the heart must always cause death Still, as we saw, the common law of Causation is most readily suggested by our experience of simple observation, and has a certain real pre-eminence because of this experience Experimentally we only follow up Cause into Effect, not Effect into Cause And thus the natural tendency is to identify Cause with Antecedent, and the common law of Causation 'Same cause has same effect' is the resulting one-sided application to Cause and Effect of the commonplace rules of the Hypothetical Judgment Of course, when we have both principles together we have what is truer than either alone; but in itself neither *prima facie* involves the other

We have seen, then, that even the incomplete or partially

known Cause can always enter into a Hypothetical Judgment either as Ground or as Consequent In the same way it is possible for Effect to be either Ground or Consequent in Hypothetical Judgment But Effect can never be Cause, unless we go back to the doctrine of *complete* Ground in which the boundary between Cause and Effect really melts away Effect can never be Cause, and yet Effect may be as inevitable, as essential to the sequence, as necessary a Ground of hypothetical nexus as ever Cause could be It is a well-known saying that we cannot conceive a storm to have been less violent than it actually was without the difference involving differences in a series of physical processes going back *ad infinitum* in the causal nexus Yet we cannot bring ourselves to treat the storm as the cause of the previous physical processes which, as we say, resulted in it. The distinction which is at the root of our inability to do so is of course the distinction of Time The operation of this distinction has never been more trenchantly stated than by Aristotle,[1] who lays down the general doctrine of Ground with perfect clearness, but in going on to deal with causation in succession doubts the security of all arguments from cause to a subsequent effect For 'in the moment between the two, it would be false to say that the second has taken place, although the first has already taken place'. It may of course be rejoined that the cause cannot have completely taken place if the effect has not begun. This rejoinder however depends on the postulated unity of the causal process and on the consequent continuity of time If we press this point of view, it takes us back to the doctrine of complete ground, which consists in exhibiting the unity or continuity of causation regardless of succession in time But we are anxious at this moment to do all we can in the way of elucidating the problem involved in the natural conception of causation as sequence, and therefore we will not simply fall back on this notion of complete ground Granted that time and sequence are continuous, yet they are also discrete There is indeed no empty μεταξύ or interval in which we can stand and say, 'The cause is past and the effect is not begun.' But unquestionably we can

[1] Anal Post II 95 a, 30 ff

make a stand at any point in the continuous sequence and say, 'So much is (or "has been") real, the rest is not yet real' And what is not yet real cannot be the cause of what is or has been real

This appears to be the root of our whole conviction about cause and effect in time Even after the entire sequence has been realised, and when all of it is alike real or unreal, as we may choose to count the past, still the objective temporal order into which we project our experiences embodies the succession of relative reality and non-reality which attached to the order in its original constitution We remember that this became real while that was still unrealised, and we therefore feel that however certainly they may reveal themselves as parts of a single whole, we can never hold the event which came after to be an element in the actual determination of that which went before It would involve to our minds the absurdity of treating the existent as caused by the non-existent

I do not mean to deny the reality of this distinction It amounts to just what it is Time is a condition, is *the* condition, is we may almost say the inmost nature, of our sensitive experience. The *first* operation of our intellectual synthesis is to build up an ideal objective order which, though itself not in time, yet contrasts as a more or less completed reality with the sensitive experience which is always passing into it It is obvious that we can only construct our anticipation of reality out of its positive content so far as known to us, and its positive content so far as known to us belongs to the past We may fill up gaps in the past out of other parts of itself, but we can get nothing out of nothing, and therefore can draw no anticipations from the future Therefore *at any given* moment we have no choice but to say that the future is conditioned by the past, and the past not by the future, effect by cause, and cause not by effect Cause, at any such given moment, is what we have, and effect what we have not. And further, taking the past as a representation of all that is, for it is the only positive content that we have to represent anything, we are right in saying that the past as a whole is the cause of the future as a whole What is, is, and will act as it

will act, and what we already know of it is the only source from which we can anticipate its action But of course the past *as a mere series* of events *is* past, it has ceased to be real just as truly as the future is not yet real; the relation between two nothings is nothing, and cannot cause anything. The same applies to particular events; it is hard to find words which describe the negative relation of effect to cause, and which do not apply equally to that of cause to effect Effect cannot be the cause of its cause, for the reason that it is absurd to find the cause of something existent in what does not (yet) exist But is it less absurd to find the cause of what is now entering upon existence in what does not (any longer) exist? Yet this is what we do if we take cause as an event and effect as a subsequent event Hence we are driven to the *second* operation of our intellectual synthesis, which is, after erecting an objective temporal order not itself in time, to strip this temporal order of the importance attaching to its successiveness, and to treat it more and more as the expression of a plan or unity *Except* as the expression of such a unity, causation, as we have seen, disappears, but *as* the expression of such a unity, the causal relation ceases to be in time, because the positive connection between cause and effect being made manifest, the two are united in the complete ground This must be carefully distinguished from saying that time may be introduced anyhow without making a difference, it does not mean that eggs boil water, or that death produces a revolver-shot It simply means that the order of succession, which has a largely negative aspect, disappears in the significance of a positive systematic connection, and that we do not in fact, in considering a past sequence, regard what came later in time as less fundamental or elucidatory than what came before

Then is not Time real? I answer that everything is real, so long as we do not take it for what it is not Time is real as a condition of the experience of sensitive subjects, but it is not a form which profoundly exhibits the unity of things And when we transfer the true judgment 'What has not yet happened must be a manifestation of the same unity which is involved in what I already positively know' to a totality

which is already in all its parts equally real, we confuse, and give 'time' a reality which it has not Such a confusion is involved in the idea that there can be no more in the effect than there was in the temporal cause, and in the tremendous power consequently exercised by historical analysis over common minds. The confusion is reinforced by another aspect of causation Practice, like sensitive experience, is in time In translating a plan into practice, the relations of succession hold good A sequence is what it is, and nothing else; and a reversed sequence would simply be different. Therefore for practice the earlier event is more important, in the sense in which the means is more important than the end For knowledge of the end does not give power to produce, but knowledge of the means does But this importance begins and ends with practice, and even there it only exists in virtue of the unity whose nature is expressed *for us* in the fabric of ideal reality—a fabric which is not in time

Thus it is easy to see the relation between Cause and Ground in the imperfect stage in which they are distinguishable conceptions Ground is a content which is perceived, by reason of any systematic relation whatever, to involve the determination of another content Cause is also a Content perceived to involve the determination of another content, and is therefore a kind of Ground, but is primarily confined to the special case in which the determining content is real and the determined content unreal But as this is merely a negative relation, even the first presumption of causation in some degree supplements it by the postulate of a positive nexus, and we know very well that in practice if no positive nexus, no continued identity of process, can be alleged, we do not allege causation. A familiar illustration is the sequence of day and night It is generally urged that if causation were mere succession, day must be the cause of night, but that really day is not the cause of night, because both are effects of a common cause, and either might very well go on without the other This is one of those trivial examples that seem hardly worth arguing about, and yet, if argued about, must be treated at length I have shown that I at least think

'Causation = mere constant succession' plainly false and in contradiction with facts, but if I must discuss this case I should like to have the terms defined The question, as I take it, is, Does 'day' in the sense in which it is here used include a unity, a system, a principle, which is continuous in and responsible for night? If so, day is the cause of night, if not, not The reason why we think it wrong to call day the cause of night is not because night has the same claim to be called the cause of day There is no contradiction here In our ordinary way of treating imperfect causation there is no reason why the daylight hours of Monday should not be the cause of Monday night, Monday night of Tuesday, Tuesday of Tuesday night, and so on The difficulty does not lie in the sequence being of this alternate nature—a single oscillation of a pendulum is certainly the cause, though not the complete ground, of the next—but *that the persistent unity which lies at the root of both phenomena does not fall within the natural definition of either* If day meant not merely the presence of light on the earth's surface, but, in relation to any given point on the earth's surface, that portion of the earth's rotation which carried that point from its sunrise to its sunset, then I do not see how it could be denied that this portion of rotation, in as far as it determined the position of the selected point throughout the immediately succeeding section of the rotation, was the cause or principal condition of the ensuing night But of course the name 'day' is applied e g to the six months' day of the poles only by metaphor, being a chronological idea which has become largely independent of the relation of a particular place to the sun's illumination, and having legal and social meanings which do not admit of an antithesis with night So the true reason why we do not like to predicate causation of this sequence is simply that, owing to their varied accessory significations, the terms day and night do not apply to successive stages of a continuous natural process, but are mere chance distinctions that are drawn, according to our shifting purposes, on the surface of that process

But though we do not allege causation where we cannot allege a positive nexus, yet, as I pointed out above, there is

a considerable distinction of degree between the objective temporal order and the intelligible unity of things The less we advance beyond the stage of perception and narrative to that of science and intelligence, the more does the negative distinction of time retain its significance. Strictly speaking, the distinction between cause and effect in time is only real at an arbitrary moment in which we draw an ideal line across the temporal process of sensitive experience, between the real and the unreal When cause and effect are both absorbed in the past, the distinction is only transferred by memory into the content of reality, which thus takes the form of the objective temporal order This order would be an intolerable chaos but for a certain presumption of causation, i e of unity, which binds it together according to some sort of system, there is no real history apart from the idea of causation Nevertheless this unity remains for the most part inchoate, i e only in some degree explicit; and so, though not itself in time, presents the scheme of a *de facto* evolution in time as a sort of extended memory, with the transferred character of determination of unreal by real, i e of effect by cause. Now it is plain from what has been said that the distinction of Cause and Effect is self-destructive It is utterly impossible to be successful in the investigation of a causal relation without reducing it to the intelligible unity of a complete ground History therefore, in the sense of the mere record of remembered fact, would seem to have for its ideal to disappear into systems of hypothetical judgment, in which complete ground should do duty for cause and effect, and the relation of time should disappear.

This conclusion is true, in my judgment, in relation to the mere phenomena of the past, and the resulting connection between causation and ground But as regards what we really mean by history, such a conclusion is repugnant to our feelings and inconceivable to our understanding The reason is plain History is not merely a name for the recorded past. A series of astronomical observations is not history, it is science, and has no value but for science, unless by chance it throws light on the observer's character or on the state of science in his time as an element in the

condition of man What we mean by History is the revelation of man's nature in action and intelligence And when we deprecate the reduction of history to a system of hypothetical judgments founded in some single abstract individuality—to a science like abstract mechanics or abstract economics—what we really mean is that man's nature reveals itself in individuals, in actions, in forms of intelligence, and we do not want to lose these realities in abstractions of relativity and necessity But if we consider that hypothetical necessary or relative judgment is entirely based upon categorical judgments, that all nexus is within an individuality, we shall see that history may be received into the intelligible unity of knowledge without sacrificing its concrete import and characteristic significance This could only be destroyed if we insisted on predetermining within *what* whole or system we should find the facts of history to be necessarily related

And no doubt a suspicion of some such prejudice is operative in the reluctance to absorb history in 'science' to which I have adverted If science meant exclusively the sciences which grow out of the one-sided forms of measurement, then we should rightly deny that there is a science of history, and, for the same reasons, that there is a science of art, of political form, or of religion We escape however from such suspicions if we remember that all connection is based on fact, and all analysis on individuality, and that the nature of the facts or of the individual whole or system with which a science deals can *ex hypothesi* be only for that science itself to determine

Thus the conception of Cause as an event in time anterior to effect gives way on analysis, and forces us back to the conception of the complete Ground, and the conception of incomplete Ground (*causa cognoscendi*) as distinct from Cause, expands into the same unity, which, as we saw, is at once the complete Cause and the real Ground, i e the relation of part to part within an actual and systematic totality. This relation of part to part, either burdened with irrelevancy as in the ordinary hypothetical judgment, or pure and relevant as in the hypothetical judgment whose terms are reciprocal, forms the content of the abstract universal judgment And

this abstract judgment is a divergence from the concrete evolution of thought, and joins with the mediate and quasi-generic judgments of the sciences which arise out of one-sided measurement [1] But it may also be regarded as an element in or aspect of the popular or transitional quasi-collective and the generic judgments which are enumerative or individual in form but analogical in meaning Analogy, as we pointed out, is compatible with systematic necessary relation

On the other hand, though a complementary aspect of the universal judgment, the pure hypothetical is destitute of finality, and incapable of standing alone It demands a reversion to concrete thought by the fact that it presupposes a self-existent whole Apart from such reversion it may become a wholly arbitrary and meaningless play of fancy, presupposing conditions which are not made explicit

As I have not yet dealt with negation, I shall leave the negative forms of the hypothetical judgment to be dealt with in the same chapter as the disjunctive judgment, with which they are closely connected

Supposition in universal judgment.

c The above appears to me to be a fair account of the hypothetical or abstract universal judgment considered from a strictly logical point of view In one important respect however it is *prima facie* at variance with conclusions which might be drawn from the grammatical shape of these predications as to the attitude of mind in which they are normally made. The point in doubt is the existential significance of the universal judgment. The account which I have given treats the existential implication which attaches—undoubtedly in very various degrees —to the different forms of the universal judgment, as cognate with the existential affirmation involved in the singular and in the perceptive judgments. But it has been frenchantly laid down by Mr Bradley that a different view is suggested by the attitude of the mind in all purely abstract judging—abstract be it remembered not necessarily in content, but only as all thought is abstract when contrasted with sense-perception I am unable to reconcile this view with the existential value of judgments about individuals designated by proper names, in which there is no direct

[1] See Scheme, *p* 86

reference to sense-perception nor to anything but a content, whose real existence is as I imagine taken to be asserted owing to its concrete nature But I proceed at once to discuss the analysis of the abstract universal affirmation, from the point of view to which I allude

It will be remembered that in the general discussion of the nature of judgment, we agreed that the ultimate subject in judgment was never an idea,—never that is to say even a logical idea or content, for the particular psychical image we found not to concern us in logic The ultimate subject in judging was always, we held, the Real, which in the act of judgment is qualified by certain logical ideas So long as the immediate subject was present perception, whether additionally qualified by significant ideas or not, all went smoothly, for the immediate subject was then simply the point of contact with the ultimate subject of judgment. Some difficulty, indeed, arose in explaining the real reference of the Singular judgment in which the subject may fall outside present perception and may have to be united therewith by a constructive process Still, however, the immediate subject was a determinate element in the whole of Reality, having individual existence manifested within the sensible series, although e g in the case of an organism, still more in the case of a man, not of a kind that could really be presented by sense-perception as such So far therefore we were, in judging, referring an ideal content to reality in some particular concrete aspect, and therefore our Judgment was still plainly existential, i e such as to become false if the concrete element of reality described had no place in the series of sensible events

Now when we come to deal with the Universal Judgment it must be admitted that *at least* as a question of form the reference to reality becomes less easy to define The pseudo-collective and the analogical judgment at any rate (dismissing the conception of the individual generic judgment, in which the singular judgment revives) are unquestionably capable of an interpretation which reduces them to the pure Hypothetical Judgment In our old acquaintance 'All men are mortal' the 'all' is too obviously *not* collective to stand

in the way for a moment We certainly might be driven to confess that in so judging we had only asserted 'If man, then mortal' or 'Where man, there mortal' Such an interpretation is involved no less in Mill's analysis of the import of propositions than in Lotze's or Bradley's treatment of the universal judgment To affirm co-existence of attributes is not to affirm existence of subjects [1] The analogical judgment has this aspect even more plainly 'An organism as such is mortal' means, it may be urged, if taken strictly and without counting implications, 'If organism, then mortal,' 'Where organism, there mortal,' 'Whatever is organic is mortal' If in consequence of such an assertion we take it that organisms are actual elements of the real world, this is implication—though very strong implication—and not assertion

According to this analysis, the essence of which is to regard the implication of existence in these judgments as outside the matter affirmed, all abstract affirmation—abstract merely in the sense of not referring to present perception or to particular sensible events—is on the level of hypothesis, has for its immediate subject an idea not a reality, and consequently has no existential import, or 'deals purely with adjectivals' [2]

The identification of the universal judgment as such with affirmation based on hypothesis being thus made, the further development of the view turns on the nature of hypothesis or supposition The essence of supposition is that it is *qua* supposition, wholly arbitrary in its starting-point. Its content is taken, not given, is an idea, not a fact (if *per accidens* a fact, is not used as a fact, i e its existence is not argued from), and is considered not in itself, but for the sake of its relativity, i e for what flows from it, for its consequences The essence of supposition is in short argument from content, and not from existence of content The consideration of any proposed legislation, e g a Reform or Land Act, with

[1] This comes out very emphatically in Mill's account of Definition, which when Real at all, he analyses into a meaning *and* a postulate of existence In this he is pretty much at one with Mr Bradley's account of universal judgment See also Mill's account of mathematical truth.

[2] Bradley's Principles of Logic, p 81

reference to its consequences, is an example of supposition. 'Suppose every adult male to have a vote, it will be impossible to maintain indirect taxation,' 'Suppose Ireland to have a Statutory Parliament, the Imperial Parliament will by this fact itself become statutory' Or again, 'Suppose beings endowed with perception but confined to a plane in the exercise of it, they must see all figures as lines or points'

The process is to select or to fabricate, apparently at pleasure, an ideal content, to think of it as in connection with some known reality, and to judge the result as a truth conditional on such connection.

It is plain that in this operation, subject to a certain reservation to be mentioned directly, the supposition selection or fabrication [1] of content is arbitrary, but the judgment proper is necessary

What then, precisely, it has further been asked, is in such a case affirmed about ultimate Reality? Plainly, *not* the existence of the content as a fact in the context of our world We may take Sigwart's instance, reproduced by Bradley. 'Si tacuisses, philosophus mansisses' Something is here affirmed, but not the actuality of the content, which is by the strongest implication denied Or again, 'All trespassers will be prosecuted' The truth of this declaration does not depend on there being trespassers, though it cannot be tested unless there are trespassers

When we have once accepted this point of view, and excluded as unessential the various implications of existence which attach to various Universal Judgments, the conclusion is inevitable Judgments so regarded do not affirm as true of Reality any explicit content or even any connection of explicit content We have seen that it is no impeachment of the judgment that its content never has been nor can be actual. 'The necessary may be impossible or non-existent.'[2] And as for the *connection* of content, though it is necessary, yet it cannot be actual unless the content to be connected is actual, and the judgment may be true though the content

[1] Selection and fabrication differ only in degree, not merely because all fabrication is selection, but also because all selection is fabrication—involves the constitution of an idea from a given or chosen point of view.

[2] Bradley's Principles of Logic, p 186

be incapable of actuality Therefore the truth of the judgment, according to this extreme analysis, depends neither on the actuality of its content, nor on the actuality of the connection it alleges within its content What then does such a judgment assert of that Real which is the ultimate subject in every judgment? Simply this, that the Real *is such* that under the ideal condition which forms the immediate subject of the judgment it will furnish the ideal consequent which is expressed in its predication The hypothetical judgment would then be *illustrative* but not *enunciative* of Reality The property of Reality which it illustrates may however be accessible to knowledge, or again, according to the view before us, it may not. Simple examples of the former case form the best explanation of the conception which we are discussing 'If you ask him for money he will refuse you' The real quality of the real man, on which this prediction rests, may be that he is a miser, then his miserliness x is the real fact, *not* apparent in the judgment, which the supposition and consequent a and b only illustrate by its effect in an ideal case; and which is categorically affirmed, but only as an unknown x, in the Judgment And it is conceivable that the property of Reality which lies at the root of the judgment may not be known, and wherever the connection envisaged as hypothetical is considered to be ultimate and not susceptible of further explanation, the property of Reality at its root *is* pro tanto unknown Such a connection would be, I suppose, 'Whatever is material, has gravity' The property x of reality which is here categorically asserted as the basis of this connection, 'If a then b,' is, I presume, and is likely to remain, unknown

The real criticism which I have to offer upon this view is contained in the whole account of judgment which has been submitted to the reader Its point and purpose have been to exhibit the aspects of fact, and of relativity or necessity, within the judging function as complementary and inseparable, but as differently predominant in dealing with different contents Consequently, the abstractness on which their respective predominance has hinged, has been treated as the abstractness of contents, and not as that

formal abstractness which is merely the mark of thought as opposed to sensuous perception And the result has been to exhibit the graduated existential implication of universal judgment as falling within and not without the import of those judgments, and as homologous with the aspect of existential affirmation in perception and in historical narrative But I propose to comment very briefly on two special problems raised by the view before us

Simple and conditional Judgments.

(1) It is easy for any one to form for himself a *catena* of universal judgments, beginning where the proper name becomes significant, as with Europeans, Christians, Peelites, and passing on through the generic conceptions of classificatory sciences, to physical and mechanical principles, geometrical axioms or theorems, and finally to imaginary and impossible but self-coherent hypotheses, like that embodied in the ingenious *jeu d'esprit* entitled 'Flatland' Such a *catena* is at the root of the view of Universal Judgment which I have endeavoured to formulate Any one who will take the trouble to follow up and to fill out with instances familiar to himself the terms of such a series, will hardly be able, I think, to avoid forming the conviction that no single type of proposition is adequate in the same degree to the content of all universal affirmations If, however, we are compelled to choose, the conditional proposition is the more adequate But it is not adequate Such a judgment as 'All Christians hold that God is a Spirit' combines collective and generic meaning with necessity It indicates not merely that the doctrine is of the essence of Christianity, i e that 'If a man is a Christian he holds' &c, but that there is *a* Christian world, realised in *many* individuals, which is united in this conviction

And on the other hand, after our too laborious consideration of the subject, it is not necessary to remind the reader that relativity is operative within the judgment from the moment of the first analysis introduced by perception into the data of sense, that is to say, even when the judgment assumes the external form of the impersonal proposition, which indicates that identifiable subjects are not yet constituted in virtue of determinate qualifications

The fact then is this Speech can express no logical relation except by making it the correlative of a word or clause But the common types of speech, which have been made the basis of logical investigation, are direct and simple They therefore embody only one aspect of the concrete logical thought, and leave all others to be guessed at from variety of context and the requirements of content. The process of recognising explicit linguistic correlatives of relations which in these direct forms are only implicit is a slow process—Aristotle did not recognise the conditional proposition on its merits—and begins, as is usual in such cases, by substituting one-sided abstraction for unanalysed confusion Therefore by adopting either the direct or the conditional mode of expression we ignore in the first case, and formally exclude in the second, elements without which it is impossible for judgment to exist The direct or 'categorical' form is used till it breaks down under the burden of an import for which it was not invented, and the conditional form then takes its place, to express the relative import, the determinate and restricted reference from part to part within a whole, which now insists on making itself predominant But both forms, not one only, are inadequate to their content The content of categorical assertion has relativity, and that of hypothetical assertion has absoluteness But categorical assertion (I am speaking of grammatical form) leaves the former, and hypothetical assertion leaves the latter to be implied The two forms however have not an equal right to the ground they commonly claim The first comer has, naturally, occupied all it could get, and more than it could adequately deal with It is against this encroachment of the categorical judgment-form that modern logicians have rightly espoused the cause of the hypothetical. The true frontier is, beyond a doubt, where the singular judgment ends After that point, if we dismiss the Individual Universal and omit to consider the Disjunctive, the purpose of assertion is relativity or sequence, and absoluteness or existence is only its presupposition. Formal Logic granted this territory, that of the pseudo-collective and the analogical judgment, to the Categorical *Judgment*, simply because it found the Categorical or direct *Proposition* in possession

And the further analysis of the irrelevant connections which encumber every perceptive and singular judgment, and which, if understood to be *generally* affirmed, immediately become false, may be regarded as a reprisal on behalf of the hypothetical judgment, which, now that attention has been drawn to the meaning made explicit in the conditional sentence, threatens to dislodge the categorical judgment even from the fields of perception and of history The degree in which such a claim should be conceded has been indicated above [a]

Thus the simple and the conditional [1] propositions are, the one of them an indeterminate and the other a one-sided type—both therefore imperfect expressions of thought The latter is borrowed no doubt from, or is at any rate most appropriate to, the extreme and arbitrary attitude of mind known as supposition. It is natural, as we have seen, that the empire of the first comer should not be broken into unless a forcible demand arose for something utterly incompatible with its type The representation which is correlative to supposition is thus the sole representation in ordinary speech of the aspect of necessary sequence within the judgment. This is why, when logic awakens to this aspect, it is tempted to find its essence in supposition.

But supposition is not the essence Supposition is inten-

[a] See p 261 above, and note Mr Bradley's later statement, 'Thus we really always have asserted subject to and at the mercy of the unknown And hence our judgment, always but to a varying extent, must in the end be called conditional' To the word 'unknown' he adds a footnote, beginning 'Hence in the end we must be held to have asserted the unknown. It is better, however, *not* to call this the predication of an unknown quality (Principles of Logic, p 87), since quality either adds nothing, or else adds what is false'—Appearance and Reality, ed 2, 362 The following discussion of supposition and the hypothetical judgment is not relevant to and does not deny Mr Bradley's general position as formulated above I retain it mainly for its treatment of illegitimate hypothesis, which seems to me an interesting point.

[1] The grammatical difference between the two is connected with their logical import in that the conditional sentence provides for an express analysis *ad hoc* of the subject-qualification of Reality, whereas the direct sentence simply indicates a subject-content by a name If we insist on the name being the *right* subject name in the context, as in Aristotle's *καθόλου*, we have an intermediate stage

tional abstraction or selection culminating in fabrication; and the essence of the judgment is not in the fact of intention, but in the logical ground and justification of the intention The interference of the will is no *differentia* in a logical process All thinking presupposes will, but the *guide* of thought can never be moral purpose, the purpose of will, but must always be logical necessity. What remains then out of the fact of supposition is that a content is taken, chosen, fabricated, if we will, with a view to its relative aspect, to its consequences. But what is presupposed in so taking it, and what are the conditions under which alone it can be so taken—whether, that is, it can be taken in the air and without either self-relation or relation to an actual system, are questions in no way touched by the mind's attitude in supposition. We shall see that speech finds the embodiment of a necessity resting on fact in the proposition which expresses the disjunctive judgment.

The basis of Hypothetical Judgment

(2) The character of Reality on which the sequence 'if *a* then *b*' depends must be in the ultimate analysis of judgment admitted to be unknown [a] This follows from the ultimate reduction of all judgment whatever to judgment which is conditional. But if this were so in the case of the hypothetical judgment as commonly understood, the back of the sequence would be broken. The *ground* would be absent Every hypothetical judgment is affirmed only within an actual system. What then are we to do with our ultimate sequence, e.g with the nexus between resistance and gravity? I cannot but maintain that, *if* we have no explicit ground to go upon, we must either surrender the sequence altogether, or affirm it categorically, i e not *as* a sequence, but as a datum, not as a coherence, but as a conjunction 'All matter (i e all that resists) gravitates' is no doubt a judgment in which we look for necessity But it does not follow that we find it It presupposes the judgment 'There is gravitating matter'. It is not adequately rendered by 'If or wherever there is matter it gravitates'. We are unable (or at least I am unable, which is all that my illustration requires) to assign any system which acts as ground and compels the sequence to be true within it. The world of

[a] See note, previous page

matter given as resisting and gravitating, as a systematic fact, is the ground And therefore it is on the other hand a true hypothetical judgment to say, 'If a material object is set free within range of a gravitating centre, it falls with a velocity accelerating as the squares of the times' For the content of this judgment is *within* the system of gravitating bodies, and the sequence is compelled by that system, whose existence as a fact is required by the judgment, but not in this form explicitly asserted. In this case the unity of gravitating matter is the x on which the sequence a–b is founded

May not the whole system be supposed, ground and all? Certainly not; and this is the fundamental point at stake Every judgment is a qualification of reality by some ideal content, and when the basis of a sequence is the content by which reality is qualified, that basis is by the judgment affirmed to be actual So far Mr Bradley and I are together, my further contention is that this ground or basis must be known, and must be indicated in the judgment, of which it forms the essence The degrees in which it is so indicated form the various complete and incomplete grounds which were discussed above But an ultimate ground must be actual, it is the fact which is judged in a hypothetical judgment. We may of course freely suppose or imagine *a* system, as complex as we please, but if we proceed to *judge* about the consequences or results of such a system, it must thus be related to these consequences within some further system and this further system must be actual In other words, you can only suppose an antecedent, you cannot suppose a consequent, the consequent must be judged, not supposed, and in judging the consequent you assert the underlying ground to be actual

This may be illustrated by the extreme cases in which we refuse to entertain a supposition. This means that we are not aware of any reality which furnishes a system such that the supposed case is capable of entering into it When supposition begins to infect the nature of the reality, we are beginning to suppose and not to judge our sequence It is quite doubtful whether the conclusions of 'Flatland' can be taken as true even *qua* hypothetical judgments When your supposition

has knocked the keystone out of actual reality, how is it to support a conclusion ?

The application of this conception would I think solve the curious cases in which a sequence is true, though it is possible for the content of supposition not to exist, or even impossible for it to exist. What must exist is a system that, subject to the supposition, necessitates the consequence drawn from the supposition. Whether the content itself exists or not depends upon whether it is an element essential to the system; and how it exists, on the nature and selfcompleteness of the system. The former condition meets the case of impossible contents, which are in every case illegitimate suppositions—i e. suppositions in which the consequent has to be supposed and not genuinely judged, because the supposed antecedent conflicts with the nature of the real system on the basis of which alone any conclusion about it can be drawn.

I will take some examples. 'Given a first cause, we can dispense with the idea of a regress to infinity.' But Cause, in any sense which it could be First, i. e in temporal relation, means an element in a system of relativity. Therefore the idea of a first cause contradicts the whole actual system to which the idea of Cause refers, and it is utterly impossible to affirm anything about an idea which begins by destroying its own basis of affirmation—the causal system Or again. 'If one man were throughout the whole period of his conscious life alone in the universe, his moral purpose could be nothing but to please himself' Here we are judging on the basis of an existing moral world—for it is only this that gives a meaning to a judgment relating to moral purpose—but we are putting a case which contradicts the nature of man as a being existent in a moral world. I do not think that in this case any judgment can reasonably be made. But the purposes of supposition in argument are so various, for its object may be in different degrees to emphasise the impossibility of the content supposed, that the limits of legitimate supposal are exceedingly hard to define. Undoubtedly its use is one of the most fallacious if one of the most effective means of controversy. 'If A. B. were to turn coward'—'But he could not'—'But I am only putting a case'—'But if you put such a case I may

put any consequence I choose as equally likely,' i e it is felt that the real basis on which judgment rests is annihilated.

Some real basis can never be dispensed with in judgment The nearest approach to dispensing with it is made when elements of reality which would conflict with the suggested case are wilfully kept out of account by an act of abstraction, which act of abstraction may be either borne in mind, or forgotten If the act of abstraction is borne in mind, we obtain such judgments as those in which mathematical science deals with imaginary quantities[1] Thus judgments are subject to the reservation implied in the abstraction from reality which enables them to be made. Yet, in as far as they are judged at all, they must rest upon, and involve the affirmation of, properties of reality. If the abstraction is forgotten, we then obtain such judgments as apply imaginary conceptions without reservation to the real world, 'A conjuror can tie a knot in a string whose ends are held, because he understands the properties of four-dimensional space'

I may further illustrate this last case by the example of artistic fiction, which I have discussed at length elsewhere It consists of judgments both singular and universal,[2] made on the basis of human nature, but subject to a reservation which separates them from the world of past sensible events While actually under the spell of romance or of the drama, we forget or half forget this reservation, but we do not and cannot forget the true and ultimate basis of the judgments, fundamental human nature, which is the ground and substance of the whole matter

[1] See p 165 ff on mathematical infinity

[2] It may be doubted how far the universal judgments in a fiction ought to partake of the character of fiction It depends on their grade of universality Judgments of a reflective order, about human nature for instance, if not dramatic, are expected to be true without reserve Judgments about parties, nations, &c, may be fictitious Dramatic sayings are yet more complicated, they are not judgments of the author at all, the author's (fictitious) judgment is that they were uttered Then their value or merit is compounded of their truth as estimated by the limited reality of the drama in which they occur (i e their appropriateness) and of their truth as estimated by real reality, i. e their weight or depth, which of course involves the whole relation of the piece to real reality See Knowledge and Reality, p 140 ff

But we raised a question not only whether the content exists, but *how* the content exists. This depends, we said, on the nature and self-completeness of the system within which it exists. Many universal judgments deal with sensible events, which are not, within our knowledge, contained in any really concrete whole in time and space. For these, according to the principle which we have followed throughout, the abstract universal or hypothetical judgment postulates such existence as they can have, according to their nature and that of the incomplete system to which they belong There is nothing which cannot ultimately be taken up into some individuality by constructive thought But as in ordinary judgment no such actual construction is operative, we have to substitute for it the mere assertion of the basis to which the sensible occurrence in question is known to have some relation. The imperfect explicitness of this relation is the note of distinction between the hypothetical and the disjunctive judgment 'Pure red is ethereal undulation impinging on a normal eye with λ vibrations to the second' Here the ground or basis of affirmation is the existence of light, which is ultimately dependent on the existence of sentient organisms in a certain relation to the material world Now this relation, though not known nor explicit in the judgment, must be taken as knowable and real. Space and Time indeed produce the illusion of endlessness, but no special positive content like sentience or light is involved in this illusion, we rather assume every positive content to have its own time, place and conditions within our actual individual system. No doubt we may be asked, Does the above hypothetical judgment assert the existence of red, or not? And can we claim to assert the reality of what is *for us* an indeterminate endless series which as a whole cannot be real? To these questions I should propose to reply 'The hypothetical judgment in question asserts the existence of light as the categorical basis of the nexus which it selects. The existence of red light is involved in the existence of normal light-stimuli and normal eyes Therefore the judgment in question asserts the existence of red light as a feature of the reality constructed by and for us, and subject to the reservations which its position in that reality imposes on it. We do not

claim to assert the reality of an endless series of sensations as such, but in the first place a positive series as referred to an actual system is already placed beyond theoretical endlessness, and in the second place it is not as an occurrence in the way of sensation that we assert its reality, but as an attribute of things in the whole of consciousness, which, as constructed out of perception, is *for us* the only reality.' This is in effect the answer which was given above to the question whether the conceptions of geometry were judged to be real, and it follows inevitably from the considerations to which we were led in treating of the doctrine of Ground. We shall see that the real Ground, when made explicit, takes us into the province of the disjunctive judgment. The element of categorical assertion in the hypothetical judgment, consisting as it does in some underlying real attribute, also presents a close analogy to the positive basis of negation, as will appear in the following chapter.

CHAPTER VII

NEGATION, OPPOSITION, AND CONVERSION

1. THE Negative Judgment presents at first sight a paradoxical aspect. We are bound to take it, *qua* judgment, as playing some part in knowledge, and as at any rate capable of contributing some factor to the ideal fabric of reality. But it assumes the external shape of ignorance, or at least of failure, and the paradox consists in this—that in negation the work of positive knowledge appears to be performed by ignorance The contradiction arises, as we have seen other contradictions arise, from the adoption by thought of a shape which at best expresses it but partially, and the retention of that shape when the aspect which it did express has come to be dwarfed by other aspects of knowledge. But of course the shape could neither be adopted nor retained did it not in some prominent aspect coincide with the requirements even of developed thought. Here then, as elsewhere, the key to our problem must be looked for in the conception of the individual mind working out its participation in reality by help of forms never wholly alien to this aim, but profoundly transmuted in proportion as it is attained. Negation and Affirmation

Negation is at first sight merely negative It appears to say nothing, but only to deny, i e. to put away some ideal content as other than reality or to express our inability to recognise it as belonging to reality The first step then towards ascertaining its import is to ask, *what* does it deny or pronounce unreal? what does it presuppose to be present before denial is possible?

It certainly does not presuppose an affirmation. Both fact and theory protest against such a view. We have not always judged a matter to be true before we deny it And if an affirmation[1] of the same content is to subsist as a con-

[1] Sigwart distinguishes 'affirmation', as the conscious opposite of negation, from 'Positive' judgment, as assertion without consciousness

dition of the negation, it seems doubtful whether a negation would not always have to be self-contradictory Although Sigwart's account of double negation suggests this view, yet he says explicitly[1] that it is an *attempted* judgment ('Ein versuchtes — Urtheil') against which negation is directed And this seems so far to agree with experience. What is the nature of this attempted affirmation? 'Suggestion' and 'question' come into the mind as possible equivalents for it It is difficult however to find a special significance for either the one or the other on purely logical ground A question is closely related to a command, and has its differentia in being addressed to the will of another person It is impossible in good faith to ask a question of oneself. The power of the metaphor by which men are said to question themselves rests on treating oneself as another person. A question is a demand to the will to reveal something known to the person whose will is appealed to, but if I know the answer, I need not ask myself, if I do not, I cannot ask myself

A question then as such has not a logical differentia, and cannot be the logical presupposition of negative judgment Still it may contain what we want It is not merely an ideal content floating before the imagination, even if we were to grant that there are such floating ideas It is an idea in some way tested by Reality A suggestion seems to be the same It is not a floating content, it is suggested *as* something, as, so to speak, a candidate for a place in a judgment already framed. That is to say, a question or suggestion as it is on logical ground, omitting any demand upon or incitement to another will, amounts at least to an idea whose content is Reality qualified in a certain way Is this all? This would not suffice to explain the import of suggestion followed by affirmation or denial. It would not explain the significance of the decision even when negative. There is something more, and it is this The content of the suggestion

of possible negation We shall see in the sequel the value of this distinction, which can only be taken as one of degree

[1] Logik, vol i, p 150, and 194 (with assent to Bradley's Logic, p. 149 ff) E Trans 119 and 149

is taken within a whole in which we have an *interest* and which is referred to actual Reality. Every suggestion enters into a rudimentary disjunction I mean by *rudimentary* disjunction an alternative whose limits are not made explicit. Where there is a question or suggestion, there is always a something—some general predicate—known to be true with reference to the matter before the mind, and an interest which we have in that something. It is within this something that the attempt to state it more precisely, the question or suggestion, falls It is hazardous to obtrude analysis on the simpler stages of consciousness, but I do not see how we can have a suggestion followed by denial cheaper than this We might even fall back on the principle which has been laid down in previous discussions, that any consciousness for which a continuous real world exists, sustains that world by a judgment When a man first doubts and then decides, on such a question as whether the river which he sees before him is safe to ford, however simple the mental procees may be, there must be in some form a positive basis of the two or more alternatives as well as one suggested alternative He must start with the fact that the river has depth, or current, which he must deal with in crossing, and within this fact the doubt 'too deep?' 'too swift?' has its meaning for him In simple cases this embracing judgment of fact is hardly traceable except through the interest in the question This interest, if looked at closely, betrays the nature of the alternative which the question involves. 'Too deep?' 'What then?' 'I cannot get home to-night,' i.e the general fact is that the river is between me and home.

Negation then presupposes an idea *suggested* as true of Reality within a state of facts *judged to be* true of Reality and interesting to us in respect of the matter suggested, or in other words, an alternative judged to be true of Reality, but only so judged as one among a set of alternatives, and therefore, in itself alone, *problematically* judged true of reality —judged as a possibility, as one among a number of alternatives, or as subject to unexpressed conditions

We are now able to decide the disputed question whether Affirmation is prior to Negation, or whether they are co-

ordinate types of Judgment. Negation is not, as such, the denial of affirmative Judgment, and therefore does not presuppose the affirmation of that which is denied So far, affirmation is not essentially prior to negation. On the other hand, Negation does presuppose some affirmation, that is the affirmation of a state of facts which, being judged true as a whole, carries with it the problematic affirmation, the affirmation as a possibility or enunciation as a conception in the world of meanings, of the idea 'suggested'. In this respect, therefore, affirmation is in the beginnings of knowledge, at any rate, prior[1] to Negation The world must have positive content judged to be real as a condition of anything following from the removal of a positive suggestion But (I cannot believe that the consciousness of a positive world could in fact exist for an appreciable time without the development of negation[2]) Further, however, it is also true that in the beginnings of knowledge negation is a degree more remote from reality than is affirmation,[3] and this character of ideality clings to the negative form through its whole development, though without debarring it from the acquisition of objective value The remoteness consists in this, that the suggestion which denial presupposes is, as we saw, not a mere floating content, but a suggested qualification of reality, in short 'a suggested affirmative relation'[4] An affirmation can be, comparatively speaking, given as fact, a negation cannot, except in quite another sense, be given It has to be made, and made by setting an ideal reality over against real reality and finding them incongruous 'That fire is still burning' involves no doubt intellectual selection and is so far ideal, but 'that fire is not out' is one remove more ideal, because it has to bring up the idea of 'that fire being out' and test it by the perceived reality, and then only proceed to judge its exclusion to be a fact We must not however exaggerate this difference. Affirmation itself, or even positive Judgment, cannot take place until the distinction between a mere idea

[1] In this sense it may be called, if we prefer to do so, 'positive' judgment

[2] See above, Introduction, p 22, on the *formal* implication of distinction in objectification

[3] Cp Bradley's Principles of Logic, p 116 [4] See Bradley, ib

and a fact of reality is recognised And with this distinction the idea of negation is given It might therefore be argued that Judgment, not to speak of affirmation, presupposes the idea of a negative relation,[1] just as negation presupposes that of an affirmative relation. The fact seems to be that affirmation presupposes the idea of negative relation in general, while negation presupposes *the idea of*[2] a corresponding affirmative relation in particular This applies to the beginnings of knowledge In complete thought we shall find the two more on a level

Thus it is true, especially in the beginnings of knowledge, that Affirmation is prior to Negation, both as one remove nearer to reality, and as supplying the reality within which alone Negation has a meaning But it is no less true that Negation has from the first its essential place in knowledge; and as Reality becomes for us an articulated system, the value of negation approaches more and more nearly to that of affirmation, with which it finally becomes equivalent This is however not to be understood in the sense that the import of negation disappears from knowledge; but in the sense that affirmation and negation alike become double-edged, each involving the other

Bare denial and Infinite Judgment

2 Negation then, in its primary shape, is the exclusion of a suggested qualification of reality The bare expression of this import, reduced to its minimum, would be found in what has been called by a mis-translation of Aristotle, the infinite judgment.

The infinite judgment was a term applied by Kant (following, I presume, the tradition of formal logic) to judgments which had for their predicate what Aristotle called an 'indefinite' or 'undistinguished' name or predication;[3] i e such a phrase as 'not-man', 'not-good,' or the like. More important however than Aristotle's expression 'indefinite name' was his distinct verdict that such phrases were not names or predications He gave them the title of

[1] Cp Bradley, Principles of Logic, pp 2, 110 I find some difficulty in reconciling these two passages

[2] Not 'presupposes a corresponding affirmative relation as judged true' That view we have rejected

[3] *ῥῆμα* or *ὄνομα ἀόριστον*

indefinite or undistinguished names or predications, 'because they may be truly predicated of everything alike, whether existent or non-existent'[1] It is plain that Aristotle's verdict is right, and that such names have no signification They are 'undistinguished' because they are undistinguishing. It may therefore be observed in passing that to attempt to read all negation as affirmation of a negative name is an unmeaning device, though possibly guided by a feeling of a true ideal, viz that negation, if it is to have a positive import, must involve an affirmative element Only by this contrivance the affirmative element is ludicrously absent We should be, as Mr. Bradley says, denying, and then affirming that we have denied

It is for this reason that the 'Infinite judgment' may be fairly represented by examples in which the denial, though undeniable, is unmeaning 'Virtue is not square,' 'The soul is not red,' 'Man is not a stone' These, *qua* negative, are fully on a level with 'A monkey is not-man,' 'A stone is not-Christian' And so in illustrating the import of negation we may disregard the pseudo-affirmative character of the latter instances Our interest in them is that, if strictly interpreted, they display to us the nullity of bare denial. 'Not-Christian' literally interpreted includes not only heathen humanity, but the fixed stars, the sea, and indeed, in Aristotle's words, 'everything whether existent or non-existent' except Christians It refers to no one sphere in preference to another, and thus says nothing definite enough to be intelligible. The point being once established that negation *qua* negation has no significance, we may disregard the attempt to erect it into affirmation which draws our attention to this fact. And we may then safely take as instances of the infinite judgment, so far as its import is concerned, the judgments typified by 'Virtue is not square'. These show the true type of bare denial, for they are the only negative propositions in which usage does not irresistibly limit the sphere of the negation. And when the sphere is limited, the denial is no longer bare.

[1] Ar Περι Ερμ 2 3 Though the reading is doubtful in the application of these words to οὐκ οἰκία, it is enough for our purpose that they are applied to οὐ ῥῆμα

Thus it appears that bare denial, whether disguised as spurious affirmation, or taken as the mere exclusion of mere suggested predicates, amounts in the strict sense to nothing The judgments by which it is typified are the exact counterpart of absolute tautology, and like such tautology, are not really judgments at all Identity and difference are inseparable aspects of all that exists or can be thought, but in these two classes of would-be judgments identity and difference fall apart, and thereby the conditions of intelligible judgment are destroyed Pure tautology aims at mere identity, and bare negation at mere difference It will be found that any meaning which in practice we attach to an apparent tautology or an apparent bare negation is owing to the introduction of difference into the former, or of identity into the latter. 'Business is business' qualifies a certain class of affairs by the principles on which they ought to be conducted, 'The soul is not a machine' qualifies the soul, not by the mere exclusion of mechanical properties in favour perhaps of absolute nothingness, but by some positive characteristic of the soul which is incompatible with its being a machine

Significant Denial

3 We have seen that denial was not to be made into affirmation by the rough and ready method of the Infinite Judgment But we started, on the other hand, from the postulate that denial, as a form of judgment, must be capable of contributing something positive to knowledge Whether positive necessarily = affirmative is a problem that will solve itself as we come to understand the full nature of negation. I use it here simply as antithetical to 'nothing', or as an emphatic reiteration of 'something' We must assume with Plato that knowledge is the knowledge of something, and if the nature of 'nothing', as e g the abstraction of empty thinking, can be known, then nothing is so far and in that sense something

All significance then is in this sense *positive* significance, and significant negation must therefore convey something positive. What is it that it does convey? We shall find the answer if we look at that which all judgment has in common, viz the *interest* or *bearing* of the judgment What is it that we really mean or wish to predicate when we make

an ordinary negative judgment? There is always, I may observe, something unreal in the analysis of isolated propositions. Apart from the context of a book or of a conversation, or from the precise standards which involve the fixed context of science, our interpretation of propositions into judgments is almost entirely arbitrary. On the other hand, it may be said that in a given context the 'bearing' which we ascribe to a proposition is not strictly within the limits of what the proposition enunciates, but is read by us into its meaning. It may be doubted however whether in actual living thought there is any judgment that is not an enthymeme, i.e. an argument with a suppressed premise or a suppressed conclusion. If we attempt to prune away from the judgment all the implied and suggested bearings of the proposition which conveys it, we shall find that we have whittled away the meaning which is the judgment itself. We must never forget the conclusion which we reached above, that the unity of the judgment does not exclude systematic multiplicity within it. The logical content employed in any given judgment is a many-sided although determinate idea, and is gripped and attached to its actual place in the logical mechanism now by some of its prominences and now by others. Some reservations will have to be made in this respect when we come to speak of judgments that deal with self-contained systems, a type which we have more or less anticipated in the individual and generic judgments. But in ordinary reflective judging we are constantly referred away and away along a series of grounds and consequences, and it is idle to attempt to reduce the judgment to simplicity. What we really *mean to mean* is only to be found in the explicit articulation of the whole system of fact which the interest of the moment covers, and all ordinary judgment toils after our interest in vain. But *prima facie* the positive judgment has an advantage in this respect over the negative. A positive content is at all events something, it is an instalment in satisfaction of our interest. If I say that the fire is burning in the dining-room, this judgment is no doubt compatible with various grounds and various consequences, and in the judgment as I mean it some particular ground and some

particular consequence are probably included Such a statement would not be made *à propos* of nothing, or if it were, it would be resented just as talking gibberish would be resented. There is some point or purpose to which it must be taken as contributing, and some reason—though possibly falling outside the content of the judgment—which serves as a ground for making it But over and above all this there is, in the affirmative judgment, the positive logical content itself, which, though modifiable within very wide limits, yet cannot be modified beyond certain limits The judgment may be in praise of some one's thoughtfulness, in condemnation of their extravagance, in contempt of their effeminacy, or in alarm at their carelessness But there is a nucleus, not indeed fixed nor free from ideal selection and synthesis, yet not quite indefinitely variable and containing a positive element of appeal to normal perception

In a negative judgment strictly interpreted as mere denial this nucleus is lacking, as the consideration of bare denial taught us Therefore we have nothing left but those elements of meaning to which the interest of predication is the sole clue Let us take a plain every-day judgment such as 'A. B is not a dishonest man'. If we enunciated this proposition in the sense of the infinite judgment, meaning e g that A B. is not a man at all, but a stone or a monkey, we should unquestionably be held to have violated the conventions of speech The meaning of every judgment is to be looked for in the attribute to which is attached the interest that guides the selection of the content used in judging. But this attribute must obviously have definite relations, at any rate for some special purpose, to the content affirmed or denied Whether it can precisely coincide with the content affirmed is a point to which I shall have to recur ; but it cannot precisely coincide with the content denied, for if so, no result in which we had an interest would spring from the denial—the whole reason of our interest would be cancelled and be put away with the denial of the attribute on which it centred.

Now I may be interrupted at this point with the objection that this is exactly what is always happening in negation. When we have suspected a man or thing of having some

attribute which interests us, and then find that we were mistaken, our interest in the individual may, and often does, fall dead at once A man may be pointed out to us in a crowd as about to be our fellow-traveller on a difficult journey, and we may regard him with some interest on that account, but if we learn that it was a mistake, and that we have nothing to do with him, we shall probably after that regard him as 'nothing to us' This is in fact almost a recurrence to the infinite judgment, for though we must know that he is a man, yet so long as he is not to be our fellow-traveller we do not care whether he is alive or dead i e for any positive quality of his humanity The judgment 'he is not coming with us' approaches then to a judgment of bare exclusion, the attribute in which we have interest being the attribute excluded

In the first place, I think, we must to a great extent admit this contention, and simply refer it to a difference of degree Negative judgment does begin with a phase inappreciably differing from the infinite judgment But yet, of course, the question 'Coming with us?' must have originated in some such judgment as 'He is the sort of person who might be coming with us', and it is within this fact judged true that the ultimate negation 'not coming' has its meaning. The interest is first awakened by the whole attribute 'such as to come', and could never have been aroused but for the presentation of such an attribute If the negative judgment were really one of bare exclusion it would apply equally well to everything in the world, and no interest would have been aroused which could have led us to make it of a particular individual But in the second place, it must be added that in as far as, in the alternative which formally arises within the attribute 'such as to come', all the interest practically falls on one side of the alternative, this is an actual defect of knowledge and morality, but has not, in the case supposed, been pushed far enough to prevent the insistance on the positive attribute 'capacity to come'[1] And this attribute

[1] It is a commonplace satire to say of a man that the universe interests him only in as far as it is what his particular whim requires What is not money, or total abstinence, or woman's emancipation, as the case

supplies in fact a positive content for the denial, though we by the hypothesis happen to be now indifferent to it. The denial really contains the judgment, 'The man is one who, we thought, might come, but is not coming' The proximate case in the ascending scale of knowledge may be illustrated by supposing that instead of our interest dropping dead on hearing that he does not come with us, this negative judgment enunciates to us a regret that he is not coming, implying that we ascribe to him some positive quality which causes us to regret losing his company In this case the positive bearing of the negation 'He is not coming with us' is primarily a *consequence* of the negation operating upon the judged content 'We should wish him to come with us' This consequence supplies the denial with a positive import, and may be the only aspect of it prominent in the mind

But there is something else to be considered. Every judgment without exception challenges the question 'Why is it so?' or at least 'How do you know it to be so?' the former question demanding the cause or real ground, the latter demanding the cause of knowledge or logical ground These two kinds of ground run into one another, as we saw, and need not be distinguished for our present purpose In denial, then, there must be *this* element also of positive import, the attribute which justifies the denial for us It is plain further that in case of the non-existence of the immediate or apparent subject, this attribute may be judged directly of Reality, which is the ultimate subject in Negation as in all Judgment. 'The house on the marsh is not burnt down' may be true because there is no house on the marsh, and although reality—positive fact—excludes the burning down of any such house [1]

may be, is nothing to him. This is a good illustration of the moral and intellectual impotence indicated by any approach to bare negation. Hegel has quaintly compared the distinction between bare and significant denial to that between crime and civil dispute If I steal, my act says 'This is not yours', without asserting that it is mine or any one's by right, i e it ignores the whole sphere of property or reasonable possession by alleging no ground for its denial In a civil dispute I say 'This is not yours, because it is mine,' i e I assert rights of property, as you do, but I deny your right on the ground of mine

[1] It may be objected that in such a case to say 'The house on the

Thus in Negation we have two positive elements which may be present together, or in various relations of prominence, or may wholly or partially coincide. The third element of import, the positive content explicitly employed in judging, falls away in the negative judgment, being replaced by a bare exclusion of an explicit content This bare exclusion is what we discussed on its own merits in the second section of the present chapter, and found to amount in itself to nothing.[a] We are therefore referred for the meaning of significant negation to the positive ground, or positive consequence, of the exclusion which forms the outward and visible shape of negation. Thus we approach the solution of the problem how knowledge can take the form of ignorance—how 'what it is' can be known through knowing 'what it is not'.

marsh is burnt down', is not so much false as unmeaning, in other words that the negative 'the house is not burnt down', has meaning only if there is a house, and presupposes or asserts that there is one I have strong sympathy with this objection, which turns on the problem of a real distinction between subject and Predication within the Judgment. But an unmeaning judgment is clearly not true The only doubt is whether its not being true justifies any negative except one which should brand it *as* unmeaning, e g. 'There is no house to burn down' The more hypothetical judgments are, the less they present this difficulty

[a] I think that Mr Keynes' objection (pp 122–3) to my account of bare denial is sufficiently met by what has been said above, especially on pp 281–2 and 285–6 I will only remark (1) that I count two elements of positive significance in the negative judgment viz. the ground and the consequence or positive interest and not the ground only, (2) that wholly to separate ground from a judgment is always to destroy in great part the meaning of the judgment—this is essential to a sound theory of inference, and (3) that our ready acceptance and interpretation of such judgments as his examples shows how far we are from bare negation when we accept them, for their natural meaning, which we accept, is conditioned by a whole system of context and positive interest which we presuppose as always present with us Literally taken 'A B did not start by that train' may mean there was no A B in the world, or the man who started had not that name, or that A B is a mountain in South America, and therefore could not start by a train How common it is for any careful speaker to say 'A B did not start—at least I looked and could not see him', thus supplying what we know the literal judgment wants You may insist, 'but it means at least the denial,' but I insist 'It does not tell you what the denial means, except by assuming a context and positive interest, which take it beyond bare negation' On the same passage in Mr. Keynes' book, see further, p 297 below.

The primary analysis of the significant negative judgment presents therefore a close parallel to that of the hypothetical judgment 'A is not B' may always be taken to = 'A is x which excludes (or which is implied in excluding) B' Or in extreme cases we may have 'Reality is x, which excludes or is involved in excluding A B' Just thus we saw that 'If A is B it is C' may always be interpreted into 'A is a factor in a real system x which given A B involves C', or in an extreme case, 'Reality contains a system x such that, if A B were placed within it, C would result' We examined at length the conditions under which such a relation as that enunciated in the hypothetical judgment could be made intelligible, and we found that a real system within which the separate terms should be interdependent was the bond of union which alone could justify such a reference of one thing to another In the same way we saw that negation presupposes a real system—a system affirmed in judgment to be actual—as a condition of its intelligibility, and if it is replied that a system presupposes negation—for it presupposes difference—we must answer, first, that negation in its pure form as simple contradiction is the abstraction of difference, and *may* be later in origin than, or at least presupposes as coeval with it, the positive differences which all thought involves, and secondly, that in thought it is possible and indeed is the rule for factors to presuppose each other, and to grow into distinctness *pari passu* In fact, Negation *is* simply the *logical, conscious* expression of difference.

Significant negation, then, like hypothesis, is intelligible within and with reference to a system judged to be actual. It is only within such a system that something can be made out of nothing by implying a positive ground or consequence in a bare denial In other words, the essence of formal negation is to invest the contrary with the character of the contradictory, or to raise mere discrepancy or positive opposition to the level of the absolute or contradictory alternative which is the abstraction of difference It is only contradictory negation which allows a conclusion to be formally drawn from the negative, contrary negation does not admit of this It is only contrary negation which allows any import to be

materially attached to the negative; contradictory negation does not admit of this. The fact that contrary negation can be given the force of contradiction, that a positive opposite can be known as a sole alternative, and that unless this is done knowledge remains inarticulate and chaotic, is simply the fact that Reality is a system. It is in considerations of this kind that we must look for a reconciliation of conflicting ideas as to the 'subjective' or 'objective' place of negation. We must ask in every case *what* negation? The negation of what and under what presuppositions? Without going further into extreme views about the objective import of negation, I am most anxious to persuade the reader that the fruitful question is not 'Can we conjure a meaning out of a bare denial?' but 'Why is it that in knowledge we cannot do without denial?' 'In your "system of differences",' it may be said, 'you put a significance into your negations, and then pride yourself on finding there what you put in.' This is true. What I want to insist on is the fact that this kind of significance cannot be put into anything but negation, and the light which this fact throws on the significance in question. Reality is a system, and you cannot have system apart from negation. This is the central fact from which all enquiry must start. The connecting link between difference, contradiction, and contrariety[1] is that difference becomes contradiction when taken as mere difference or as the abstraction of difference, that is, as expressed in a negative judgment which (like the infinite judgment) professes merely to exclude a given idea, or deny a given affirmation. If the denial were within a self-identical system it would carry a positive bearing. As *ex hypothesi* it has no positive bearing, it embodies mere difference without identity, or the abstraction of difference. Such a denial is the pure contradictory of the affirmation which it denies. On the other hand, difference becomes discrepancy or contrariety when not the formal abstraction of difference, but positive differents claim the same place, and the same place means the same relation to the same system. Such contrariety exists between 'A is B_1' and 'A is B_2'.

[1] See below, II, chap. vii, on the 'Formal Postulates of Knowledge'.

Thus the articulate arrangement of differences under their systematic relations is the root of positive discrepancy The system as determined by one relation excludes, under that relation, the system as determined by any other, and the system as a whole identifies any one of its *positive* relations with the mere difference from, i e the bare exclusion of, certain other positive relations A is either B_1 or B_2 This is the combination of Contradiction and Contrariety

It is not essential that the positive ground and positive consequence of a denial should be different Under conditions of precise knowledge they must be the same We shall see that where only two alternatives are possible, and they exclude one another, either of them is denied by the affirmation of the other, and affirmed by its negation Under such conditions the denial of one alternative has the affirmation of the other both for its ground and for its consequent 'He will either dissolve or resign' permits us to understand under the denial 'he will not resign' the affirmation 'he will dissolve' alike as its ground and as its consequent. It is plain that in actual knowledge there are degrees of this coalescence. The ground may be imperfect or extraneous, it may be replaced by any positive quality that excludes the predicate denied, i e in this case, excludes resignation This positive quality might be even difficult to formulate precisely, and might run as close as possible to a bare denial, we might feel sure that 'he is not the man to resign', which means that his general character as we understand it precludes the idea that he will resign Such a general *ground* would be at once reinforced in its cogency and restricted in its result by the *consequence* of the denial, the necessities of the case being as supposed above The two in their coalescence, 'he is a man who will not resign,' and 'not resigning, he must dissolve', would in that case form the complete analysis of the denial 'he will not resign'.

It has been necessary in this discussion to anticipate the account of disjunction which will be given in the next chapter Perfect disjunction is of course a late form of knowledge. But it is an ideal inevitably involved in the nature of negation All significant denial—all denial, that is, which rises above

the level of the infinite judgment—corresponds to this ideal in two respects at least Every denial has its meaning within an attribute or set of attributes judged to be real, and every denial affirms some positive matter which affects and is affected by its relation to such a comprehensive attribute This may be illustrated in another way, which will also serve as a recapitulation of the stages of negation

Negation is rooted in the fact of difference, but difference is not enough to warrant negation Mortality is a difference within the identity of man, but we do not therefore deny that man is mortal Significant negation begins, we said, when positive differents claim the same place in the same system—in the simplest case, when differents claim to be identical Man is not mere mortality, i e does not coincide with mere mortality in the system of reality As thus conflicting, which of course can only happen *in a judgment*, differents are contraries or opposites, and the assertion of one is the negation of the other Now these contraries or opposites may be of any number There is nothing to limit them Any colour is the contrary or opposite of black, if asserted in the same relation, and so would any sound or taste be, *if asserted in the same relation*, which seems impossible for a sane man If then we consider negation at this stage as embodying contrary opposition, what follows from it? 'This surface is not black' has indeed a ground, viz that it is some particular other colour, but what about the consequent of the negation? There is none, except that the surface is *some* colour other than black, and therefore we lose by choosing the negative expression rather than the positive, and the significance of the negation *qua* negation is absent What follows from the absence of a consequent as definite as the ground? Nothing less than this, that negation cannot be explained on the basis of mere exclusion of contraries It is indeed possible to deny intelligibly on such a basis, in virtue of the general consequence of negation, but no reason can be assigned for in such a case preferring negation to affirmation For in the case supposed we should be concluding from 'A is red', through 'A is not black', to 'A is of some colour other than black'—a manifest loss

Negation can have no bearing, i e no interest or *raison d'être*, unless the contraries are limited so that something follows from the negation In other words, Negation always *involves* Contradiction between contraries and not merely Contrariety There may be any number of alternatives, but unless the number is limited or falls within some positive characteristic however vaguely known, nothing can follow from the denial either of one or of any number among them Therefore though negation originates in difference, which it raises to contrariety by embodying it in definite relations, and though it does not arrive at formal contradiction between opposites till long processes of thought and language lie behind it, yet I am unable to comprehend how any negation can have interest apart from being taken within a positive whole, however vague, which is of the nature demanded by the relation of contradiction when established between opposites Mere contradiction as between 'he is good' and 'he is not good' is given in the nature of negation from the first; and its development consists in filling this unmeaning form with significant opposites, so that from 'he is not good' we may be able to infer something more than that 'it is not true that he is good'.

Significant Negation then combines in itself the absolute Contradiction which was illustrated by the Infinite Judgment, and the Contrariety which arises between differents when referred to the same place in the same system Without contrariety negation would have nothing that it could mean, but without contradiction it would not have in itself the power to mean anything [a]

[a] The above account was modelled so as to respect the distinction made in Formal Logic between Contraries, both of which may be false, and Contradictories, one of which must be true But in common usage the relation between Contraries is called Contradiction, no less than that between Contradictories And it is important to realise that Contradiction in the metaphysical sense, i e a deadlock which reason cannot acquiesce in, is rooted in what we defined as Contrariety, i e the unmediated reference of different predicates to the same subject See Bradley, Mind, 72, 496, and Appearance, ed 2, 562 I think that the above account of Contrariety and my account of Contradiction in the Companion to Plato's Republic, p 149 ff , are in harmony with Mr Bradley's statement The ultimate nature of Contradiction is philosophically very important

Opposition and conversion of Judgments.

4 The rules of the opposition and conversion of Judgments, which have come down to us almost as Aristotle formulated them, are founded on the classification of judgments according to 'quantity' They therefore lay down the relations to one another of all classes of judgments which this principle recognises, in as far as the truth or falsehood of any one judgment affects the truth or falsehood of any other which deals with the same content But when we attempt, as we have attempted above, to distinguish the *kinds* of truth which various types of judgment embody, then the relations between the various classes of judgment cease to be a matter of mechanical rule-of-thumb, although not hard to understand if we pay attention to the actual significance of the judgments with which we deal

Opposition

1 The principle from which we must set out is that everything which can be affirmed can also be denied In some cases the denial will be confined to a shape closely corresponding to that of the affirmation, and in some there will be two kinds of denial which will fall apart

Between Singular Judgments

α The Singular Judgment cannot be treated for the present purpose as a case of the universal It has, as we saw, a universal character, but not in the sense of referring to an aggregate of individuals If 'Caesar crossed the Rubicon' is true, 'Caesar did not cross the Rubicon' is false, and if the negation is false the affirmation is true Thus the relation of Contrary Opposition, according to which the Universal Affirmative and the Universal Negative of formal logic may both be false, falls away in the case of a Singular Judgment in our sense of the term In this type of judgment we have the simple relation of affirmation to formal negation which is contradictory, i e presents an absolute alternative The reason of this we shall have further to consider in treating of Double Negation

Between Plural Judgments

β The Judgments of Enumeration, Plural or Particular Judgments, including the Collective Judgment, present the relations towards each other with which we are familiar in the common scheme of opposition The peculiarity of these relations is that in them an absolute alternative or contradictory opposition is only to be obtained by opposing judgments of

different quantities The reason of this is well brought out in a phrase which Aristotle employs in his account of opposition, where he states the contradictory of 'All are' to be 'Not all are'—instead of using the expression of our text-books 'Some are not'; or when he says that the contradictory of a proposition which affirms a predicate universally, is one which asserts that of the same subject the predicate is not universally true What is affirmed in a collective judgment is the homogeneity in a certain respect of an aggregate still regarded as an aggregate of enumeration, although endowed with sufficient unity to warrant itself as a completed whole The denial of the judgment is the denial of this homogeneity, and is rather a consequence of than identical with the partial counter-enumeration which our logic-books take as its type If, for instance, we assigned a number to the counter-enumeration, the contradiction would no longer be complete, and we should find ourselves in contrariety instead To say that 'All these books are German' and that 'Two of them are not' cannot be a contradiction pure and simple, for the falsehood of 'All these are' is compatible with the truth of 'Two—three—or all of them are not' This is enough to show that 'some not', if we take it as a sign of counter-enumeration, is less safe in contradictory opposition than 'not all'.

As regards the 'contrary' opposition of 'All are' and 'None are', the doctrine of formal logic is true so long as we confine ourselves to Judgments of Enumeration. 'None are' asserts a complete counter-enumeration or its equivalent based on some other ground, and besides this assertion and its contrary there are as many alternatives as there are individuals in the aggregate, *minus* one

The particular or incomplete collective judgments 'Some are' and 'Some are not' (sub-contraries) are of course compatible with each other so long as we do not determine them numerically, and continue to be so then if we consider that incompleteness of enumeration debars us from all reference to a sum total If on the other hand we permit the comparison with an assumed sum total, we pass at this point into calculation. The general conception under which we are enumerating always, it must be remembered, implies a total, so that

calculation lies very near to judgments of this type. 'Six men were killed and ten wounded (but not killed)' are judgments perfectly compatible with one another if the number of men on the ground was sixteen or more, but if there were only fifteen or fewer, then the two judgments at once rise into contraries. Both may then be false, but one must be. This however is calculation.

This account of the matter includes all that need be said of the Singular judgment if used in opposition to the Collective judgment. In such a case the Singular judgment takes the place and follows the rules of a judgment of Enumeration. It must however undergo a transformation, even if only implied, in taking on a relation to the basis of enumeration. 'How many Liberals voted against the Bill?' 'A B did, C D did,' &c., &c. These singular propositions are read off into enumerative judgments, 'One Liberal, two Liberals,' &c.

The further judgments which arise out of the Judgment of Enumeration and Measurement follow the characteristics of those forms of the true universal judgment to which, whether as generic or as hypothetical, they severally approximate.

Between Generic Judgments.

γ. I will now point out shortly the characteristics of these judgments themselves when placed in opposition. The tendency of the higher stages of knowledge is, as we saw in the last section, to fuse contrariety and contradiction into one. This is obvious, for instance, in the individual generic judgment, for the same reason as in the singular judgment itself. And even in the analogical judgment the tendency to fusion is strongly marked. The allegation of exceptions against a generic character, whether in form of expression positive or negative, must either be insignificant as when the exceptions are apparent and not real, or else tend by analogy to establish a contrary alternative or *positive contradictory*. In the former case the judgments which emphasise the exceptions must be taken to be not generic judgments at all, but mere enumerative judgments, which therefore cannot touch the essence of the generic judgments they appear to oppose. For we shall find the dominant principle in these relations to be that a judgment of one type α cannot deny a judgment of another type β, although the former may suggest a judgment

of the type β which will constitute a full denial of the other judgment of that same type[a] Under the head of such mere

[a] Cp Keynes 101, 122 3, and 259 and p 369 below See also Essentials, 116–17 Objection is taken to the views (1) that the Particular in categorical shape is unscientific, (2) that a judgment is in principle effectively denied only by another of its own type

The governing principle of these views is the conception of the double nature of judgment, which makes every actual judgment a battle-ground between the existential and the necessary character No form, I take it, is quite unambiguous (see Preface to ed 2) but the separate types of import do tend to find appropriate expression, and it seems only right in logical theory to insist on this tendency And I think it a counsel of perfection for practical life

Thus (1) if what you want of the categorical particular is to contradict the hypothetical or scientific universal, you are (p 300, below) fighting a theoretical principle with unanalysed judgments of perception and enumeration Do not nearly all popular errors rest on this practice? And you have an appropriate mode of denial in the modal particular, which is intended for the expression of incipient analysis, when it falls short of justifying a contrary universal

If what you want of it is to assert existence then you are falling back on the assertory or existential side of judgment, and letting the aspect of necessary connection slide into the background But every judgment includes in some degree both Yes it does, and that is why we are always confusing them with each other Ust hoc, ergo propter hoc' is a more general formula than Post hoc— and expresses much the same fallacy In Logic we might at least try to avoid both

(2) If I were to say that a scientific judgment can only be denied by a scientific judgment and an existential by an existential, that might seem obvious, and would be all I want A doctrinaire denies facts on the ground of laws ill understood An empiric denies laws on the ground of facts ill conceived M Keynes says (p 124) that I admit that exceptions can overthrow a law, and the rest of my argument is *ignoratio elenchi* But of course I mean exceptions interpreted in terms of a law How am I to know that Volvox is not an animalcule, when Ehrenberg says it is, unless I can tell more or less to what kind it really belongs? I need not know what precise kind, but I must know 'some kind excluding animalcule And from that it is only a step of degree to knowing precisely what kind

As to denying every judgment only by its own type, there is a peculiarity in the disjunction (cp vol II, 194) It is (see p 328) a unity of hypotheticals and a categorical, and, further, represents the whole system which itself gives meaning to a negation So, taking the last point first, I should say that it is hardly natural to deny it, for in doing so you must presuppose another disjunction to give meaning to your negation The place for negation is rather under than of a disjunction The natural course would seem to be to let it alone and begin again Cp Essentials p 125 and below, 301

But if you are able to specify and deny the categorical basis e g to

enumerative judgments would come all observations of artificial forms, mutilations, abortions, &c, in organic and spiritual beings; and all accidental juxtapositions rendered by judgments whose subjects have no connection with their predicates The generic judgment 'Man is a creature with a sense of justice' is not invalidated by instances drawn from dead men, lunatics, or idiots, nor even, perhaps from criminals, if there are such, whose conscience is obliterated by lifelong war with society But I shall be told that a scientific law has no exceptions This is just what I am maintaining, I am saying that exceptions are either apparent or real, and in the former case, that now before us do not deny the law, in the latter proceed positively to indicate another law In the second case, when the law is really impeached by the bearing of the exceptional instance, this bearing must have a positive import, which may not amount to a suggested law, but must be in the same region of essential individuality in which the characteristic that is denied has its import

If I say that all exogenous trees are dicotyledonous, I am opposed by a real, not merely apparent, exception in the case of the Coniferae, which though exogenous have in many species more than two cotyledons Here we have one Generic judgment (I use 'generic' in the logical sense which I have explained) opposed to another It is not merely that here and there an aborted or mutilated instance is to be found in which a part of the plant is wanting, but that an enormous natural order with marked unity of habit, and in the strictest sense[1] sharing the characteristic on which the analogy is based, does not display the character required by analogy.

say, in denying 'X is red or green, 'X is not a colour, that is one way of disposing of the whole disjunction, or if your objection is to its exclusiveness or exhaustiveness you can deny that one of the two special hypotheticals involved which alleges the necessity you desire to impeach In all this categorical is opposed to categorical, and hypothetical or at least modal particular to hypothetical

[1] Here an error is possible The wood formation of Coniferae, though proceeding from a cambium ring outside the old wood, has differences from that of Dicotyledonous Exogens If these differences were of importance the exception might break down as not a case under the rule, Coniferae not being true Exogens

Now I am not aware that any importance has been attached to determining the connection between exogenous wood-formation and the number of seed-leaves or cotyledons; but it is plain that the conflict of generic judgments so far as we have followed it does not destroy the idea of such a connection, which would be the effect of a mere contradiction, but suggests that it may be characteristic of plants with two *or more* seed-leaves Supposing that monocotyledons were also found in some cases to form wood externally, the question would still arise whether any principle of development could be traced according to which the characteristics under discussion might arise together or owing to connected causes, as if e g there was a point at which the one natural kind approached very close to the other If so, the supposed further exception would still lead to a positive principle or contrary, and not to an empty contradiction Even if the first generic judgment were a sheer blunder and confusion, as has been the case from time to time with judgments propounded in science, it is scarcely possible to rectify the confusion except by substituting for it the true positive conceptions that arise out of the cases which overthrow it An example in point is Ehrenberg's inclusion of a group of confervoid Algae (Volvox) under the class of Infusorial Animalculae, or again, probably, his alleged detection of highly-organised structure in the Infusoria proper. Enumerative exceptions are futile in such cases, what is needed is a re-interpretation of the character of the group as such Such a re-interpretation is at once contrary and contradictory to the mistaken judgment which it corrects. But in the process of interpretation it may and perhaps must pass through a stage which may best be described when we are speaking of the hypothetical judgment

Between Hypothetical Judgments

δ. The *contrary* of the Hypothetical Judgment is as usual a judgment of the same type The complete contrary of 'If outdoor relief is refused the workhouses are crowded' would be 'If outdoor relief is refused the workhouses (*caeteris paribus*) are not crowded'. This denial means that the condition expressed in the hypothesis 'If refused' is not merely inoperative to cause pressure on the workhouse, but is actively operative to decrease that pressure I insert the

limitation *caeteris paribus* merely to secure the judgment being taken as truly hypothetical, because in such concrete matter as this there is a tendency to interpret the judgment as collective in the sense that 'Every case of α is a case of β', so that it may be objected to on the score of accidental instances In abstract matter, e g in geometry, where the hypothetical judgment has an unambiguous import, these objections are understood to be inadmissible Parallel lines are taken *qua* parallel when it is said that they do not meet, and so forth

This contrary is the form of negation to which all precise thought aspires If the condition is irrelevant and wholly unconnected, then indeed the entire type of knowledge to which we have aspired is a delusion and a snare, and the mere contradictory which will express our ignorance must be found in a judgment of a lower type But if the condition is relevant it must operate somehow, and we can only choose between one view of its operation and another Such a choice is expressed by embodying mere contraries in an absolute alternative or contradictory, by considering 'If A is B it is not C' as the contradictory of 'If A is B it is C', as it is the *only* way of denying it *by a hypothetical judgment*

But if we are to destroy the hypothetical judgment itself as an expression of ground and consequence, we must aim our negation precisely at its form The enumerative particular would not help us here When we have said that 'If (i e in so far as) a man is good he is wise', it is idle to reply that some good men are not wise This is to attack an abstract principle with unanalysed examples. What we must say in order to deny the above-mentioned abstract judgment is something of this kind 'If,' or 'Though a man is good, yet it does not follow that he is wise,' i e 'Though a man is good, yet he need not be wise'. The particle 'though' introduces the condition as a supposal, but by its adversative force prepares us for a denial that it has any connection with the consequent

This same form, which may be called the modal particular, is the appropriate contradictory to a generic judgment which has to be altogether surrendered and cannot be corrected by

a positive contrary Its meaning is however not as clear as that of the Generic or Hypothetical judgments. For it does not assert a positive relation, but drops down into an unanalysed quality of exclusion, and thus into the confused concrete of phenomena It may therefore mean that the supposed condition is inoperative and irrelevant, or it may mean that it is weakly operative and liable to be overcome by normal counteracting conditions, or that it is operative as asserted in the hypothetical judgment, but liable occasionally to be overcome by exceptional conditions. This last is a common meaning, but is not the true negation of the opposed abstract hypothetical judgment, and ought to be discarded from science, though there is no means of relieving the modal particular negation from it How then are we to treat the correlative case, in which we deny 'If A is B it is not C' by 'Though A be B yet it may be C'? This seems to confirm our view that the character of mere negation is incompatible with that of the hypothetical judgment. For the former of these two contradictories is of course the same judgment which we have already considered in the light of a contrary and *positive* negation, and the latter seems therefore to retain under its affirmative form the essential character of mere denial 'Though A is B yet it does not follow that it is not C' expresses the only sense in which the affirmative modal particular is a true or mere contradictory to the negative hypothetical judgment [1] This sense is not in itself satisfactory if taken as the import of possibility, and real possibility demands a nearer advance towards the affirmative contrary—a recognition of some real operative condition making for the connection alleged possible That this is so only confirms the suggestion made above that in significant negation contrary and contradictory tend to become one

I may anticipate the case of Disjunction so far as it here concerns us by saying that the denial *of* a disjunctive judgment, though formally possible, is not a problem that naturally arises in logic. The disjunction is the presupposition and the goal of negation as an *organon* of knowledge By denying it as a whole we sweep away the fabric of knowledge relative to the

[1] See below, on Privation and Affirmation.

matter in hand, and must begin over again from the beginning, as there is no sphere left within which anything can arise from the denial of the disjunction To deny a disjunction in this sense we should not trouble ourselves with the alternatives, but analyse them into their common basis and deny that. An impersonal judgment, as expressing wholly inarticulate knowledge, is the appropriate form for such denial, *unless* we are denying one disjunction under another. 'The soul is neither square nor round' may be denied by 'The soul is not in space', for this denial has a positive ground and consequence founded on an implied disjunction, 'either in space or intellectual' But if we are presented with a number of alternatives about a matter which seems to us to have no basis in reality nor relation to actual knowledge at all, then we may reply with the impersonal negative as the form of thought most suited to mere absence of positive content 'The disembodied spirit in its earthly presence is revealed either by contact or by signs.' To this an entire unbeliever would probably answer, 'There is no earthly presence of disembodied spirits,' and here he approaches, not wholly by his own fault, the infinite judgment. He has merely said that reality is without the matter alleged, and his saying has no positive import beyond what arises from the imputation to reality of a character—whatever that may be—not necessarily incompatible with, but rather undistinguished by, the presence of disembodied spirits.

Specific denials of a disjunction on the ground of incompleteness or superfluity[1] are of course either under a further disjunction which the denial tends to make explicit, or in the second case, *prima facie*, under the disjunction to be denied itself, with the result of showing that a different one is needed.

Double Negation

11. In treating of the contradictory relation, that namely between two judgments of which one simply denies the other, so forming an absolute alternative, we saw the results that spring from *double negation* We appealed to double negation for instance in the case of the singular judgment, such as 'Caesar crossed the Rubicon'. This, with its simple denial 'Caesar did not cross the Rubicon', forms an absolute alternative, although the denial has a meaning and is not a mere

[1] See following chapter on this case

infinite judgment. So far as this is the case, the opposition between the two is at once contrary and contradictory.

Do the two cases of inference from falsity in contradictory opposition stand on the same footing? The one is from the falsity of the affirmative, the other from the falsity of the negative. The former amounts to single, and the latter to double negation.

The explanation of single denial which has been given above may be briefly restated here. The abstraction of difference, taken in respect of its contrast with identity, and so as mere non-identity or otherness, is employed in negation as the vehicle of a positive contrary, which contrary is thus invested with the full alternative force that belongs to otherness as such when contrasted in the abstract with sameness as such As regards the history of early thought on this question, of course we are not to look for determinate abstractions in primitive minds. But primitive minds probably are abstract, though they do not deal with abstractions. We must look for the germ of contradictory negation in mere repugnance or repulsion, which, although a positive state, has a peculiar aspect of negativity to which the inarticulate abstractness of the primitive mind lends an aspect rather of contradiction than of contrariety. Every one who has watched children must have noticed the remonstrant 'No' without any *expressed* content, which is a sign of aversion to something done or suggested The absence of *indicated* reference to any particular matter is often surprising, and impresses an observer by the difficulty of finding either the bearing or the ground of the negation. Here it is rather the positive contrary that is undeveloped and latent than the mere rejection or contradictory Therefore I cannot but think that the absolute alternative—mere generalised otherness or rejection—makes itself explicit by the side of the positive contrary at a very early stage of thought

Thus though I do not take every negation to be necessarily aimed against an affirmation of the same content, yet it seems to me that the pair of judgments which form a contradictory opposition embody an ultimate fact of knowledge. Single negation is in form the substitution of *mere* difference, or

nothingness, for the combined identity and difference which alone have meaning This form is the basis of the alternative in contradiction It is the alternative antithesis between something and nothing That 'nothing' is furnished with a meaning *in hunc effectum* does not appear from or affect the form of contradiction. Single negation, then—the passage from the falsity of 'A is B' to the truth of 'A is not B'—must be regarded on its formal side as the abstraction of a universal characteristic of knowledge. It means that A is, under the conditions and for the purpose of the judgment in question, in a relation of *pure* otherness to B

The case of double negation—the passage from the falsity of 'A is not B' to the truth of 'A is B'—is in my judgment accounted for by the preceding remarks The true problem, to my mind, is not how negation should be the absolute alternative of the corresponding affirmation, but rather how, being in its exterior form and vehicle such an alternative, it should become possessed of positive intelligible import The fundamental nature of negation, thus understood, may no doubt be embodied in the principle of Excluded Third or Excluded Middle, which asserts that of two contradictory enunciations one must be true and the other false Thus stated, the principle is merely formal, because the question immediately arises, 'What are contradictory assertions?' The definition might be made plainer by substituting for the phrase in question an expression such as 'the assertion and denial of the same content', but no definition can relieve us from the task, which I have attempted to perform in this chapter, of explaining what a mere denial of an assertion really is Excluded Middle is thus merely the abstract case of Contradiction or simple negation, and the proof of the principle lies in the analysis of negation

I may illustrate this view, in the case of double negation, by two conceptions that deserve attention The affirmation that admittedly results from double negation may be ascribed to the reappearance of an original affirmation at which the first negation must have been aimed, so soon as that first negation is cancelled by the second I take this view to be true in substance, but false in the fact which its expression

postulates It was pointed out above that not every negation presupposes an affirmation Sigwart, in treating of double negation in his second edition, correctly postulates as the condition of denial only an *attempted* affirmation But it is true, and this is what Sigwart throughout meant to convey, that every negation bears on its face the nature of an alternative, so that, though we may not in fact have proceeded to it by denying an explicit affirmation of the same content, we are yet able to go from the negation of one member to the establishment of another It is one thing to say that every negation *is* preceded by a corresponding affirmation,—for we may not have judged on the subject at all,—but quite another to say that every negation bears on its face that, if we judge, it is the sole alternative to the corresponding affirmation This principle appears to me to be of the essence of the matter. It is this that gives double negation its distinctive precision and emphasis

In opposition to Sigwart's idea, erroneously expressed as I admit it to be, of re-establishing an original affirmation, Mr Bradley has maintained that the warrant of double negation simply consists in this, that in order to deny a negation we must already be in possession of the corresponding affirmation. We can only, he contends, deny A is not B on the ground that, within our knowledge, A is B. This allegation is made with good reason We can indeed deny A is B on the ground that A is x or y, each of which excludes B; but we cannot assert A is B on the ground that A is not x or y, each of which excludes B We cannot deny the consequent not-B on the strength of denying the antecedent x or y The old rule of the hypothetical judgment, 'Affirm the antecedent or deny the consequent,' forbids this

But on looking closer we shall observe that this impossibility is based on the imperfect view of the hypothetical judgment which assimilates its rules to those of the judgment of enumeration This view disregards the possibility of a connection at once synthetic (i e not tautologous) and pure (i e free from irrelevancy) For in this case the denial of the condition is the denial of the consequent, and it is this which has been before us throughout as the essential and ideal

meaning of judgment This assertion of a pure connection between condition and consequent becomes, in the case of a negative consequent which is now before us, identical with our position that all intelligible negation takes its meaning from contrariety, though its form may be that of contradiction. To say that in 'If A is x it is not B' we cannot go from 'not x' to 'not not-B' (i e to B) is to say that there cannot be contradictory opposition between contraries [a]

Contradictory opposition between contraries [b] can of course exist only if the possibilities are limited by means of a disjunctive judgment, and in this case Mr Bradley admits that the double negation may be got at otherwise than by the corresponding affirmation If we divide all Liberals into Unionist Liberals and Gladstonian Liberals we can go from 'That Liberal is not Unionist' to 'He is not not Gladstonian'. So too in a pure nexus with a negative consequent 'If powder is slow-burning (and in that case only), it does not strain the gun unequally' Under this judgment, if we know that the powder is not slow-burning, we are able to say that it fails in the quality of not straining the gun unequally We can unquestionably get this inference whenever we approach the knowledge of a pure ground involving a reciprocal character in the hypothetical judgment, which makes it, if either member is negative, include the same cases as a disjunction [c] The pure ground and the limitation of cases are merely different aspects of the same form of knowledge The ground is the fundamental and operative character by which a system imposes certain relations on its parts; the limitation is the external and formal consequence of these relations, which may be mimicked for the purposes of common logic by an arbitrary or conventional restriction of alternatives

My only difference from Mr. Bradley consists therefore in the view which I have maintained throughout, that apart from

[a] The negative character of not-B might raise the question of two negative premisses (Muller in 'Mind' lxxx) But *assuming reciprocity*, the hypothetical major has disjunctive force, and that adds a new factor See reference in note c

[b] See p 293, note *a*, above The Contradictory opposition here spoken of is not Contradiction in the wide sense, but the relation between Contradictories as recognized by Formal Logic, cp p 333, note, below

[c] See p 329, note, below

some limitation there is no intelligible negation—nothing but the infinite judgment, and therefore in strict logic no negation at all. Under such a limitation we can always go from denial of a positive quality to a positive result which we may as a matter of both theory and of fact approach from the side of double negation, although of course the identity of double negation with affirmation is in a reflective stage of culture too immediate to admit of the two being really distinguished. The reason for which I am anxious to insist on this not very practical distinction will appear more clearly when I come to speak of induction It is, in brief, the importance of the negative instance; that is, of approaching any positive content from the side of its limit, of the exact boundary at which it ceases and some other content begins For this boundary is a negation by denying which we not only affirm the content that lies within it, but affirm it in its conditions and genesis, at least for knowledge. 'If a nation has no true art it is not religious' is a judgment that gives the analysis of a group of 'negative instances', which analysis passes into an affirmation supported by those instances, in the form 'A nation which has true art is not not religious'. The conclusion thus obtained, 'This nation (having true art) is religious' may be *bonâ fide* arrived at through the double negation I have described, and may be at first unsupported by the direct observation 'This nation is religious'.

Conversion.

III Conversion is usually treated with opposition under the head of immediate inference It is primarily a transition from one grammatical form to another which introduces no new elements into the content Whether, or in what cases, it really involves inference, i e a passage from one *judgment* to another warranted by the first, is the main question which arises in treating of it, and of course includes the problem *what* inference, if any, is involved

The unity of the judgment, it will be remembered,[1] does not exclude a considerable measure of diversity. It is often a mere chance whether a range of affirmative thought is condensed into one proposition or comminuted into several. And inference is working through the whole of such a range

[1] See above, chap I

as the judgment gains depth and width, and defines its edges Thus, if we mean to say anything definite about the point at which we pass from one judgment into 'another', we must look at what is *bonâ fide* implied as the essential import of the judgment-forms that we discuss, and not at the actual transitions which attend our interpretation of any proposition, and which vary with mental endowment in the interpreter, and with the lucidity of expression of the given sentence. That is to say, we must distinguish interpretative inference from substantial inference—transition within the judgment from transition between two judgments

The question may seem indeed to settle itself at once. The unity of judgment is determined *prima facie* by the unity of the Content judged A new subject or a new predication, i e. one not related as part within whole to that of a given judgment, is needed to constitute another judgment But in immediate inference there is no new content—at any rate no new positive content Can there then be a new judgment?

The real interest of Conversion lies in the discussion which it provokes of the precise relations imposed upon its content by any given judgment, and of the boundaries which separate the *bonâ fide* meanings of the various judgment-forms In arriving at these relations or at this meaning we use interpretative inference; it is only when we find ourselves able to go from relation to relation by re-applying the same form of judgment, or from meaning to meaning by passing from a judgment of one type to a judgment of another, that we are really employing substantial inference It does not much matter, however, where we elect to draw the line of transition from judgment to judgment, so long as we understand the connection of the implications concerned

Simple Conversion of Singular

a. I will begin with an accidental case of simple conversion, not recognised in the text-books because the content of predication in singular judgment is not necessarily of singular reference 'The Duke of Cambridge is Commander-in-chief' I think it is beyond question that this proposition might be met with 'The Commander-in-chief is the Duke of Cambridge' in a tone which would give it the force of a criticism or retort, although its content would be war-

ranted true by the proposition as first enunciated I am not prepared to abolish the distinction of subject and predicate within the judgment, and if we retain the distinction, then the mere transposition of the two makes a difference. We understand the subject primarily as a designation and the predicate primarily as an attribution The idea of concrete individuality clings to the subject, and that of specially selected determination to the predicate. The second of the above propositions might certainly be understood to mean that the qualities of the office were limited in the relation in question by those of the individual, and not those of the individual adapted to those of the office

If it is replied that this may be possible enough, but that really one who commits himself to the former proposition has bound himself to know and judge the concrete synthesis of qualities which the predication constitutes along with its subject, I cannot deny that this is so And I am therefore content to rank such inference as is illustrated by the above transition under the head of interpretative inference, i. e. inference that falls within the logical unity of the judgment as *bonâ fide* expressed by either propositional form. It is true that in some simply convertible judgments, e g 'A = B in weight', the order of terms can make no possible difference of import. But the reason of this is not that the proposition is simply convertible, but that the content is of a highly abstract character which annuls all individuality, and thus destroys any significance that might attach to the difference between subject and predication A similar character belongs in some degree to all quantitative judgments

Conversion by Limitation

β. The Universal Affirmative Judgment (under which, in ordinary logic-books, the Singular Judgment is comprehended) is not admitted to be convertible *simply*, i e. by mere transposition of subject and predicate, but is supposed to be convertible 'by limitation', i e. by transposition together with reduction to the level of a 'particular' judgment

With reference to the first part of this rule, the whole course of our investigation of judgment is a comment on the degree and *rationale* of its truth It is equally certain that the prohibition of simple conversion is warranted by common

usage, and that the 'pure case' (which does not mean mere tautology) is an ideal operative throughout the judging activity. I am confident that Quantification of the Predicate and the Equational Logic owe much of their success to their recognition of this ideal, though their forcing it upon the ordinary judgment by truncating the meaning of the latter is a blundering *anticipatio naturae*

The second part of the above rule, the 'limitation' of the converse, has different values as applied to different classes of judgment which correspond to the so-called 'universal affirmative' If we take a judgment of the collective type and argue e g that because 'All houses in this street have gardens' therefore 'Some houses which have gardens are in this street' we do not seem to gain anything by the re-arrangement. And we certainly lose something, for we cannot recover the original judgment by re-conversion of the particular thus obtained. Of course re-conversion can only give 'Some houses in the street have gardens' But we know, to begin with, that all the individual houses in the street unite with their other attributes that of having gardens We seem, then, only to have advanced to a doubt of what we knew There is however a shade of difference suggested, as in the singular judgment examined above, by the mere transposition of subject and predicate The denomination of the individuals is less emphasised than the content enunciated of it. The former fills the place of pointing with the finger to an object of perception, the latter that of the significant ideas by which the perceptive judgment qualifies such an object But this transition seems to fall within the interpretative inference, and not to amount to substantive inference

In the case of the generic judgment of either type the import of the change becomes more emphatic 'The dahlia is one of the Compositae' tells me that this regular flower, apparently a mass of petals like a garden rose or peony or hollyhock, is really an aggregate of little florets like those of the daisy or dandelion [1] In short, this judgment distinguishes

[1] In the double dahlia—the form till lately commonest in gardens—every floret is developed into a one-sided corolla like those of the florets which form the ray of the common daisy.

the dahlia from flowers externally not unlike it, but having no structural affinity with it whatever 'The Compositae include the dahlia,' or 'Some Compositae are dahlias,' presents us with the typical or generalised structure of a Composite flower determined and differentiated by the peculiarities of a dahlia The rose or peony would in this case never come into our heads; we should be occupied with some such individual as the daisy or the thistle, and the object of the judgment would practically be to distinguish the dahlia from other Compositae, not to distinguish it from flowers which are in no way akin to it The one judgment qualifies the external appearance of a dahlia by the internal structure of the Compositae, the other qualifies the diagrammatic type of a composite plant, as admitting, among others, of the specific peculiarities of the dahlia

I have but little doubt that this account represents the meaning of the two propositions *in fact*. But it may be said that we have no *right* to separate them, and that we ought to demand the explicit recognition of both these determinate affinities as essential to the meaning of the generic universal judgment itself, and it may even be insisted that if the second proposition contains more than the first in any respect—e g in the more concrete appreciation of diversities of composite structure—then it cannot be warranted by the first. This latter suggestion would make short work of all conversion whatever beyond the rank of grammatical change, unless—*πλέον ἥμισυ πάντος*—mere selection can make the old into new, or unless some principle is appealed to that goes beyond the judgment itself. The former part of the objection may be met by saying that undoubtedly the original judgment *may* be made with the whole significance of the two propositions, but that when so made it is somewhat artificial, and that the distinction of a specific adaptation within the Composite type is naturally a different process from the qualification of a given shape by the abstraction of that type I may put the antithesis thus: the generic judgment, which we took as the *convertend*, if completed into identity, would determine the composite type by limitations restricting it to the case of the dahlia, while the particular, in this case the *converse*, if

similarly completed, would extend the range of species by disjunctive enumeration till they expressed the whole content of the natural order of Compositae.

Modal Conversion

γ This conversion of a generic judgment may be taken either as interpretative or as substantial Inference Tested by common usage it is, I incline to think, a substantial transformation, resting on some such principle as the possibility of treating every content in turn as quasi-individual Tested by the logical ideal, it is a mere phase in the interpretation of the judgment, which involves reciprocal determination in the affirmation of a nexus In either case it contains the only fundamental principle on which conversion can really proceed, viz that every content can be exhibited as a quasi-individual element in a system. This is, for instance, the principle of which 'modal' conversion is merely a corollary In the typical example employed above modality has no *real* application, modality only appears where the disjunction is one of ignorance The particular judgment 'The type of the Compositae includes among its alternative modifications that of the dahlia' *may* be read into 'A composite may be a dahlia', i e the fundamental structure C is a real element compatible with the modification D But this is a mere consequence of feigning ignorance when we have knowledge, in order to get the possibility we have to imagine that conditions are unknown which in fact are distinctly known and enter into the content of the judgment

The pure Hypothetical Judgment differs from the Generic by disregarding individuality, and therefore the above principle is disguised in converting a Hypothetical, so that we appear in this case to obtain pure modal conversion 'If arsenic is taken in such and such a quantity, it will cause death with such and such symptoms', which gives the converse 'Death with such and such symptoms *at least may* have been caused by arsenic' This is a motived possibility or real possibility, not a mere possibility, and forms a substantial conclusion and one warranted by the convertend But this result is still rather a corollary from the converse, than the converse itself The example just quoted is one hardly deserving hypothetical form It involves no traceable modification such as to set

up a clear nexus between antecedent and consequence. Thus we cannot assign any precise conditions under which the consequent is related to the given antecedent, and the possibility that such death may result from such poison is merely a fact *plus* unknown alternatives In a true hypothetical there is more than this Grant that it is not simply convertible, still we can bring the possibility home to the nature of the consequent. 'If straight lines are parallel, they do not tend to meet' The real converse of this is, 'Straight lines *in the same plane* which do not tend to meet are parallel' But the limiting condition is not given in the convertend Nor do I mean that we have a right actually to state it in the converse, but yet we must think of the converse where such a condition can be suggested as exhibiting a real if undefined attribute of the content which is now the subject, in virtue of which this content includes the case mentioned in the antecedent of the convertend The affinity between generic judgments and the more valuable hypotheticals is very close, and we lose all hold on the generic element in judgment if we insist on reducing a definitely determinable content to one real alternative in an unknown number of unknown ones I cordially agree therefore with Mr Bradley's distinction between *real* and *mere* possibility, and only insist that the true content even of a modal converse is the positive nature in virtue of which the subject-content is variously determinable, and not a mere conjunction of attributes *plus* other unknown conjunctions

Simple Conversion of Universal Negative

δ We are now to speak shortly of conversion in which the negative is employed Every negation rests, as we have seen, in its purely formal aspect, on the ultimate or absolute disjunction But it does not follow from this that a real process of substantial inference takes place where negation intervenes

I begin with the simple conversion 'A is not B,' therefore 'B is not A'; 'No negroes have straight hair,' 'No straight-haired man is a negro' I should certainly prefer representing this transition as a true process of argument to illustrating it by a diagram of two separate circles, because by the latter means we destroy all idea of the structure of the judgment But yet I cannot think that in our present stage of reflection

the argument if any is more than interpretative, i.e more than we are always using in arriving at the full meaning of any sentence. We can formally trace out the process of inference, but in using it we are like an engine running free and doing no work No doubt the formal steps are, beginning from the ultimate disjunction between 'is' and 'is not', to say 'No negroes have straight hair', 'Straight-haired men are either negroes or not', 'If negroes, then they are both straight-haired and not straight-haired', 'Therefore it is false that they are negroes', 'therefore they are not negroes' But all this is given in the meaning and practice of any civilised language, and though it might be possible to stumble in using this converse,[1] that is no more than may happen in reading any sentence however simple The elementary meaning is simply that the two contents are in the relation of abstract otherness to each other, and refuse to be brought together in the modified or concrete otherness which subsists within the affirmative judgment

Contra-position

ϵ *Contraposition* seems to contain a more remote conclusion It goes from A is B, to Not-B is not A, e g from 'Every negro has woolly hair' to 'A man who has not woolly hair is not a negro' I remark on this process, 1 that the skeleton argument with symbolic letters seems far more remote and obscure than the intelligible example, 11 that we are much hampered in the apprehension of our inference by being forbidden to conclude that 'Not-A is not-B', which the ideal of judgment demands as the expression of the negative instance, 'Just not-A is just not-B,' e g —the affirmative being 'True freedom is virtue'—'If you fail to produce freedom, you fail so far and for that reason to produce virtue' And, in substantial inference is less likely here, for the unity of the judgment is much greater in affirmation than in negation, and when we have just, so to speak, dipped the object *a* in the colour *b*, it seems idle to ask on the basis of that judgment whether what is not of the colour *b* can be the object *a* We are in fact no longer dealing with *a* and *b*, but with *a b as including all a* We may say of course, 'Oh yes! not-*b* is

[1] As e g if any one tried to argue that 'All not-straight-haired are negroes'

not-*a*, because if not-*b* is *a* then it is *b*' But this remark is made ready to our hand, we have just qualified *a* by *b* and therefore are at least entitled to '*a* is not not-*b*', which brings back our problem to the last head, that of simple conversion

In short, then, in both these cases we employ the absolute alternative, but this is given in the *form* of negation or contradiction, and needing no true limitation of alternatives, can only rank as a formal principle of intelligible thought and speech, not as a real addition to content of inference Transition by help of such a formal principle I call Interpretative Inference

Privation. Affirmation and Exclusion

5 The external shape of negation belongs as we saw to ignorance, and significant negation is knowledge disguised as ignorance For bare denial would be devoid of meaning Now this external shape of negation seems really appropriate where positive knowledge fails, i e in the region of what has been called Privation,[1]—the mere absence of positive determination But this region is also the region within which there falls the limitation of knowledge, a matter of the most serious and positive import Where knowledge simply fails us, and consequently we seem to have nothing left but privation or bare denial, how are we to pronounce on any suggested possibility, *a* by way of affirmation, *β* by way of exclusion?

Privation and Real Possibility

The case *a* is the more complicated of the two The privation or bare denial is in this case the bare denial of an incompatibility with reality, i e of an impossibility *Exclusion*, as we have seen, must rest on a positive quality, a ground of negation Here, *ex hypothesi*, we can find no *exclusion* of the impossibility We are supposed to know simply nothing, either *pro* or *con*, about the positive matter whose possibility is in question Are we therefore bound to admit that it is not

[1] 'Privation,' *Privatio*, used by Sigwart as equivalent to Aristotle's *στέρησις*, *στερητικος*, and as contrasted with opposition, *ἐναντιοτής*, and distinct from negation, *ἀπόφασις* *Στέρησις* seems to be applied to any negative enunciation, *ἀποφάσις* only to the denial of an affirmation, see Aristotle's Organon, 38 b, 13, with Waitz's note Unluckily the distinction of privative and negative terms as given e g in Whately's Logic has just the reverse meaning to that of privation and negation in Sigwart and Bradley 'Privative' in Whately indicates a positive opposite, and 'negative' a mere absence The association with active 'deprivation' implying a *loss* appears to be the cause of this usage.

not-possible, and, as a consequence of this double negation, that it is possible? If we are led to do this—and the trick is often attempted, especially in popular theology—we feel that we have been cheated. A possibility, in the usual sense of the term, is something. We are foolish if we do not keep it in view and let it influence our deliberations in any way which its nature demands. Yet this something has here *ex hypothesi* been created out of nothing. But in knowledge at least nothing can come of nothing, and we are trapped in a contradiction.

What we have to remember is that our denial of the exclusion of the positive content is or approximates[1] to a bare denial, or in other words, rests upon no positive ground. We do not *exclude* the impossibility; we only fail to find it. And therefore our denial is meaningless or nearly meaningless, and amounts to nothing. In other words, the possibility which we are asked to infer from mere not-impossibility has as so inferred no foundation in positive reality. A real possibility of any result consists in something given as actual, which, under conditions of known nature and not known to be impossible, would give that result. A gun forms a real possibility of shooting, if there is no reason to suppose that cartridges are not to be had, an acorn is a real possibility of an oak, if we know of nothing to hinder its being planted and growing. But in the case supposed we have nothing like this, we have simply nothing—a failure to find incompatibility. Therefore we ought, strictly speaking, to conclude not that it is possible, which is an affirmation, nor even that it is not impossible, which borrows the form of intelligible denial, and therefore presupposes a positive ground of denial, but simply that we do not know it to be possible. This conclusion gives its true value to the form of bare denial by making the ground of negation what in the case of ignorance it really is, an actual state of our own minds which excludes the knowledge in question.

[1] 'Approximates,' for a suggestion, if intelligible, contains *some* ground in its mere conceivability, and thus affords material for intelligible support or refutation. But by far the larger element in the importunity of many suggestions is drawn from the fallacy of inferring from non-impossibility to real possibility.

Instances of assertion resting mainly on this confusion are to be found e g in expositions of the so-called modern Buddhism, the elaborate dogmatic fabric of which is chiefly protected from criticism by the impossibility of discovering any ground on which it may be taken to rest, and against which, therefore, a positive objection can be raised

To put the point in other words, in the case before us it is not true that double negation is equivalent to affirmation. For the double negation in question is founded neither on the affirmation itself, nor on the denial of a specific alternative to the affirmation. If there were such a specific alternative to the affirmation, it could only be denied on some positive ground, and such a denial would not be the bare denial of which *ex hypothesi* we are treating

Or again, without appealing to the difference between privation, i e bare denial, and exclusion—a confusion between which processes however is the root of the fallacy—we may simply lay it down that a real possibility is something actual, and that a bare denial affirms nothing as actual, and therefore a bare denial cannot affirm a real possibility

And it must be added with reference to remote suggestions generally, that a failure to demonstrate impossibility can almost always be secured by a high degree of remoteness or of abstractness in the suggestion itself Thus, if accepted as really possible because not demonstrably impossible, such a suggestion would profit by its own wrong. Reality cannot, for us, contradict a suggestion that has no point of contact with reality Things in themselves, according to the popular notion of the Kantian doctrine, are the content of such a suggestion They made no claim to affect knowable Reality, and therefore knowable Reality can present no quality which excludes them As a rule, to disprove the grounds on which a fact is advanced is not the same thing as disproving the alleged fact, denying the antecedent does not amount to denying the consequent But in the case of unverifiable suggestions the grounds which are implied in the suggestion are for a given state of knowledge the sole grounds conceivable, or at least the grounds which can be stated are capable of exhaustion, and the disproof of them may be taken as for that

state of knowledge disproving the suggestion. It must be remembered that denying the antecedent does deny the consequent *qua consequent of the antecedent denied*. At any rate it is clear that in such cases as those under discussion the failure to prove impossibility which arises from the emptiness of the suggestions themselves must not be taken as amounting to the establishment of real possibility.

Privation and Impossibility

β. We cannot go from bare denial, or privation, of an impossibility, to real possibility, because *privation* of impossibility does not involve positive affirmation as the *exclusion* of impossibility does. It may seem therefore that we have decided in the negative by anticipation the question whether privation or bare denial can ever justify exclusion. We denied the claim of privation to establish real possibility, on the ground that it could not exclude, as one form of the ground that it could not affirm. Thus when we omit the incompatibility or impossibility, and consider merely the exclusion of any positive content, it seems that we cannot predicate exclusion on the strength of privation. Up to a certain point common sense and experience support this result. Gold has never been found in Northumberland, but that alone does not prove that it never can be found there, unless the geological formation forbids, i.e. is a positive ground of exclusion.

Still, the two transitions, from privation of impossibility to affirmation of real possibility, and from privation of actuality to exclusion of possibility or of actuality, do not stand on quite the same level. The former is explicitly a passage from denial to affirmation, or, as we saw, from nothing to something. The latter retains its negative form and *vehicle* unchanged, and leaves the change of its *ground* and *bearing*, the two other elements of meaning,[1] to be moulded by the context. Any mere denial or privation when expressed intelligibly is given a ground and bearing with reference to our cognitive state, as we saw in dealing with the instance, 'We do not know that there are things in themselves.' What is a bare denial with reference to reality is a positive affirmation about our knowledge. We have agreed that we cannot transform 'We do not know it to be impossible' into 'It is possible.' Can we

[1] See above, p. 289.

transform 'We do not know it to exist' into 'It does not exist'?

It is plain, I think, that many beliefs become abortive and cease to be regarded because we become convinced that reality does not constrain us to accept them And it seems an unworthy shirking of a theoretical difficulty to treat these beliefs as shelved but not denied If, whether in serious speculation or in grave practical deliberation, some idea cognate with the matters then before the mind has ceased to exercise the slightest bias on thought or on action, it seems idle to say that upon such an idea we have suspended judgment as upon something that may be true in itself but is nothing for us It seems clear that upon such an idea—I may instance the Swedenborgian hierarchy of spiritual beings—we have in fact sate in judgment and have condemned it as unreal But I admit that the speculative expression of our relation to such conceptions meets with serious difficulty from the necessity of basing denial upon a positive ground On what positive ground can I base a denial that there are exactly seven heavens, or that there are just seven orders of superior spirits? I unquestionably do deny it, that is to say, 'I do not believe it.' The habitual use of such phrases as this,[1] which refer grammatically to a fact of my intellectual state, but actually serve as negations of something ascribed to reality, bears witness to the connection which I am attempting to point out.

Incompatibility in the ordinary sense depends on the system of reality Differents which claim the same place are incompatible, and, in short, everything is incompatible with reality which, while not conforming to our ideal system which stands for reality, is yet without the power to modify it. But, as I pointed out above, all this falls to the ground where the system does not extend Where 'we do not know' in the sense of having no knowledge—not merely in the sense of lacking complete knowledge—we can say nothing, and *ex nihilo nil* 'Then,' it may be said, 'neither acceptance *nor rejection*' I cannot follow this Knowledge is positive, and acceptance and rejection are not co-ordinate alternatives We

[1] Compare οὔ φημι, which means 'I deny', or our common phrase 'I don't think that'—which is really equivalent to 'I think that—not'

doubt, where we have a basis of fact to go upon, and presumptions that appeal to that basis; but where we have nothing to go upon we cannot doubt

The only conclusion that I see open to us is of this nature. Where privation seems to warrant exclusion, we must look for the positive ground of exclusion in the determinate completeness of our ideal Reality. Such a ground may be hard to state, and may amount in positive content to little more than our experience of the persistency of the privation But its nature must be that the ideal Reality by its organised completeness excludes the matter which attempts to introduce itself. This must not be taken to mean that we are nearly omniscient It means that the general plan and growth of our knowledge is such as to afford no basis of attachment for the proposed accretion, although for this very reason we are unable to specify any definite antagonism between the content suggested and the positive contents already accepted as part of Reality Reality is not especially incompatible with seven heavens, it could be so only if we accepted some kind of heaven as a reality and were prepared, on the basis of our knowledge about it, to reject either the particular number seven, or the application of number to such a subject at all What we really have to say is that we do seem able to trace, however imperfectly, something like a development of the sensuous into the spiritual world, and that the main lines of this development appear to have a completeness of their own, without growing out into a duplicate and quasi-material world

It is not enough to destroy the grounds on which a suggestion is explicitly based, unless we can show that they form not merely the sole ground alleged, but the sole ground possible. And in a region beyond our knowledge this *ex hypothesi* cannot be done It is often possible to show by what logical fallacy or by what psychological tendency a suggestion was generated; but this is not a logical refutation. It may however grow into a refutation if, besides the tendency which caused the error, we can exhibit the satisfaction which reality offers to the rational necessity embodied in that tendency. We thus not only destroy the *raison d'être* of the error, but show a presumption that there is an excluding ground

Exclusion by Privation then rests on a conviction, won by persistent lack of affirmation, of the *true* negative limit and external contour of knowledge, which limit, *qua* the true limit, must be held true of reality. A privation cannot ultimately be referred to our knowledge only. If persistent in the history of thought and justified by the tendencies of knowledge, it must sooner or later be taken as true of reality.

At best, we must remember, negation is always negative The last step from the positive ground to its formal expression by means of denial, retains the form of privation, i e. of ignorance. This is what the old saying means, 'you cannot prove a negative.' The negation is not after all quite the same as the positive opposite latent under the negation You cannot prove that parallels never meet In order to do so, you would have, like the Irishman, to 'be there when it did *not* happen'. You can only prove that they always do this, that, or the other which in virtue of your geometrical experience you take as equivalent to not meeting That is to say, *assuming* your geometrical system to be *ad hoc* exhaustive, then your failure to see, on the basis of that system, how parallels can meet becomes knowledge though it retains the form of ignorance. It expresses a limit or outline essential to geometrical science Thus the cases of persistent privation and of true positive exclusion (genuine denial, not bare denial) differ simply by the nature of the positive ground which underlies them respectively. In privation this ground is general, drawn from the character and tendencies of Reality; in true exclusion it is special, drawn from a system within which the alleged reality would fall. It would seem fair to concede to the former somewhat more and to the latter somewhat less finality than common theory recognises.

CHAPTER VIII

DISJUNCTION AND THE STATEMENT OF CHANCES

The Disjunctive Judgment.

1. THE Disjunctive Judgment combines in an explicit form the characteristics of the Generic and of the pure Hypothetical Judgments

Its Genesis

1 It is needless however, and would be artificial, to lay down rules for the precise mode of transition from these judgments to the complete Disjunction The whole assertory state within which the simpler forms of judgment, at any rate from Comparison upwards, have their import, is from the first of a disjunctive nature Reflection may therefore be stimulated to the explicit formulation of this type of knowledge by very various occasions But the common ground which must operate in all these occasions is the discovery of differences into each of which the identity underlying all of them enters as a whole, and in all of which taken together its manifestations are exhausted. Every difference has the former of these characters in some point of view A conjunctive judgment, or conjunction of judgments with an identical subject, can always be made disjunctive by wilful abstraction. A diamond is carbon, and crystalline, and very hard, and highly refractive This is a conjunctive judgment or set of judgments[1] But if we limit the underlying identity, the nature of the stone, by the several conditions under which it exhibits these several predicates, then each of these predicates may be regarded as not conjoined with but exclusive of the other attributes enumerated A diamond may be considered either merely as an element, or merely as a transparent substance exhibiting crystalline structure, or in its power of scratching other hard substances, or in its effect upon light This is disjunction—arbitrary and subjective, if we please, but still disjunction Any distinguishable attributes may be regarded as reciprocally exclusive by our simply refusing to

[1] See above, chap i, on the Judgment in Time

attend to more than one of them at once *x* may be both *a* and *b*, but qua *a* it is not *b* But in another respect also the above instance of disjunction is bad and arbitrary. It makes no serious attempt to exhaust the attributes We stop enumerating them simply because we do not care to go on.

It is clear that a disjunctive judgment may originate with a conjunction of judgments like the above, which turn out to have assignable relations to one another, or, again, with the discovery of an error in a generic judgment, e g 'Cereus is a night-flowering plant' 'No *Cereus grandiflorus* is a night-flowering plant, but there are a hundred species of *Cereus*, and not all are night-flowering' Such considerations would force upon us the disjunction, 'Cereus blossoms either at night, or in the early morning, or &c, &c' Again, the attempt to convert a generic judgment naturally leads to a disjunction [1] And so does the challenge thrown down by the specific condition and consequent of a pure hypothetical judgment 'If we catch the train this morning we reach London to-night' In presence of such an assertion it is not in human nature to abstain from asking, 'And if we miss the train, what then?' so as to make explicit the disjunction, 'We either catch the train or miss it,' and probably some such further consequence as, 'We either get to London to-night, or have to sleep at Crewe'

All these are merely different ways of giving utterance to the interest which attaches to some pervading identity and compels us to pursue it throughout its modifications. Such an interest, as we have seen, environs every genuine judgment, and makes it an element in a system. And in proportion as such a system is made explicit, negation enters into knowledge For in every system the parts have an aspect of negative relation to one another

Thus the immediate occasion on which we form a disjunctive judgment may vary But the characteristics of true disjunction do not vary By true disjunction I mean a judgment in which alternatives falling under a single identity are enumerated, and are known in virtue of some pervading principle to be reciprocally exclusive, and to be exhaustive

[1] See above, chap vii, on the Conversion of generic judgments.

This is the disjunction of which I shall treat, being convinced that what seem to be different kinds of it are in fact nothing but imperfect examples

Imperfect Disjunctions.

ii I will mention three of these.

Not exclusive

α. First, there is the so-called disjunction in which the alternatives are exhaustively enumerated, but are not taken to exclude one another. 'He is either knave or fool' does not, it is said, exclude the possibility that he may be both On the question of the genuineness of these disjunctions I must refer to Mr Bradley's detailed discussion,[1] which appears to me to show decisively that we never *really* mean to take into consideration under our judgment the conjunction of the alternatives specified in our 'Either'—'or' Without following him into the study of grammatical details, I content myself with remarking that any Disjunction in which the alternatives are not reciprocally exclusive must of necessity fail to be exhaustive—the case or cases in which any of them are conjoined being *casus omissi* If, for the purpose of the disjunction, such a case may be reckoned under one of the other alternatives, then indeed the disjunction may be reckoned to be perfect, but then the case of conjunction does not rank *as* conjunction, but under one or other of its component elements Thus, 'To go by train you must have either a first, second, or third class ticket' A man may of course buy all three, if he pleases, but the possession of them does not constitute a fourth case of liberty to go by the train He goes by the train in virtue of one or other, though he may change carriages at every station if it amuses him. The conjunction of all three tickets forms no separate alternative as a particular way of going by train, and therefore is rightly disregarded in the disjunction It is not indeed implied that 'If he has a first-class ticket he has not a second or third', but it is implied that 'If he goes in virtue of a first-class ticket he does not go in virtue of a second or third'.

Disjunctions of Ignorance.

β Secondly, there are the troublesome cases often taken as the true instances of Disjunction, which may be called 'Disjunctions of ignorance' The essence of these is that they refer to an individual (actual or supposed) and not to

[1] Principles of Logic, Book I, chap iv

an individuality, and consequently express doubt or indecision rather than knowledge

'A triangle is either isosceles, scalene, or equilateral' A triangle here *can* mean any individual triangle you may pitch upon—'any given triangle' And with respect to such an individual triangle the disjunction says that it must belong to one of the three kinds mentioned, but that we do not know to which. Lotze, for instance, appears to accept this expression of indecision as the final type of disjunction But it seems obvious that this uncertainty is purely dramatic or fictitious, and is a mere corollary from the true disjunction, which is, 'A three-sided plane figure as such must have all its sides equal, or two only equal, or all unequal' Or we may take a case where the doubt is real, as often in common life; but here also it is a mere application of or inference from the true disjunction of knowledge 'Being an Oxford man, he is either a University College man or a Balliol man, or &c.' This judgment, which is a real expression of doubt or ignorance, is based of course on the positive knowledge that the conditions of University life require the student as such (generically) to attach himself to some one of the corporate bodies enumerated in the judgment. With disjunctions of this type we must class the commonest of all expressions of doubt or ignorance 'He is either angry or jealous,' 'He has either measles or scarlet fever' These, like the above, differ not in principle but only in perfection from the ideal disjunction What operates is something we know, and know to contain the specified alternatives We do not however specify our knowledge in detail—it may consist in a content hard to define—and we merely point to the concrete individual, in whom it is embodied and from whom it takes its interest About this individual, as his complete state goes beyond our knowledge, the judgment takes the shape of doubt, just as in the case of 'any given triangle'. The higher logical form may be imposed on the lower content; this is a possibility which follows from the nature of reason, and which makes abstract distinction appear in some aspects so valueless. Every thinkable content has in miniature all the characteristics of reason, and in the abstract you can

two of them are together greater than the third' Such an attribute might be called the categorical or positive basis of the disjunction, [a] but it is an illusion to suppose that a basis or ground is *necessarily* more real or more primary than its consequences, and that therefore the implied predication in disjunction is more categorical than the explicit 'either—or'. The ground is to its consequences as whole to part, but if the consequences are fully stated in a connected system this distinction falls away, and in disjunction such full statement is the ideal

The disjunction is therefore the only judgment-form that in strict theory can stand alone. All connection is within a system; and only that judgment is self-sufficing which affirms at once the system and the connections within it. I do not say that every disjunction is thus ultimately self-dependent, but relatively to a number of hypotheticals which have their truth within it every true disjunction has a substantive character. Thus the disjunction which lays down the nature and kinds of the triangle contains the ground and basis of all the hypothetical judgments which expound the properties of that figure In other words, if completed and made explicit, any one of those hypotheticals would result in that disjunction; which however itself falls within the ultimate judgment that would expound the nature and modes of space The above then is the generic or substantive element in disjunction

The Hypothetical Judgment.

β But we need in addition the reciprocal relations between the forms which constitute the explicit development of this predicated universal For these relations we must go to the hypothetical judgment, and to the hypothetical in a very late and artificial form, viz that in which the negation of one content is known as the ground of the affirmation of another content, and the affirmation of the one content is known as the ground of the negation of the other content. The perception of the relations which these two types of hypothetical judgment embody is essential to the exhaustiveness of the disjunction and to the reciprocal exclusiveness of its members. In order to know that the alternatives enumerated are

[a] See Appendix to this chapter

reciprocally exclusive, we must be able to say (using as an illustration the simplest case in which there are only the two alternatives B and C), 'If A is B, it is not C' And in order to be sure that no possible alternative is omitted we must be able to say (in the same example) that 'if A is not B, it is C'. According to a rule of Conversion, or rather of Inference, accepted for the case of Hypothetical Judgments ('Deny the consequent') the equivalent or converse judgments, 'If A is C it is not B' and 'if A is not C it is B' are involved in the two corresponding judgments above mentioned. If this Conversion or Inference is disputed, then we must say that all four cases, 'If A is B—,' 'If A is C—,' 'If A is not B—,' 'If A is not C—,' must be perceived independently before the predication of disjoined alternatives is justifiable [1]

[1] I insert here some details which are legitimate matter of curiosity, but would needlessly overload the text

1 It might be urged, on the analogy of the argument employed above (p 324)—that a disjunction which is thoroughly exhaustive cannot but have its members reciprocally exclusive—that the hypothetical which *prima facie* secures exhaustiveness (If A is not B it is C) ought to affirm the *reciprocal* exclusiveness of the antecedent and consequent, i e to exclude the case B C If it did so, on the other hand, it would become at a blow equal to the disjunction 'A is either B or C,' and would include in itself the case 'If A is B'—with its converse In other words, it would become a *reciprocal* judgment, correlative to a definitory affirmation, and as such would admit of conversion or inference *by denial of the antecedent*, just as if we were to infer from 'A is B' that 'not-A is not-B' This, as we have seen all through, is the ideal of the judgment, and a hypothetical judgment with negative condition or negative consequent, that fulfils this ideal, coincides already with the disjunction But usage does not warrant the ascription to the hypothetical 'If A is not B it is C' of the meaning 'If A is not B-without-C it is C-without-B' As commonly employed, therefore, it lays down a certain outer limit, but does not exhaust the subdivisions within the limit This is just the point of contrast between the hypothetical judgment in its ordinary signification and the complete disjunction But there is a certain tendency on the part of the former to advance towards the latter It is plain that the reasons which induce us to give prominence to the alternatives mentioned as the only ones to be specially considered may readily transform themselves into reasons why only the alternatives mentioned *can* be considered, or, perhaps, can exist We have such reasons *just* warranting a disjunction in the instance given above (p 324), where the case formed by the combination of the alternatives considered exists, so to speak, in fact but not for the law

11. It may be worth while to point out that disjunctions with more

The reference to these two or four hypothetical judgments has its value in elucidating the nature of the system which a true disjunction embodies It exhibits in the plainest light the indispensable function of negation in articulated knowledge, and the positive import with which in virtue of that function the negative is invested We have already seen the nature of this import in the analysis of the significant negation, where, however, the positive ground and consequence of denial were matters of tacit understanding and inference from context In explicit disjunction, on the other hand, we find them after they have been developed independently and distinctly in the hypothetical judgment, and affirmed as actual attributes within a system that is alleged to exist in reality

When are parts disjunctively related?

iv But, it may reasonably be objected, it is not in every system that the parts are disjunctively related As a rule the parts of a system are predicable in conjunction and not in disjunction

Apart from the case of intentional abstraction by which any conjunction can be turned into a disjunction—for in the last resort within every system every part involves the whole nature of the system—this criticism is just. A human

than two alternatives must be treated, as regards the hypothetical judgments involved, as a succession of dichotomies The hypothetical judgments of each type involved in such a disjunction would therefore be equal in number to the disjoined members, i e each alternative must be made in turn the positive and negative condition of an hypothesis, with a compound consequent, the disjunctive nature of which cannot appear in the hypothetical judgment A is either B, C, D, or E Then we have the negative conditions, 'If A is not B, it is within C D E,' 'If A is not C, it is within B D E,' 'If A is not D, it is within C B E,' 'If A is not E, it is within C B D' The positive conditions will correspond severally to the above negative conditions

Here we see a second defect of the hypothetical as compared to the disjunctive judgment It can only handle one reciprocal relation at a time, and cannot master a whole system of such relations in a single view In the above analysis the hypothetical judgment does not enable me to express more than a single contradictory relation, as between a particular A and its not A The idea of a pervading contradictory relation, characterising *any* one part as against all the others, cannot be expressed in any one hypothetical judgment By saying If A is not B it is *either* C, D, *or* F we should be pressing the hypothetical type beyond its powers

body is made up of trunk, limbs, and head, not of *either* trunk *or* limbs *or* head The government of a civilised nation consists of the legislative *and* the executive power, not of the legislative *or* the executive A genus, again, may be said to be identifiable with *all* its species, not merely with *either* this *or* that, though here we are on more doubtful ground Triangles are isosceles, scalene, *and* equilateral Men are white, black, *and* yellow We could hardly say however that '*the triangle is* isosceles, scalene, and equilateral', or that 'man is white, black, and yellow' The difference between conjunctions of the kind here brought forward, and true *dis*junctions, is formally speaking a difference of the aspect in which a real system is regarded, but materially, therefore, has an intimate dependence on the actual nature of the real system in question, which may be such as to throw one aspect or another prominently forward Every universal may have its differences conjunctively enumerated, whether they are in time or in space, or merely distinct in thought But in so far as the universal itself enters as a whole into each difference, which it can do in very different degrees, so far each difference, if imposed as a condition on the universal, excludes all the other differences A man's having a hand does not interfere with his having a foot But a man's having a feeling in his hand does begin to make a claim on the universal, the man himself, which is to a certain limited extent incompatible with his having a feeling in his foot or elsewhere And when we come to consider such acute interest or feeling as occasions the absorption of the whole mind in the perceiving or suffering member, then it is true to say, 'The man perceives or feels *either* with eye *or* with ear *or* hand *or* foot,' as the case may be. So again if we think of a triangle as a mere abstract generality that describes a heap of various figures, we may say that it includes, or that—in a collective judgment—its individual instances *are*, this, that, and the other But if we think of it according to its complete conception as an individuality that must necessarily take individual shape, and if we follow the process by which such shape must be determined, then we can only express our insight by the use of the disjunctive 'Either—or'

The conception of the whole as conditioned by one of its parts takes the place of that imaginary reference to an atomic point of time which has been supposed to be of the essence of disjunction 'A moving object is either here or there' means 'if here, not there' and 'if not here, then there'. It does not mean 'Within the indivisible moment in which I am judging a moving object can be in one place only'. For I cannot judge in an indivisible moment, nor can I refer to a present that is an indivisible moment In any extended time a moving object traverses space, and our 'present' is always an extended time And so the disjunction if referred to our 'present' time would not be true, and the moving object would be, like Sir Boyle Roche's bird, 'in two places at once'

Is the disjunction reducible to Hypotheticals?

v When we have understood the nature of disjunction there is not much profit in asking whether the disjunctive judgment can be 'reduced' to two or more hypothetical judgments The mere fact that the hypotheticals in question[1] are separate judgments, and that the disjunction is a single judgment, is enough to show that we have in the latter a *combination* of unity with reciprocal exclusion which we have not in the former. I have endeavoured to express this unity by representing the disjunction as a combination of the generic and the hypothetical judgments But it must be remembered that at best we are dealing with grammatical types which are only the symbols of states of knowledge, and it is most probable that any one who is able to make the two complementary hypothetical judgments 'If A is B it is not C' and 'If A is not B it is C', supplies out of his own mind the systematic relation which the two taken together involve, in a judgment equivalent to 'A is either B or C'

Not to dwell longer than I can help on formal points, I merely remark in addition to what was said above,[2] that in any case these hypotheticals themselves presuppose *the* ultimate or formal disjunction, 'A is either B or not-B,' by their introduction of a negative relation into knowledge; and that further, if we wish to take the two hypotheticals above mentioned as implying the two which follow from

[1] See footnote, p 329

[2] p 329

them by denying the consequent, we are once more relying on this formal disjunction, which is essentially involved in such a transition [1] The material importance of the whole question lies in the process by which the *form* of disjunction, in itself on a level with or consisting in the Law of Excluded Middle, i. e. of contradictory opposition,[2] acquires the material significance of Disjunction between positive contraries I have tried to show above that these two elements, the bare rejection and the positive contrary, are probably to be regarded as distinguishable from the first, but as tending to coalesce, and not as later and earlier phases respectively of the same movement Or if, in history, earlier and later, then the later, the abstract formal negation or bare rejection, is to the earlier, the actual choice between positive alternatives, as a separable outgrowth which consciously reunites with it in the region of reflective intelligence No anthropological doctrine can affect—though it may elucidate—the above logical analysis of the relation between the negative and its material import as made explicit in the judgment

2 The statement of chances is a case of the Disjunctive Judgment. The statement of Chances

1. The title which I place at the head of this section indicates the limits of the question which I propose to treat in it. The calculus of chances, like all mathematical reasoning, has at its root an inference that can be expressed in ordinary language. In treating of inference it will be incumbent upon us to discuss the differentia which separates calculation from ordinary reasoning, and we shall find the outward and visible sign of this differentia to consist in the enormous abbreviation of reasoning processes by their condensation into the import of recognised symbols It is a further question in virtue of what peculiar nature an inferential process can submit to such an abbreviation, and also to what extent the abbreviation has the effect of substituting something else, e g application of a rule-of- Limits of the problem in present work

[1] See the account of Contraposition, above, chap vii. the process by 'denying the consequent' is essentially the same with this

[2] It is an unlucky confusion that the so-called law of Contradiction only explains Contrary opposition, and that it is the law of Excluded Middle that lays down the principle of Contradictory opposition See Book II, chap vii

thumb,[1] for the reasoning process so 'abbreviated' But the value of any such abbreviation must ultimately rest in a logical sense upon the reasoning which it represents, and this reasoning must be in its nature explicable in language like any other reasoning[2] Thus in the calculus of probabilities, though I am obliged from mathematical incompetence to omit much that might be of interest to an expert even from a logical point of view, yet the principle of the statement of chances is not a matter of technical method, but of fundamental postulates of knowledge And also, no doubt, it illustrates the necessity by which totalities of a certain degree and kind of abstraction become subject to numerical manipulation

Affiliation of abstract disjunction

11 The statement of chances rests upon a species of disjunctive judgment, but not on what we have spoken of as the true or real disjunction—which might also be called, in contrast with that which we are about to discuss, the concrete disjunction The concrete disjunction, in as far as it reaches its ideal, embodies differences that are distinct and individual modifications of the underlying system, and provides in the nature of the common subject a complete account of the conditions which determine it to each of these differences. In using such a disjunction we know precisely how and why the whole or real subject must enter into each of the differences which constitute it And though it may be said that we do not or need not know when or how far each *condition* involved is or can be *real*, yet we must know something of the relation of such conditions to the reality of the system which they affect (because their reality is partly relative to its reality), and at least there is no sort of reason

[1] I have in my mind as an instance the use of tables of logarithms It does not appear formally essential that any one who uses them should understand the reason of the rules he applies, i e (I presume) the nature of Indices

[2] A *practical* reservation must here be made in considering the higher mathematical processes from this point of view, because they may presuppose a number of stages consisting of subordinate processes, and be inexplicable apart from these latter But to explain the whole complication in ordinary language might involve a lengthiness that would make it harder to follow such an explanation than to master the proper mathematical language

for supposing that the reality of these known conditions is to be taken as an equal amount in the case of all the several alternatives The variety of the world and of all reality throws the whole presumption the other way The idea of equally grounded alternatives is a negative idea, and can only exist by a defect of knowledge, or by an abstraction from what we know

The abstract disjunction, on which alone a statement of chances can be based, cannot be a system of alternatives whose conditions and relations are thoroughly understood It is rather allied to what was spoken of above as the disjunction of ignorance The affiliation to the hypothetical judgment is indeed the same in all disjunctions that are formally perfect, i e both exclusive and exhaustive. But on its other side the abstract disjunction does not, like the concrete, descend from the generic judgment with its penetrating and dominating individuality, but rather from a judgment of enumeration such as the collective judgment, with its homogeneous parts which do not occupy individual or distinctive relations to the containing identity It is true that to give meaning to any disjunction, or to a statement of chances founded upon it, the parts or members of the whole must be distinct as well as homogeneous But the distinction is in this case mere distinction, interesting as a result, like the differences between the six sides of a die, but on the side of its relation to the whole not rooted in any known heterogeneous modifications of that whole In other words, the *number* of parts, or the fact that each is *one among so many*, is the primary fact, and their *nature* is secondary. We may illustrate this by contrasting an example of a concrete with one of an abstract disjunction 'The constitution of a modern nation,' it may be said, 'is necessarily either democratic or plutocratic' Here the fact that the species assigned are two in number is of no importance No one would think of trying to infer anything from it as to how many nations were likely to be democratic and how many to be plutocratic The whole weight of the judgment rests upon the component elements implied in 'modern nation', and upon the development of those elements which the judgment indicates to be

necessary. But if we take such a judgment as 'A die must turn up one of its six sides' we here regard the individuality of the several sides as indifferent with respect to the probability of their recurrence, though not with respect to its results. The important matter is the *number* of the alternatives. For either we are unable to estimate the operative causes which determine one alternative rather than another, or we wilfully abstract our attention from them for the sake of falling back on a more general process of estimation We are to suppose then that as the basis of our statement of chances, we have before us such a formal disjunction as the above, closely akin to the judgments which arise in the process of enumeration, but with the addition of those known relations between the enumerated parts which are embodied in the hypothetical judgments with negative antecedent and consequent respectively

Essence of the statement of chances

111 We are then in a position to enter upon a process which I can only describe as taking stock of our knowledge by arithmetical methods We know that the die has six sides and no more, and that as the result of a single throw it must turn up one of them We know that we do not know of any cause operative in the nature of the die or in the conditions of the throw that should favour any particular side, nor of any grounded presumption whatever in favour of any one side in particular. In the case of the die, which herein differs from many cases of the statement of chances, we may say that we *know that there is not* any permanent operative cause either in or outside the die that can favour one event in particular. Therefore—and here is the all-important step which really constitutes the statement of chances—we go from 'no known inequality of the grounds for affirmation'[1] to 'equality, *so far as follows from this knowledge*, of the grounds for affirmation' of the several alternatives Having made this step, i e having placed the grounds for each alternative on a level as equal, we are of course free to treat them as units, and a

[1] Judgment about the future, *if* we judge, is of course affirmation as much as judgment about the past But it is not essential to the statement of chances to refer to the future They can be stated on given premises about any event, although its real issue be known.

ratio expressing the relation of each to all follows as a matter of course Each alternative counts for one, and none for more than one In other words, the ground for the affirmation of each, assuming the reality of the common subject, is represented by a fraction of which 1 is the numerator and the total number of alternatives the denominator. We do not attempt to say what the ground is, but we say that, by the terms of the disjunction, it is one out of so many equal and reciprocally exclusive grounds. This transition from indifference of formulated knowledge to equality of grounded affirmation, and so to the relation of units within a sum total or fractional parts within unity, is the logical foundation on which the statement of chances rests

iv. All its developments are applications of this principle. Applications of the statement as calculus Alternatives and Results

α For instance, it may happen that alternatives which are separate units in respect of the ground for affirming their reality are identical in respect of that result of which the chances are being stated It follows that the chances of this result are represented by as many units out of the sum total as there are equal alternatives which produce it In other words, the chances of the common consequence of a number of alternatives whose chances are known is the sum of the chances of those alternatives If you bet with me that you will throw a six first throw with a single die, then all five alternatives from one to five inclusive have annexed to them the consequence that I win the bet The total number of alternatives being six, I have thus $\frac{5}{6}$ of the chances in my favour, or the chances against you are 5 to 1.

The further processes of the calculus must always reduce themselves to obtaining a correct enumeration of the equal alternatives, and a correct estimate of the number of those equal alternatives which have annexed to them the result whose chances are to be stated

β The combined chances of independent events illustrate both these principles very simply. Physical alternatives

To find the physically different throws that are possible with *two* dice, we must take into account which is which of the two dice It is obviously possible in a single throw for any side of the one die to concur with any particular side of

the other die, i. e. writing down the six sides of the one die as headings, you will have to write down all six sides of the other as possible cases under each of these headings This relation is of course expressed numerically by multiplying the whole number of sides of one die by the whole number of those of the other: $6 \times 6 = 36$. This gives the correct enumeration of the alternatives that are physically possible, the chance of each in case of a single throw being $\frac{1}{36}$. Generalising this process, we may say that in two or more independent sets of alternatives the chance of the concurrence of two or more particular events, as many as there are sets concerned, is determined by the product of the numbers of alternatives forming each separate set multiplied together. That is to say, the chance of any particular concurrence of events, consisting of one out of each independent set, is 1 divided by the product in question

Interesting results.

γ But to return to the example of the two dice, it may be that the 36 possible concurrences would not all count as different, because e. g the throw 2 with the die *a* and 1 with the die *b* may be treated as the same with the throw 1 with *a* and 2 with *b*. Therefore the results which are the alternatives according to this mode of counting have not all of them the same chances in their favour, i e do not severally contain the same number of physically distinct alternatives. Each of the six throws 1 and 1, 2 and 2 &c. up to double sixes inclusive corresponds to one physical alternative only out of the possible 36, and therefore has only the chance represented by $\frac{1}{36}$ On the other hand, each of the fifteen 'throws' 1 and 2, 1 and 3 to 1 and 6 inclusive, 2 and 3, 2 and 4 &c. to 5 and 6 inclusive, is a result annexed to two physical alternatives (1 and 2 or 2 and 1, &c), and therefore counts as two equal cases or units, and has a chance represented by the sum of the chances of these equal cases, viz by $\frac{2}{36}$ instead of $\frac{1}{36}$ These 15 'throws' then, answering to two actual alternatives apiece, exhaust the 30 real cases that remain after deducting the six doublets, and the whole 36 alternatives are thus accounted for. Or again, if we take into account merely the number of points thrown at each throw, without regard to their distribution between the two dice, we get six com-

binations of throws that will give seven points, five for eight points and six points respectively, and so on to one combination for two points and twelve points respectively. The chances of throwing 7 are therefore $\frac{6}{36}$, and of throwing the other numbers $\frac{5}{36}$, $\frac{4}{36}$, and so on down to $\frac{1}{36}$ respectively.

What does statement of chances represent?

v. Logicians are not agreed as to the proper description of that which is expressed by the ratio that embodies a statement of chances Their disagreement arises more from the subtlety of the distinctions involved, which makes description difficult, than from a substantial difference of opinion as to the relation between reality and the cognitive act in question It is not unnatural, for example, to say that the ratio expresses our subjective expectation But this is an obvious slip, because the whole process of the statement is undertaken in order to correct and control our subjective expectation, and is futile unless it does so. The complete counterpart of this idea would consist in maintaining that the ratio expressed an actual behaviour on the part of real things I do not know that this suggestion has ever been made in this extreme form. Something of the same kind however is commonly believed with respect to the realisation of chances in a series, which I shall speak of directly.

The ratio of chance seems really to express the amount of ground, *which is afforded by the knowledge formulated in the disjunctive basis of the calculation,* for affirming the reality of the result whose chances are in question, on the assumption that the general case, which forms the subject, is realised. Instead of 'amount of ground' it would be more usual to say 'degree of probability'. And by avoiding the expression probability we do not really escape the tautology which it would introduce. For the idea of a measurable amount of logical ground, like the idea of a measurable degree of probability, is only intelligible with reference to the statement of chances.

'Expectation' sounds more like a term which might elucidate the definition, and if we say that the statement of chances represents *the expectation which is justified by the premise,* we might not be far wrong But I do not feel sure that anything can be meant by a degree of expectation except

the mood, whatever that may be, which is founded upon a statement of chances So that the chance would not come from measuring the expectation, but the expectation from measuring the chance Even if we identify expectation with judgment—a proceeding which is more than doubtful—we cannot say that the chances necessarily represent the degree of certainty *with* which we judge[a] in a statement of chances, because the statement, giving due numerical weight to every equally-grounded alternative, asserts in the proper ratio the reality of *all* the alternatives, and is in this aspect a judgment with its conditions made fully explicit, and therefore necessary or apodeictic Not that a statement of chances is usually or ever true in fact, but this divergence from fact, or abstract character, affects the categorical aspect of the whole disjunctive basis of the statement, the truth of which basis is in no way affirmed or impeached by the ratio of chance founded upon it The chance that one side of a 6-sided die will turn up is $\frac{1}{6}$, but this says nothing about the certainty that there is a 6-sided die Paradoxically enough, the statement of chances seems to measure, if anything the degree of certainty of a problematic judgment made without knowledge of or in abstraction from the statement, as to the probability of a single alternative out of a number without reference to the remaining alternatives Not that a statement of chances can govern the meaning of a judgment made in ignorance of it, and it is indeed hard to see any meaning in degree of certainty apart from measurement by the enumeration which gives a ratio But on the other hand, the moment we have the ratio, we have with it the *whole* consequence of the assumed reality, and the judgment which asserts the ratio is mere arithmetical necessity, the nature of the assumed reality being given. The judgment that 'the odds against any one side of a (6-sided) die turning up at one throw are

[a] It is rather 'the degree of certainty *which* we judge,' i e affirm It is beside the point to discuss the probability of the truth of the judgment which states the chances It is the degree of certainty of a particular alternative happening, not that of the judgment which states the degree of certainty, that is represented by the chances This is the view of the text in the end, and I let the discussion stand, because it illustrates a possible error

5 to 1' has not a probability as 5 to 1, but on the premises is necessary But it determines what I *ought* to mean, when I say, in ignorance of or not considering the calculation, 'it is unlikely that a six will turn up the first throw' If I reflect and say, 'It is unlikely (with an unlikelihood of 5 to 1) that &c.,' then my judgment has ceased to be problematic, and has become necessary, i e the conditions of its probability are analysed and made explicit Thus probability as a character of judgment disappears when measured.

The ratio of chance then expresses the amount of ground for affirming, that follows from the knowledge formulated as the disjunctive basis of the reckoning. We may possess knowledge that does not conform to the conditions of the statement of chances, or at least that is not relevant to the special disjunction which we are able to employ Such knowledge may cause us to distrust the reckoning slightly or wholly as it affects some particular case. I may think for instance that the calculated risk of being run down by a cab in the streets of London does not apply to one man who is in the prime of life and habitually alert, or to another who is bedridden and never goes into the streets at all But none the less the probability of cab accidents for each individual of the population *on the data that are taken* is a mere question of calculation and can only have one correct answer. My private notion that I have supplementary data which ought to be considered, or that a more careful distinction should be made between classes of persons in the data, does not in the least affect the probability which flows from the premises in any particular calculation.

The calculus of chances, in short, bears the character of the judgments from which it is derived Like the pure hypothetical judgment, and the greater part of the abstract judgments derived from the judgment of enumeration, it affirms of Reality indirectly and conditionally. Its truth is a truth of necessity, a consequence that follows from a selected or fancied character when taken as real Such a consequence is not subjective or arbitrary Given the premises, it can only be drawn in one way, and every other result from those premises is wrong But yet it does not express actual

concrete fact It expresses a truth necessitated by the nature of Reality, but not as it stands embodying a fact of Reality. It is simply an arithmetical consequence of a highly abstract disjunctive enumeration.

Chance and actual series

vi. In every statement of chances we admit our partial ignorance If this were not so, the statement would involve a flat contradiction. For our grounds for affirming reality are equal in the case of all the alternatives, and yet our statement of chances is based on the assumption that only one of them can be realised.

Fallacies relating to series

a. But if we do not bear in mind the proportion of ignorance which enters into our data, we are tempted into two fallacious attitudes

In the first place, we consider ourselves justified in being astonished at the realisation of the alternative which has very few chances in its favour

And in the second place, we palliate the apparent contradiction between equal grounds for reality and unequal realisation by affirming that the statement of chances has genuine truth only in an actual series which realises all the equal alternatives equally To realise all the equal alternatives equally is of course the same thing as to realise all the interesting results[1] in the ratio prescribed by the statement of chances

First, then, we have very small ground for being surprised at the actual occurrence of that alternative which had fewest chances in its favour ; and absolutely none for being surprised at the occurrence of a marked or interesting alternative which has against it enormous odds, but only the same as against every alternative which can possibly occur In the former case we are cherishing a private presumption that the knowledge embodied in our premises represented the actual operative causes which determined the realisation of one or another alternative, and this is *ex hypothesi* not the fact In the latter case we are misled by a special interest into comparing, as if they were cases of which the chances should be equal, cases which are not 'equal alternatives', but 'interesting results' comprising unequal numbers of equal alternatives,

[1] See above, p 337

viz. on the one side a single case which is in some way remarkable, e g. a hand at whist consisting solely of trumps, and on the other side all other possible hands whatever, which we implicitly contrast, as a single case, with the opposite 'interesting result'. We are therefore surprised at the immense adverse odds in spite of which this result has been realised, not reflecting that there are precisely the same adverse odds against any one of the alternatives which occur in everyday experience, though not, of course, against all of them together.

And secondly, the realisation of the ideal alternatives in a series of real cases is confessedly a fiction unless we stop the series at an arbitrary limit—say, for instance, the actual limit of individual objects or events in question in space or time —and even within this limit the series is not what we want.

There are under this head of an actual series two possibilities which *prima facie* at least must be distinguished. We may have to deal with a natural or deductive cycle of alternatives, or with an arbitrary or inductive cycle.

In what may be called a natural cycle the alternatives are ideal cases that follow obviously from the nature of the general subject, and are distinct from the real cases, the actual instances or events, which may or may not continue to present themselves beyond the one real case which is postulated in the statement of chances. The sides of a die or of a coin furnish ideal cases, the throws of a die or tosses of a coin are real cases. The natural cycle is the lowest number of actual events in which the ideal alternatives could be all realised equally, i.e six throws in the case of the die, two tosses in the cases of the coin. Beyond this natural cycle there is nothing to suggest a limit to the series of real cases. We have therefore to ask, in considering the verification of natural cycles by experience, whether the real cases correspond to ideal cases (i) within every natural cycle that is observed, and (ii) in the series as a whole continued without limit.

In what, on the other hand, may be called an arbitrary cycle the 'ideal alternatives' are derived from the enumeration of real cases. The population of Great Britain at a given date would be such a collection of real cases, and that these real cases are identical with the equal ideal alternatives is

shown by the fact that their number forms the denominator in the fraction that states the chances If e g the population were 30,000,000, and 600,000 people died in the year, the chance, assuming constancy of the average,[1] that any one individual taken at random would die within the following year would be $\frac{600000}{30000000}$, viz $\frac{1}{50}$ The analysis of such a statement of chances appears to me not quite simple, and I doubt whether Mr Venn, in maintaining that chance essentially refers to series, has identified its elements rightly. The individual human beings composing the population in question, in whatever order we choose to take them (say, in the accidental order of our enumeration), must correspond, I think, both to the real case [2]—the throw of a die or toss of a coin—and also to the ideal cases—to the sides [3] of the die or of the coin considered as possible alternatives And the two general cases of dying or not dying within the year cannot, I think, correspond to the equal ideal cases or possible alternatives, but must be treated as those combinations of alternatives which arise when several possible 'ways in which the event may happen' have an identical consequence in which we are interested Or, in short, we might put it thus, following in part Mr Venn's interesting discussion —English humanity is the 'event', each individual is 'a way in which the event may happen',[4] dying and not-dying or male and female [5] are general consequences or results each of which emerges from a large number of 'ways in which the event may happen' Thus, as is right in theory, the ratio of chances

[1] Or following what was said above of the independence of calculation and real event, we may neglect the constancy of averages, and say, referring to the same year for which the enumeration is taken, 'the chance that any particular individual will have died in that year' On the same premises, the chance is the same after the event as before it

[2] This is what Mr Venn describes (Logic of Chance, chap i, sect 6, chap iii, sect 33), in the ordinary language of the theory, as the 'event', and in his own language as the collection of attributes

[3] This is what Mr Venn describes either as 'a way in which the event may happen', or as the 'occasional attribute'

[4] Or a particular modification of the collection of attributes by occasional attributes

[5] Mr Venn, Logic of Chance, chap i, sect 6 Male and female are not the 'ways in which the event may happen' This would give the chance of each as $\frac{1}{2}$

is determined by the number of ways in which the event may happen, which in these arbitrary cycles = the number of cases in which the event does happen It is a consequence of the inductive character of the cycle that this number of ways has no obvious and necessary meaning, but is a mere inference from the number of times that the event does happen. It is as if there was a die of unknown structure thrown 30,000,000 times, and exhibiting a white side 29,400,000 times and a black side 600,000 times If then we chose to assume that we had before us a die with 600,000 black sides and 29,400,000 white sides and to estimate all further chances on that basis, we should be in an analogous position to that which we adopt when we calculate chance on the basis of a ratio observed *de facto* in a cycle of cases The chances of white and black respectively are then $\frac{49}{50}$ and $\frac{1}{50}$ on the basis of this real but arbitrary cycle, regarded as the foundation of an ideal cycle of alternatives identical in content with itself

Now the arbitrary cycle itself, the 300,000,000 individuals and 600,000 deaths, being given as real, there is no question of its correspondence with a pre-established ratio If we ask whether the ratio of $\frac{1}{50}$ which it prescribes is realised in experience, we can only mean to enquire whether the distribution of the deaths is regular within the observed cycle, or whether the same ratio prevails outside this real cycle, which latter question may be followed up if we please by the same question as before about the regularity of occurrence I am not quite sure that in speaking of the uniformity which attends large numbers of instances our writers always remember that such uniformity demands the comparison of two or more cycles The ratio of results within a single group, even if embracing all hitherto observed instances, is in the absence of an antecedent rule a mere fact and not a realisation of anything Every ratio is definite, and any two numbers have a ratio, so that it is a truism to speak of a definite ratio as prevailing between classes of cases in a single group.[1]

[1] Logic of Chance, 1 6 The distinctive characteristic of probability is that occasional attributes as distinguished from permanent ones are found on extended examination to exist *in a certain definite proportion of the whole number of cases* The italics are Mr Venn's How could they exist in anything but a definite proportion ?

Within a single large group if we speak of adherence to a ratio, we can only mean regularity of occurrence. And in the case of individuals that are not events in time the significance of such regularity and its existence must depend simply and solely on the order in which we consider and enumerate them.

Thus the two kinds of cycles or groups of instances seem to be quite differently situated with respect to empirical verification 1 A natural cycle from the first corresponds or does not correspond to its antecedent law, within the limits which that law spontaneously prescribes *If* we throw all six sides of a die in every six throws there can be no doubt that up to the point when we break off the series is an empirical verification of the chances as we stated them It shows that the unknown causes operate equally, and thus produce the result, which we anticipated by neglecting them But in treating of actual experience we may practically disregard this kind of correspondence, which is not common in fact and at all events could hardly repeat itself through an extended number of observations

11. We are thus referred to the second question which we mentioned as capable of being proposed respecting a natural cycle, viz whether the law which it presupposes (e g. every side to turn up once in six throws of a die) is realised in the long run It must be observed that to speak of realisation in the whole set of actual cases, whatever their number may be, cannot furnish a standard in this question, because this number is constantly varying and is very different in different subject-matters There is nothing between realisation in every natural cycle and realisation in an infinite series if we keep clear of causal presumptions which do not belong to the reckoning of chances pure and simple There is no doubt that the law presupposed by a natural cycle may be realised in a great and increasing number of observations, and that inferences may with the aid of causal presumptions be drawn from this realisation. But for all that, it is simply nonsense to speak as if the true and only true realisation of a ratio of chance was in the series of real instances continued *ad infinitum.* Mr Venn, who is consistent in regarding this as the

solution of the antithesis between equality of ground and inequality of reality, denies, as I understand him, that the formula $\frac{1}{6}$ [1] has any meaning as applied to a single real throw of a die, except by association with the idea of a series in which *all* sides should equally be exhibited Here we come into the province of fiction. There is no reason, in the cases before us, that such a series should be a fact at all. And in these and in all other cases alike it is impossible that the infinite series could be a fact And yet, if not a fact, it fails to solve the antithesis as a solution of which it is propounded. It is not in fact possible to go on trying for ever, and it is not in theory true that supposing we did go on trying for ever (abstracting from the contradiction involved) every alternative must be realised according to the ratio The ratio may be justly erected according to our grounds of knowledge, even if some of the alternatives are absolutely impossible and therefore could never occur although, *per impossibile*, the series of trials should be prolonged to infinity

In the case of the arbitrary cycle the answer to the question is still less favourable The primary interest of the arbitrary cycle is just in that statement of chances affecting individual real cases which is suggested not to be the true meaning of the ratio. There is in this case no antecedent law between which and the real cycle a correspondence could be observed. The real cycle itself is and prescribes the law And although cycles of real events that fall outside it, or minor cycles within

[1] Mr. Venn seems to have been influenced by considerations such as those of p 340–1, above It is true, as there shown, that the judgment which measures probability loses *ipso facto* that isolated reference to a particular alternative which marks the genuine, or at least the natural problematic judgment In every statement of chance we have an apodeictic judgment involving the entire content of a disjunction in the bearings of its members upon one another (as condition and consequent) So far Mr Venn and I are together The question therefore reduces itself to that of the purport with which the reference to the remaining alternatives is charged, whether it depends on the idea of a completed series, or can be explained by the (assumed) equal claim which each alternative makes on reality in virtue of the (assumed) equality of their grounds The latter view seems to me to be demanded by the nature of the abstraction on which the whole idea of stating chances depends.

it, can be tested and compared with it in respect of the ratios they display—as we compare the ratios of deaths to population in successive years—yet it is not easy to say on what ground the first cycle that we happen to observe should furnish a rule to which subsequent cycles are expected to conform The whole group of observed events or individuals is a comprehensive fact, within which the ratio of the ways in which they happen or of their classes is also simply a fact Any comparison which we may make of the ratio exhibited in minor cycles within this entire group has only the interest of the comparison between different groups of actual occurrences or individuals[1] On the assumption that the operative but unknown causes are not changing progressively, or that we can allow for their progressive change, we may no doubt expect to find that a ratio which we elicited from case 1 to 1000 will hold good for cases 1001 to 2000. But if what we want is merely the serial form, we have it already in cases 1 to 1000; while if what we want is the multiplication of observations, why should the 2000 cases—a wholly arbitrary number—be especially satisfying? The fact is, that when we have the serial form given to begin with, as in these arbitrary cycles, or groups of instances limited only by our ceasing to enumerate them, we see that it omits just that peculiar transition which is the essence of the statement of chance. It only presents us with this transition in so far as, surrendering its serial form, it becomes the basis of a fractional expression which summarises our knowledge, drawn from the series, with reference to some instance or instances whether within or external to itself This criticism applies to the serial form as such The equal realisation of alternatives considered as equal (i e apart from regularity and irregularity, which are equality and inequality as judged by minor cycles) destroys the peculiar relation of equal knowledge to unequal fact, which is the ground of chance

[1] Mr Venn, Logic of Chance, II 8, seems thoroughly to accept this result, and to conceive that *all* probability, even in what I have called natural cycles (in his phrase '*a priori*' probability), is at bottom this and no more I cannot but think that if probability in a specific case means anything, it must, even though dependent on an arbitrary cycle, be stated as above in the form of a natural cycle

On the infinite series, or approximation in the long run, I can say no more than I have said above, and others have said before me The thing is simply a fiction, and the reference of the realisation of a ratio to it proves, if anything, that it is ultimately necessary to admit that chance is independent of a real series

Causal Inference from Series

β. The true bearing of a series on the verification or corroboration of a ratio expressing probability must consist in its relation to the causal presumptions which dominate our judgments about reality All judgments that deal with fact assume, though they may not explicitly assert, causation Statements of chance do not proceed by following causation into its ramifications, we should thus have concrete knowledge and not equality of alternatives. But when the results of experience coincide with the predictions of the calculus, this suggests to us *not* that we knew the right causes or any causes at all, but that the actual causes at work have a character compatible with the results which we obtained through the indifference of ignorance If, on the other hand, the results of experience deviate widely, so far as experience goes, from the ratio suggested by the calculus, then *though this deviation can never amount to a flat contradiction,* yet it suggests an arrangement of causes incompatible with the results which were generated by the indifference of ignorance If we cast six twenty times running with the same die, we have no right to say that this theoretically speaking contradicts the ratio of chance (unless we take as a standard, which no one would ever do, the natural cycle of six throws), for in 120 throws the balance might be restored In other words, no sequence is impossible in such a case, nor is one more improbable than another, but of course any one sequence is immensely improbable beforehand as against the whole remaining mass of possible sequences And in any marked or so to speak identifiable sequence this improbability strikes us as though it had occurred in face of some enormous probability in favour of some *one* other sequence, all the less identifiable sequences counting as if one in number, though $n-1$ in probability. This is a partial account of our surprise, but as has been well explained by Sigwart, there is in it the further

element that whereas the twenty consecutive sixes are on the assumptions from which we started but one among an enormous number of equal possibilities, they happen to be of such a nature that on another assumption, incompatible with those, they would follow with absolute or all but absolute certainty We know that if the die is cogged it will always turn up the same side, or to speak generally, if there is present an operative cause which necessarily produces one alternative, that alternative will always be produced This suggests the comparison between one ratio, that with which we started, which gives a very minute probability for the result found in practice, and another ratio, formed on a different assumption, which gives the observed result with something like certainty Nothing binds us *absolutely* to either, but it is plain that *so far as experience has gone* the probability is with the latter in the proportion in which it gives the result observed with greater probability than does the former It must be carefully remembered that here, as all through this discussion, we are dealing with hypothetical judgments only The probability I speak of is only on the data taken into account If I am playing with a most respectable friend who says he has got the die from a good shop, I may prefer to believe in the reality of a peculiar case rather than in a fraud.

Observed and calculated series *may* coincide.

γ. I do not see that it is inevitable, as has been maintained, that an observed series *must* deviate from the calculated ratio, as it passes through fragments of a fresh cycle Of course its coincidence with the ratio will not be demonstrable while any cases are wanting to finish the cycle; but if we shrink from saying that the observed numbers can coincide with the calculated numbers in a half cycle or less, we must not, I think, say that they deviate, unless coincidence at the next natural cycle is already impossible Five complete cycles of sides and three different sides in thirty-three throws of a die surely form a case which should be distinguished from the same five cycles of sides plus three repetitions of the same side.

The series a reality for some purposes

δ In speaking of the truth of chances based on statistical averages, we may illustrate what has been said by the different positions of an insurance office and an individual customer

of the office. To the individual whose expectation of life is in question the chance of life gives but little information, at all events so long as it is large Whether he gains or loses by insuring his life is for him practically a mere uncertainty He knows what it is reasonable to expect on the general data of reasoning, but he has no sort of ground for being surprised if it does not happen For the office, on the other hand, so long as the averages are constant, the fate of individuals is wholly indifferent except in so far as they are more lightly or more heavily insured If the office could be sure that in each class of customers (ranking them by the amount of their insurances) the average of deaths would be maintained at the same figure, it would make no sort of difference to it who in particular died. Thus it is true that in a real cycle the ratio of chance *may* in a certain sense become a fact What is not true is that in becoming a fact it remains a chance,[1] and that if it fails to be realised in the short run it must be realised in the long run.

Probability of judgment in the absence of knowledge.

VII. Before passing to the subject of modality I will mention an interesting point in the theory of chance, which is cognate to the above discussions on Privation, Affirmation, and Exclusion.[2] What statement of chances expresses the attitude which we ought to adopt towards an affirmation in the absence of all knowledge? The accepted answer appears to be $\frac{1}{2}$, 'for if we make it less than this we incline to believe it rather false than true,' or putting I suppose the same ground into mathematical language, 'If we grant that the probability may have any value between 0 and 1, and that every separate value is equally likely, then n and $1-n$ are equally likely, and the average is always $\frac{1}{2}$' I am not prepared to deny this conclusion, which of course follows from its data, but I think that it may be instructive to discuss these data, which appear to me somewhat superficial. It appears that the symbol $\frac{0}{0}$ has also been proposed, on the ground that 'the *a priori* [formal?] probability derived from absolute ignorance has

[1] Cp. Adam Smith's attack on lotteries You may see, he said, how much the chances are against you, by the fact that if you take all the tickets you are sure to lose

[2] Chap. VII, supra See Jevons' Principles of Science, p. 212.

no effect upon the force of a subsequently admitted [real ?] probability'[1]

It cannot but strike the looker-on that these two suggested values $\frac{0}{0}$ and $\frac{1}{2}$ seem to correspond with the conceptions of non-impossibility and of real possibility respectively, and that to take probability as having the value $\frac{1}{2}$ in the absence of all knowledge[a] is analogous to conjuring a positive favourable presumption out of an absence of counter-presumptions The question is, according to the analysis of chance which has been stated above, whether the two alternatives 'true' and 'false' are *sole* and *equal* alternatives. Interpreting absence of knowledge as Jevons interprets it, to include entire ignorance of the meaning of an enunciation, I do not see that they are *sole* alternatives 'If I ask the reader to assign the odds that a Platythliptic coefficient is positive, he would hardly see his way to doing so unless he regards them as equal'[2] But to a reader who does not know what the words mean—and this I suppose is what Jevons intended—there is no judgment conveyed The alternative 'unmeaning' must then be allowed for in addition to 'true' and 'false' This, it may be said, makes nonsense of the problem The unmeaning is not a judgment, and the problem is only about judgments. Granted, but then I must not include the very large division of the unmeaning in the statement of chance without recognising it as a separate alternative And if I am to exclude it altogether I must either be given a sentence which I am able to recognise as a judgment, or the problem must refer to any judgment as such without considering whether I know beforehand what it is The latter case is not that in question, and could be treated perhaps better through statistics of error in the sciences, than by deduction from the alternatives 'true' and 'false' It includes in the reckoning all judgments known to be true, whereas the present problem says nothing of these, but only of judgments on which being presented to me I am unable to return a verdict based upon positive grounds.

The real question then is this Given a judgment which I can understand, but which I have no positive ground either

[1] Bishop Terrot, quoted in Jevons, l c

[a] Cp Appearance and Reality, ed 2, p 505, note. [2] Jevons, l c.

to affirm or deny, what are the chances in favour of its truth and falsehood respectively? The conditions of this problem cannot of course be actually realised, because to understand the meaning of a judgment theoretically involves some consciousness of *pro's* and *con's*. Yet there are in the world so many almost arbitrary judgments, that the question has some importance Truth and falsehood *are* in this case, the case of intelligible judgments, *sole* alternatives, but I cannot think that they are, under the supposed conditions, *equal* alternatives I cannot think, that is, that every separate value of probability between 0 and 1 is equally likely For the judgment being *a* form or indeed *the* form of knowledge, the hypothesis of ignorance, in this case 'absolute ignorance', on which the statement of chances is erected, has a peculiar relation to the content of such a statement when that content is the judgment. If we knew there was a certain motion below a certain limit of velocity, but had no further clue to the velocity of the motion, it might be true I suppose that every degree of velocity below that limit was equally probable But if we know that there is a judgment made, or proposed to be made, and have no clue to any degree of positive probability in its favour, then for us the zero of probability is the fact, and if we were to make the judgment in question it would in our mouth be false even if in reality true. Thus, on the basis of my individual knowledge, such a judgment *qua* judgment is by the hypothesis *prima facie* false But my knowledge is not all reality, and therefore I dare not say that falsity holds the field as an absolute certainty. The possibility however drawn from the mere difference between my knowledge and all reality, is an unmotived possibility, for there is at least no antecedent likelihood that my knowledge is always wrong And I am not entitled to raise this unmotived possibility into an alternative having equal grounds with the *prima facie* falsity which follows from the hypothesis.

I am not prepared to suggest any way of representing these chances in numbers. Without equally grounded alternatives we cannot state chances, and I do not see where in this case these are to come from If one read 'doctrines' for judgments, so as to restrict the question to matters of some depth

and importance, one might obtain interesting enumerations out of the history of science bearing on such relations as those of false anticipations compared with true discoveries But it would all amount to very little I only desired to point out that the suggested symbols $\frac{0}{0}$ and $\frac{1}{2}$ seemed to lie in the track of the fallacy discussed above[1] To say that objecting to a judgment we do not know to be true is as unreasonable as accepting a judgment we do not know to be true—and to say that truth and falsehood have a chance of $\frac{1}{2}$ each is to say this—appears to me to be a sophism in the vein of Sir Anthony Absolute[2] If you have no reason for accepting a judgment, you must decline to accept it If you only decline provisionally, and say that in future, or to the knowledge of wiser minds, the judgment in question may perhaps be proved true, then you unquestionably are cherishing some distinct though general presumption in favour of the judgment, and it is not one of those whose chances 'in the absence of knowledge' we are discussing We do not treat really arbitrary suggestions with so much respect It seems to me monstrous to say that half the equal grounds are for truth and half for falsehood when the fact is that you *have* no ground to think the thing true, and *require* none to think it false

This brings me to one further distinction The reader ought to reply, 'You do need a positive ground of denial in order to deny, and in stating the chances as $\frac{0}{0}$ you are denying, which *ex hypothesi* you have no right to do' But I suppose that by the symbol $\frac{0}{0}$ we do not so much deny the judgment as refuse to state the chances It is only the logical interpretation of this refusal that brings something like a denial into the matter You cannot obtain a denial out of a pure privation, i e a mere profession of ignorance, but then in view of the positive mass and far-reaching presumptions of knowledge as

[1] See chap vii, p 316

[2] 'Absolute "Sure, Sir, this is not very reasonable, to summon my affection for a lady I know nothing of"

'Sir Anth "I am sure, Sir, 'tis more unreasonable in you to object to a lady you know nothing of"'

Sir Anthony wishes to represent the chances of attachment and non-attachment to any unknown lady as $\frac{1}{2}$ each, or even as more than $\frac{1}{2}$ in favour of attachment This is really not a bad parallel to the view criticised in the text

a whole, no privation however *complete*[1] can be quite *pure*,[1] i.e quite free from positive grounds of denial. A complete and persistent privation must always, as I have tried to show above, verge upon an exclusion But if we had at command a direct and positive ground of denial, then I imagine that we should not restrict ourselves to refusing to state the chances, i e to $\frac{0}{0}$, but should employ the symbol o Or, in order to indicate that a case has no chances in its favour because all the possible chances are absorbed by another case which is certain and which excludes the former, we ought I should think to use the expression $\frac{0}{1}$, i e negation grounded in positive certainty, which is the remainder of $\frac{1-1}{1}$, and so represents the total certainty available for the two cases as entirely absorbed by the one. Thus we should have three symbols representing ideas which we ought not to confuse, $\frac{0}{1}$ for demonstrable impossibility, $\frac{0}{0}$ for absolute ignorance (privation alike of real possibility and of impossibility), $\frac{1}{2}$ for a conflict of proofs, such that truth and falsehood are equally grounded alternatives, which in presence of absolute ignorance is not the case

APPENDIX TO CHAPTER VIII

ON SOME RECENT DISCUSSIONS OF DISJUNCTION

Are disjunct members exclusive?

Mr G R T Ross (Mind, N S , 48) and Mr Keynes (pp 277 ff) have recently supported the view that members under a disjunction ought not to be considered as reciprocally exclusive. I will comment on some of the arguments that have been used, and will try to explain my own position further.

Is there really anything to argue about? If the whole question is one of usage, and that is arbitrary, it would concern grammar rather than Logic But I think that three points of interest for Logic are involved *α* the sort of guidance which a logical instinct can follow in common speech , *β* an

[1] By a ' complete ' privation I mean one in which we know absolutely nothing *in favour* of the matter of which we deny all knowledge, while I call a privation ' pure ' in as far as we know nothing positive *against* the matter of which we deny all knowledge And my suggestion is that, looking at knowledge as it really exists, wherever we have the former case it is almost impossible to have the latter

application of the theory of incompatibility, contradiction, and negation, γ a substantial though not formal derivation of the exclusive from the exhaustive character

Choice involves incompatibles

α One must not expect to find a definite theory governing usage If a moral is to be drawn from it at all, one must show some simple character in the matter in question which could serve as a guide through feeling and habit And I think Mr Ross has helped me to put this in form, though his argument is directed against my view His phrase, 'the nature of the predicates,' is suggestive I believe that usage favours 'Either—or' when predicates are coming which are felt to be incompatible They are the particles consecrated to choice, and choice means incompatibles, i e that you cannot have it both ways Choice is always a special attitude, whether in action or in judgment When we feel it coming we adjust ourselves to the situation by a special form of speech No doubt the area of choice—the space within which a right choice must fall—is of interest to us This is the exhaustiveness of the 'Either—or' = 'within these cases the right choice must fall' But the primary character, that which determines the very existence of a need for choice at all, is the presence of incompatible alternatives—the exclusiveness of the disjunct members.

'Alternatives,' whatever else they may be are surely incompatible No one was ever heard to say in presence of a pair or set of alternatives, 'I choose them both or all,' unless he means that it was an error to propose them as alternatives And it seems to me plainly erroneous to speak of an alternative proposition while suggesting that the alternants may both be true (Keynes, 277, 279) If you may take both, you are surely not in presence of an alternative

A conjunction cannot include two operative alternatives

β Now incompatibility has degrees And I wish next to point out that there is a certain marked degree of it which is present in the cases under any disjunction whatever, even on the hypothesis that exhaustiveness is the primary assertion which is intended Take, for example, Mr Ross's comment (loc cit) on the argument (Bradley's Logic, p 124, above, p 324), that if the alternatives are not exclusive the disjunction cannot be exhaustive the case of their being conjoined being

a *casus omissus* 'We might ask how it is that *bc* can be exclusive of *b* and *c* when as a matter of fact it includes them.' For the possibility that *bc* may really go under *b* or *c* (either *b* or *c* being inoperative in it) I refer to my argument on p 324 with the example of the railway tickets. In instances of that type it is needless to mention *bc*, but this is because it already falls under *b* or *c*, and therefore the cases are already both exclusive and exhaustive (This disposes of two of Mr. Keynes's cases on p 279—the candidate for an appointment, and the peer The membership of more than one University, in the first case, and the possession of more than one title, in the second, are not special cases of eligibility and of peerage In either instance only one can operate, and the possession of more than one constitutes no separate case under the Disjunction)

But if *bc* is to be a distinct case under the disjunction, then we have an interesting point *bc* then cannot include the cases *b* and *c*, and for this reason Each of these cases, *qua* case of the disjunction, is proposed as capable of meeting the requirements of the situation by itself and therefore completely. Take it as a cause (Keynes, p 278), (it might also be an explanation or a consequent, this makes no difference). 'He has either used bad textbooks, or has been badly taught.' Each of these suggestions is propounded as stating an adequate cause of the existing situation The mention of it as a separate case makes this certain, even if we do not deny the possibility of the fact contained in it being combined with another fact, such as bad education In the case put (the case *b*), the boy's ignorance, or whatever is the matter, is accounted for by bad textbooks Now the case *bc* cannot include this case *b*, though it may include part of the fact suggested in it. In *b*, bad textbooks are *the cause* In *bc* bad textbooks are not the cause, but a co-operant cause. You cannot take two causes out of a 'plurality of causes', and treat them as co-operant, without completely changing the operation of each It is a flat contradiction to predicate conjointly, without special explanation, two predicates which are capable of being predicated alone When my builder gives me three explanations of the same overcharge, it is not possible for me to

accept them all, and this follows simply from the fact that each claims to be sufficient taken by itself The facts alleged in them may in some way be combined, but the three sufficient explanations cannot possibly be true together, though the builder is inclined, like my critics, to say so *b* and *c* are each exclusive of *b c* So with the well-worn case of rogue and fool. If *X* is both rogue and fool, that excludes the cases of simply rogue and simply fool The man is different in every fibre of his being So if 'the witness is perjured or the prisoner is guilty' (Keynes, 279) you mean to say that you see two possible explanations of the witness's evidence, constituting incompatible cases You can choose that which seems most probable, but they are rival explanations, and you cannot, having regard to the facts before you, accept them both The witness may be perjured and the prisoner nevertheless be guilty, but that must be on further grounds, not present in the facts so far to be explained

Incompatibility of cases, then, is the primary meaning of the words 'Either—or' I will analyse Mr Ross's curious example (Mind, loc cit), pp 492–3 The point of it is that if disjunctions are both exclusive and exhaustive, '*A* is either not-*b* or not-*c*,' and '*A* is either *b* or *c*', mean exactly the same thing, e g if you say 'a man who says he has seen a ghost is either not sane or not truthful' you can go from that to 'a man, etc is either sane or truthful'

I note first, that this result is sound in the main It looks absurd, because contrast calls attention to the positive aspect of the terms Pronounced *avec intention*, it would be an example of the phrases in which mere precision constitutes effective irony, 'He is *either* sane, *or* truthful,' i e certainly not both I think that the effect of precision here really proves the exclusiveness of the disjunction to be essential

And secondly, the instance with Mr Ross's comment itself shows clearly that what we primarily mean is the incompatibility of the *positive* terms In a word, we mean to say 'He cannot be both sane and truthful'. But there is a curious point in the argument which might obscure this fact, and I will take it to pieces

Starting from the *negative* terms, not-sane and not-truthful,

what we want to assert is primarily exhaustiveness. But the reason of this is that exhaustiveness as between them is the same as exclusiveness between the two positives

X is either not sane or not truthful, implies (1) If *X* is not (not-sane) he is (not-truthful)

The type is, If *A* is not *b*, it is *c*, the hypothetical which ensures *exhaustiveness* This is clearly the right analogy, expressing the meaning of the original statement, and keeping the negative terms as such, and I think, from his arrangement of the propositions on p 492, Mr Ross recognises this

But, being 'simplified' by cancelling the double negative (whether it is a true double negative is questionable), this proposition results in

'If *X* is sane, he is not truthful,' a proposition of the type, If *A* is *b* it is not *c*, which is the hypothetical that secures not exhaustiveness but exclusiveness And this is undoubtedly what the speaker meant primarily to assert, viz exclusiveness (incompatibility) between the positive terms, though this took the form of exhaustiveness of the negative terms. Again,

(2) If *X* is (not-sane) he is not (not-truthful) This is of the type, If *A* is *b* it is not *c*, the hypothetical which secures *exclusiveness*

This, 'simplified' as above, gives

If *X* is not sane, he is truthful, of the type, If *a* is not *b*, it is *c*, the hypothetical which secures not exclusiveness but exhaustiveness

This judgment, perhaps, as Mr Ross says, we do not want, and the disjunction 'Either sane or truthful' is excessive by that factor And it arises, as he implies, from assuming exclusiveness But the point is that the exclusiveness assumed is between the negative terms, and we have seen that this is the same thing as exhaustiveness of the positive terms But it is this exhaustiveness which is the superfluous element What we asserted from the beginning and meant to assert all along, was incompatibility, exclusiveness of the positive terms, and it was only because we saw it meant this that we asserted exhaustiveness of the negative terms What we meant to say was simply 'He cannot be both sane and truthful', and the 'Either—or' saddled us with an unnecessary element, not in

the assertion of exclusiveness of the positives, which was the whole point of our saying, but in the assertion of their exhaustiveness which is more than we needed to say. Taking the positive terms, then, which are what we are really concerned with, the argument proves my point, that a disjunctive proposition is primarily concerned to affirm the incompatibility of the cases set out under it

Now to predicate without distinction or reconciliation (e g without explaining how and in what degree they co-operate) separate predicates of the same subject is the definition of contradiction. And therefore it is plain that the cases set out under a disjunction, and *prima facie* meant to be separate and sufficient for the situation, are not predicable conjointly without special explanation and defence of their relations in the conjoint case, which, if the several factors of it are really operative, cannot possibly include the other full cases as they are when separately predicated. The appearance to the contrary has been shown to mean that the alleged conjoint case satisfies the disjunction in respect of one of its elements and no more

The whole significance of the theory of negation lies in showing how these independent cases, incompatible when you offer to predicate them conjointly become consistent when arranged as the import of a disjunctive system (see p 289 above) Every significant negative takes its meaning from such an arrangement implied or expressed

Exclusiveness follows from independence Other criticisms

γ. I am convinced, then, in the first place that 'Either—or' implies choice, that choice implies incompatible alternatives, and that incompatibility or mutual exclusiveness of the cases (otherwise than as the content of an ordered system) is the primary meaning of disjunction But in the second place, supposing it is maintained that the primary meaning is 'one or other at least and, it may be, both', I submit that the above-mentioned considerations suffice to establish exclusiveness as a logical consequence of the disjunctive attitude, even if it is a consequence not always foreseen (as in the case of my builder). If you assert 'either this or that', even intending to concede the possibility of both together, you clearly mean that this or that by themselves are at least possible indepen-

dent cases You mean that each is a possible cause or explanation or course independently of any other

But if so, the nature of incompatibility or contradiction determines the sheer impossibility of predicating two such cases together The joint predication can never include two cases of the disjunction which have been offered as independent. We have seen the two modes in which the appearance of such a joint case may arise, and how in each of them we really have no joint case of *b* and *c* at all

I pass to other criticisms of my view of Disjunction.

Mr Keynes (p 283) criticises a statement in Essentials, p 124, that the proposition, 'The signal shows either red or green,' asserts it to have 'some colour', which is not asserted by either of the hypotheticals entering into it Mr. Keynes objects that 'if not red, then green', plainly makes this assertion The passage in Essentials is obscure from its brevity. I refer for my meaning to p 328 above and the following pages The disjunction unites the two characteristic hypotheticals and the system within which they are true and significant in a single survey and affirmation You cannot represent an assertion like this by two separate judgments, each of them partial and hypothetical (cf Essentials, 116, and Bradley, Logic, 121–2)

On my general view of the logical rank of Disjunction, to which Mr Keynes (p 283) objects that it gives an unreal importance to classification, see besides 328–9, Vol II, p. 194 ff These passages contain, besides an estimate of Disjunction, a special reference to the subordinate importance of classification It is not classification, but the system of reconciled incompatibles in which every universal when made explicit is seen to consist, that I hold to be best exhibited in Disjunction. It depends on the principle that in every system the nature of the whole, differently conditioned, is in every disjunct member. This seems to me a fundamental character determining the explicit formulation of every universal, and therefore necessarily prominent in logical theory.

Mr Ross's argument that from a simple constructive dilemma you can only get a conclusion of an exhaustive, and not one of an exclusive nature (p 495), seems to me to tell us nothing

about the disjunction The reason of the failure is plain. According to the ordinary convention you can get no inference from *A* is *d* (under the proposition if *A* is *b* it is *d*), nor from *A* is not *c* (under the proposition if *A* is *c* it is *f*) You have barred the disjunction from transmitting its characters If you recognised reciprocal hypotheticals you could go from *A* is *d* to *A* is *b*, from *A* is *b* (under the disjunction *A* is either *b* or *c*) to *A* is not *c*, and from *A* is not *c* to *A* is not *f*.

The instance of the man at the top of the burning house (at p 495) seems to me to be spoilt by considering whether the man's view is true or not All we are concerned with, surely, is what his view is, as expressed in the proposition ascribed to him, 'If I jump from the window I shall break my leg, and if I do not I shall be burned to death' Whether he is right or not is a thing no one can possibly tell But that the whole motive for formulating such a proposition is the intention to make a choice between alternatives taken as incompatible, is surely undeniable

I think, as I said above, that Mr Ross is right in saying that it is the nature of the predicates which makes them incompatible But it seems to me that this is the guide of our instinctive usage of 'either—or', that when we feel a choice between incompatibles before us we confront it with that form of thought, although we may not always realize that it debars us from conjoint predication, in the strict sense, of the cases which are offered to our choice as severally independent and sufficient solutions of the situation. I use 'the situation' as the most general term for the fact or idea with which we are confronted, and which forms the disjunctive heading under which the alternatives of our affirmation or action have to fall.

CHAPTER IX

MODALITY

1 I PROPOSE to conclude the discussion of the judgment with a short treatment of Modality For Modality, if it exists at all, is simply the degree in which individual judgments participate in the certainty of that permanent and all-embracing judgment by which the individual intelligence sustains those qualifications of the Real which for it constitute Reality Our account of Modality must therefore resolve itself into a recapitulation of the principal types of judgment, having for its object to bring together in a single view certain of their characteristics which have already been noticed. The question before us is whether and in what sense there are degrees of logical certainty , not merely of habitual conviction, or of readiness to act on a belief, which are psychological and not logical,but of that characteristic which forms the differentia of judgment, and which may be described as *logical* assertiveness This logical assertiveness itself indeed includes a psychical or psychological element which must be carefully distinguished from the purely logical or rational element of assertiveness

Kant's view fundamentally just.

One preliminary difficulty meets us on two sides We find Kant[1] maintaining that modality affects only the copula in judgment, and that therefore, though a measure of assertiveness, it is indifferent to the content affirmed And we find it maintained against Kant that modality has no reference to the copula in judgment, nor, consequently, to the assertiveness of assertion, but is a peculiarity of the content affirmed which does not affect the essential act of affirmation.[2] Both of these

[1] Kant, Kritik der r V , p 97 (Hartenstein), ' Von der log Function im Urtheilen '

[2] Sigwart and Bradley certainly agree in this Lotze seems to say it of the old Modality, in which ' "It must be so" counted as apodeictic ', but does not distinctly say it of Modality as understood by himself (and subsequently by the other moderns above mentioned), which requires

views, it will be remarked, by separating the assertiveness of assertion from the content asserted, represent judgment as an arbitrary and irrational activity It is not surprising that in the 'Grammar of Assent' ecclesiastical interest should have thrown itself zealously on the side of such a conception.

The view which I have attempted to explain in the present discussion is incompatible with both of the above-mentioned ideas Every judgment, as we have seen, unites in it two elements of certainty, the formal or psychological element, which consists in a reference of its content in virtue of a perceived identity to the individual's personal world of perception and experience, and the material or logical element, which consists in the attachment of this content by rational necessity to the organised nature which the Real possesses as already qualified by the individual's knowledge The former element corresponds to memory and the latter to reasoning All reasoning is in the medium of memory, but memory as such does not involve reasoning The two activities however are one in origin and in *ultimate* nature

Now the psychological element of certainty does not vary The unity of a content with the individual perceptive self admits of no degrees If it appears to vary in degree, we are remembering *by* something, i e we are eking out memory by reasoning Memory as such is dumb in presence of questions or comparison of grounds It tells us nothing beyond the mere content which it recalls But material or logical certainty depends on reasoning, and is therefore capable of more and less, and is the chief element of the assertiveness of judgments. It follows that modality *is*, as Kant said it was *not*, a characteristic of the content affirmed, but is also, for that very reason, as the moderns say it is *not*, a measure of the assertiveness of assertion. Whether modality must be said to affect the 'copula' at all, or the copula only, depends on what we mean by the copula If the copula is formal and empty, an unexplained act of reference like that of memory,

hypothetical or disjunctive form for apodeictic judgment In his suggestion as to Modality Lotze was of course anticipated by Hegel. Sigwart and Bradley deny the superior logical certainty of the apodeictic judgment, and Sigwart even disparages it on the ground of its mediate character Here, at any rate, he commits a gross blunder.

then modality does not affect it, or rather it does not affect modality If, on the contrary, the copula is the act by which grounded necessity is recognised, then it is the essence of modality The formal and indifferent copula of traditional logic *is* the psychological copula, and when treated as the logical copula becomes a logical copula indifferent to the logical content,[1] which is absurd

As a matter of organic principle, therefore, I shall follow Kant, although by what I must regard as a confusion, he refers modality to the form of judgment and not to the content, and although his 'problematic judgment' is rightly pronounced by later writers to be no judgment at all His summary runs thus 'Because, then, in this aspect [i e in modality] everything incorporates itself with the intelligence by degrees, so that one begins by judging problematically, and subsequently takes the matter to be true assertorically, and ultimately affirms it as inseparably united with the intelligence, i e. as necessary and apodeictic, it follows that we may confer on the three functions of modality the further appellation of so many moments of thought as such'[2] Here as throughout philosophy it has been the task of later writers to realise in the concrete a conception enunciated by Kant, but by him only half liberated from the formulae of obsolete conventions. Hegel's analysis of the hypothetical and disjunctive judgment, adopted by Lotze and subsequent writers, is the realization of Kant's idea of modality as progressive incorporation with the understanding

2 In order to begin at the beginning we must start from Kant's 'problematic' judgment, which, *as he describes it*, is not a judgment at all The problematic judgment according to him expresses mere logical possibility,[3] not 'objective' **The Problematic Judgment**

[1] Lotze does not fall into this trap But he seems only to avoid it at the cost of separating off a particular fragment of content to be identified with the copula, at least when he is explaining the traditional view But as against this the traditional view was right. The copula must have to do with all of the content or with none. In Lotze's own view of Apodeictic Judgment it has to do with all

[2] Nearly equivalent to what I have often spoken of as non-impossibility See below

[3] Kritik der r Vernunft, p 97 (Hartenstein's ed)

possibility, and he gives as examples of it the antecedent and consequent in hypothetical judgments, or the isolated members of a disjunction

Nature of Suggestion or Question

1 But such elements of thought are of course by themselves not judgments, i e as Kant himself says, they are not assertory. It may be doubted however whether such elements of thought exist by themselves at all, and whether they do not of necessity enter into some judgments Their nature is no doubt the same as that of the genuine [1] Question, which Sigwart has the merit of having discussed I cannot think however that his psychological expressions help us to grasp the logical differentia of question or doubt It is nothing to say that the idea merely floats [2] before the mind It must, in order to rank as a definite doubt or question, make some specific claim, be a candidate for some definite place, in the permanent judgment of consciousness There is a nearer approach to something intelligible in the suggestion that a problematic judgment is a judgment about oneself, saying that in the matter before us we are unable to judge But this, though often true in fact, is an evasion of the theoretical difficulty There must be some definite logical situation in virtue of which we say this When we judge that a judgment is merely possible we must judge that its content has mere possibility The reduction of judgment of possibility to possibility of judgment is an attempt to take refuge in psychology from a logical difficulty

[1] The question which expresses a real doubt or ignorance, not one to which we know the answer but ask in order to force the interlocutor to give it This latter is an imperative, as Sigwart says (vol i, 231) But then the former hardly perhaps has the differentia of the Question, which is just this imperative significance So the genuine Question is perhaps not a Question at all but only a state of knowledge *What* state of knowledge? Obviously one that presses for an advance, so probably a disjunction of ignorance Kant's problematic judgment includes all supposition *qua* supposition although known to be false or known to be true But I do not think that Sigwart's complaint of a want of distinction between these cases and that of genuine doubt is well founded, because a supposition *qua* supposition is considered by an act of abstraction apart from the relations which constitute its falsity or certainty Therefore as subject to this abstraction, any supposition may rank as a doubtful judgment

[2] In der Schwebe Sigwart Logik i [illegible], E Tr i 177.

Although then the mere idea of a judgment, floating before the mind, is not, even if it exists, a judgment at all and therefore not a problematic judgment, yet every genuine question and every judgment made subject to a 'possibly' or 'perhaps' represents a peculiar logical situation, and not merely a psychological incompleteness of the act of judgment. It is not true as a rule that we begin with floating ideas and advance from them to judgments I doubt if such a beginning is possible,[a] it is certainly not normal Our thought consists in the continuous modification of judgments—I had almost said, of a single judgment. The question, considered as a state of knowledge, is a disjunctive or hypothetical judgment used as the premise of an inference A groundless question is as unreal as the 'infinite' judgment I cannot ask 'Are you going home?' except on the basis 'You are possibly or probably going home', which means when analysed either 'If x is the case you are going home', or else 'Either you are going home, or x is the case' The question does not mean 'I have a floating idea in my head of you as going home,[1] and want to know if I am to refer this to reality'. It means rather, 'I judge true of reality a definite situation in which some conditions of your going home are included, I want to qualify reality by this situation more precisely defined' or to qualify a further element of Reality, viz yourself, by it Thus Kant is not so far wrong in identifying a problematic judgment, or, as we may call it, a genuine question, with an isolated member of a complex enunciation, but it is only as a corollary from the complex thought or as an inability to make its outline precise, and not as a mere isolated member of it, that the problematic expression can be a judgment at all For in its isolation, *ex hypothesi*, it is not referred to reality.

Problematic and Apodeictic Judgment.

11 Possibility, then, as Mr Bradley has told us, is a species of necessity, and it seems to follow from this that the problematic judgment is a form of the apodeictic judgment, and that any series of gradations in which the two have separate

[a] Ultimately, there are no floating ideas See above, pp 4, 70, and Bradley, Mind, 60, 445 ff.

[1] The idea is not '*of* you as going home' but '*that* you may be going home', i e something *is judged*, which may result in your going

places—such gradations as Kant laid down—must be in contradiction with the nature of the case If the judgment of possibility is the first form under which matters of knowledge attach themselves to the understanding, then it can hardly be a species of the final form, and ought to be verifiable in early thought

We must begin by admitting the difficulty indicated by the last objection. All judgment, I have said, is in one respect assertory It is probable that very early thought may present no other aspect The distinction between memory and intelligence is a late distinction The old man in Homer [1] who 'knows ten thousand things' cannot but remind us of the schoolboy whose friend 'knows an awful lot'. Whatever is in the mind, such expressions seem to suggest, ranks alike as knowledge The *asseveration*, indeed, may be supposed to begin as soon as man feels the danger of deceit, and this form of speech recognises a distinction in degrees of certainty, and attempts to raise one matter of knowledge to the standard of another But on the whole, the distinction between memory and intelligence, and therefore that between mere reference to perceived reality and systematically grounded insight, would probably be found a vanishing distinction if we could examine the earliest phases of the human mind Possibility, mere assertion, and necessity, as they exist for the civilised mind, are based upon differences that concern the material logical or systematic element in judgment. We have to remember however that logical facts exist long before the technical names for them, and we must not limit the existence of modalities by that of words like possibility and necessity, but only by that of speech-elements bearing the power of 'may', 'must', 'shall', or 'would that'. And we must add that we cannot see how judgment should exist apart from all sense of rational necessity A mere instinctive identification with reality, wholly without why or wherefore, is rather a theoretical limit below which judgment cannot be taken to exist, than a historical phase of the judging faculty

[1] Homer is of course not primitive, but poetry is very conservative, and Homer is full of ideas which are derived from very early strata in the mind's formation

The next difficulty is that possibility appears on the view taken above to be a species of necessity, and yet to be prior to necessity In order to explain this we must refer back to the doctrine of opposition and conversion Strictly and properly, a judgment can only be denied by another judgment of the same nature, a singular by a singular judgment, a generic by a generic, or a hypothetical by a hypothetical. But a very strong implication of denial can be conveyed by a judgment which being of a different type from that contradicted denies the right of this latter judgment to the type which it has assumed If, however, the denial is to be prosecuted in earnest, the judgment of the lower type must be capable of maintaining itself on the level of that which it assails Possibility, if not derivative or calculated, represents such a first effort at denial, directed against a necessary judgment, and may or may not go on to assume an explicitly necessary form

Particular Judgment Exception and Instance.

a Thus possibility as prior to necessity follows the meanings and development of the Modal Particular, which, owing to the continuity in the evolution of thought, on which we have so often insisted, has its roots far back in the quasi-singular or particular enumerative judgment That is to say, the consciousness of possibility begins—I do not mean in every case, but in its most rudimentary logical form—with a rule and an exception or with a positive instance suggesting a rule. And it passes into a further type with the essentially negative, or again with the essentially positive *modal* particular The latter, indeed, the positive modal particular, is hardly intelligible apart from the explicit recognition of necessity. But in accordance with a principle to which we shall have to recur it is quite customary for thought to employ a derivative and secondary judgment which is dependent upon a primary principle remembered as a rule Many judgments of possibility —those current in systematic thought—are after this fashion corollaries from judgments of necessity, or rather from the fact that certain judgments of necessity are accepted as true. I now proceed to illustrate the development just described.

Possibility is at first negative Bare negation indeed is nothing, but possibility in its simplest case comes very near bare negation Such a possibility when veiled under the

equivocal form of the particular judgment rests on an *exception*. We must not suppose that the possibility is positive because the exception is a positive case. The content of a mere exception as related to the rule which it impeaches is purely negative, i.e. only contradicts, and suggests no contrary principle. Suppose we have a half-collective and half-generic judgment like 'All English railways are well managed'. Suppose that then we come upon an English railway which is not well managed, and embody our observation in the judgment 'There is an English railway which is not well managed'. This may readily be interpreted as a judgment of negative possibility, amounting to no more than this, 'It is not true that English railways are in every case well managed,' or in other words, 'It is not impossible for an English railway to be other than well managed'. The principle is just the same if the rule is negative, e.g. 'No crows are white'. An exception would contradict this, but only by establishing little more than mere possibility (not-impossibility) that a crow may be white: 'There is a case in which a crow is white,' 'It is not true that no crows are white'.

The *instance* makes the step from negative to positive possibility. Like the exception, it implies a generalisation, at least incipient, but it supports this generalisation instead of contradicting it. Supposing, what is always ultimately the case, that the *exception* is a latent *instance*, the judgment which expresses the exception will change, in coming to express an instance, from a mere contradictory to a contrary which is also contradictory. Let the original rule be 'No secular education can be spiritual' and the exception be 'In the case of literary education we have a secular education which is spiritual', with the negative result 'It is not true that no secular education can be spiritual'. Then when we go on to treat the exception as an instance[1] we get a result

[1] Exception presupposes rule, and rule presupposes positive instance, it may be said, so that our idea of negative possibility coming first gets us into a circle. The fact alleged is true, a positive rule must come from somewhere, and probably from a positive instance. But prior to the idea of denying the rule the generalisation is direct or naive, and does not pass through the stages of modality except on the embryo scale on which it also implies negation.

hardly differing from the former in words, but charged with the material distinction that we now see reason to think that 'secular education as such may also be spiritual', i.e. It is possible that—or 'There is real ground for supposing that secular education, &c., &c.'

Negative and Positive Possibility.

β When we advance to the modal particular we have the same distinction in a purer form The modal particular has been treated above as the converse by limitation[1] of a hypothetical judgment, and at all events may always be treated as the contradictory of another hypothetical judgment. 'If A is B it is C' becomes when converted by limitation 'If A is C it may be B', which latter is *at least* the contradictory of 'If A is C it is not B' Where the modal particular really originates by the conversion of a hypothetical judgment, it is of course an inference or corollary from a principle with reference to another and opposite principle. But as usual the actual or historical modes of initiation of the judgment are one thing, and its logical essence another Obviously the modal particular may be generated either by inference from explicit principles, or by the suggestion of rules through instances

In any case the judgments 'A may possibly be B' and 'A may possibly not be B' have two degrees of meaning analogous to those of the exception and the instance They may be *mere* contradictories of the hypothetical judgment to which they are respectively opposed, or they may be contradictories growing into contraries If they are *mere* contradictories, corresponding to *exceptions*, then the judgment 'A may be B' merely means to overthrow the principle that 'A cannot be B', that is to say it asserts that if or though A is, yet it does not follow, from that, that B is not It is easy to give the corresponding significance to 'A may not be B'. But precisely the same judgments 'A may be B' and 'A may not be B' are capable of corresponding to the *instance*, and then meaning then is that there is some positive connection between an unspecified condition *x*, which is fairly conceivable of A, and B or the negation of B as may happen Under these circumstances, even though we do not know that A*x* itself is

[1] If the hypothetical judgment has a negative consequent, it is of course needless to limit it in converting.

actually found in experience, yet we have ground for saying that there is a rational connection or antagonism between A and B. Many degrees of these connections are to be met with. If x were the entire ground of B, and we knew x to be true of A, then we should no longer have possibility but certainty But if x is part of the ground of B, and we know x to be true of A, then we have a degree of real possibility varying with the relation of x to B Or again, if x is not the ground but a consequent of B, and we know categorically that A is x, we have in effect an inference, from the hypothetical 'If A is B it is x' converted by limitation into 'If A is x it may be B' I will give a concrete example

Plants (A) may possibly possess sentience (B) Taken as the mere guess of an unscientific mind—prepared to say the same no less of stones and metals, air or water—such a judgment would represent a mere negative possibility, or in other words, it would express no more than the fact that having considered the universal judgment 'Plants are not sentient' the individual mind happens to see in it no sequence of reason and consequent, and therefore pronounces that 'There is no ground for asserting that plants are not sentient'

But the same judgment 'Plants (A) may possibly have sentience (B)' is capable of conveying a more positive meaning. It may rest on the conviction that 'If plants (A) have irritability (x) they must have sentience (B)' Assuming ignorance as to actual observations of irritability in plants we nevertheless have then a certain congruity [1] to go upon in saying that A may, in a sense that has a certain basis of reality, be B But still more strongly, if we could say 'If plants (A) have sentience (B) they must have irritability (x)', *being able to supplement this with the fact* 'There are plants (A) which have irritability (x)', we should be able to affirm There are positive grounds for maintaining that plants A have sentience B'

Possibility may therefore mean (1) the inability to make

[1] We should not e g attach any such weight to the judgment that if a plant had poetic genius it must have sentience I claim throughout that congruity is essential and not accidental in supposition If we go outside the system of fact which is our basis of supposal, we get results analogous to the 'infinite judgment'

a certain hypothetical judgment, so long as we bear in mind that *mere* privation or inability is a limit which the judgment must not *actually* reach if it is to retain significance This possibility is non-impossibility Or it may mean (ii) the inference from a hypothetical judgment, whether explicit or suggested in instances, which (*a*) assigns an intelligible condition (or makes us believe that there is such a condition), which would if *real* establish the consequent whose possibility, assuming the reality of the subject of judgment, is in question, or (*b*) assigns a certain logical consequent to the attribute or event in question considered as a logical antecedent, which consequent is known to be real This logical consequent may be either an effect with alternative causes, or a consequent with alternative reasons The ideal or reciprocal form of the hypothetical judgment excludes mere possibility and therefore does not concern us here

Essence of the Problematic Judgment

γ It is plain from the above examples that the terms 'possible', 'probable', 'may', 'might', and 'must', stand for more or less reflective estimates of certain kinds of knowledge. The essence of the problematic judgment is the substitution of such an estimate for the concrete steps of inference really involved in an affirmation Possibility results in referring to reality, *without transition*, but *subject to an estimate*, what is only connected with it by transitions When the whole transition is made explicit, the allegation of possibility is superseded [1] The judgment which has all its conditions and reservations fully assigned to it is of the apodeictic order; possibility arises from effecting the reference to reality apart from the conditions The idea of 'possibility' is our substitute for the omitted conditions Obviously such an idea may emanate from all degrees of confused perception or of reflection We may be silent about the conditions either because we cannot clearly grasp them, or because we are explicitly abstracting from them But an estimated indirectness of transition there must be if we are to judge problematically. Disjunction can be treated in the same way, owing to the hypothetical inter-relations of its members, and thus the statement of chances is a clear and extreme case of the estimate

[1] Cp p 341, supra

in question Its essence is to burden the reference to reality of one alternative with a hindrance drawn from the number of other alternatives All possibility indicates a similar tendency But the statement of chances has measured its own cloudiness and made certain of its own uncertainty[1] It is therefore no longer problematical It supplies a definite predication of a hindrance to reality and not a hindrance to predication

Thus the true problematic judgment is a judgment with a peculiar and reflective content, which interferes with its assertiveness It is a hypothetical or disjunctive judgment in disguise All judgment whatever is within a real system, but the problematic judgment has its relations to its real system peculiarly obscured or neglected In the question, for instance, we only make explicit a part of the intellectual state, ultimately affirmative, on which our desire for further knowledge depends A question indeed often vanishes when we insist on its being clearly put

Assertory Judgment

3 The other forms of modality may be briefly dismissed Every judgment may be called assertory, as we saw, in virtue of its psychological reference to self-feeling If any judgments are to be called assertory in a strictly logical and material sense, they must of course be the singular judgments which depend on union of attributes within the concrete subject of the judgment, and not on their necessary connection in a larger subject falling outside the judgment Such judgments are even *logically* assertory in as far as the concrete subject is merely individual as a synthesis of differences not connected by abstract necessity So far on the other hand as it displays individual *character* and lends itself to analogical affirmation, it stands for the present purpose on the same footing with the subjects of necessary and apodeictic judgments. The assertory judgment has a higher degree of assertiveness than the problematical judgment as such, because

[1] This is the reason why the 'Thermometer of Probability' (see De Morgan, Budget of Paradoxes, p 416) would not be of very general application What it measures is the ratio of the whole number of equal alternatives arising on a certain condition to the number of desired alternatives so arising But in concrete knowledge we have no security of finding equal alternatives

its reference to reality though not apodeictic is direct and open. It would be impossible to maintain Kant's view of a progressive incorporation of contents with the understanding, if he meant that every trivial judgment, say, of perception, was preceded by a recognition and estimate of uncertainty. But all experience supports his contention that imperfect judgments in which only one element is clear, and in which this clear element is attached to reality through others which are not clear, belong to a less complete phase of knowledge than judgments in which the reference is clear and complete If the problematic judgment arises by intentional abstraction from precise knowledge, this makes no difference. We are returning on purpose to an imperfect form of judgment from a more perfect one, in order to exhibit a net result which the more explicit form will not display We must stand by the result which we thus obtain. We cannot eat our cake and have it.

Of course where an instance indicates a possibility, the assertory affirmation of the instance and the problematic affirmation of the possibility do not refer to the same content. 'This Drosera shows irritability' is a singular judgment of perception. The problematic judgment 'There are conditions though unknown in detail under which the nature of a plant develops irritability', i.e. 'A plant as such may have irritability' arises from the analysis of the above instance into a general suggestion. Such an analysis is probably concurrent with the perception of individual identity on which the Singular judgment rests, but the two are not identical. Imperfect insight into necessary connection may affect the same content which is being erected into a thing, but is not one with this process of erection. Assertory assertion and problematic assertion may be and must be conjoined in every problematic judgment, but they refer to different elements in the content affirmed, the former to the system as a concrete real whole, the latter to some element of the system as related by abstraction to the other elements When the latter aspect of the judgment is dropped or superseded we have the assertory affirmation pure and simple The assertory judgment has no degrees of assertiveness except in so far as in virtue of its

specific content the problematic or apodeictic judgments inevitably show themselves within it

The Apodeictic Judgment

4 Judgments of apodeictic character, i e hypothetical and disjunctive judgments, lay claim in virtue of their form to a higher degree of assertiveness than either problematic or assertory judgments The reason of this is that their form has for its differentia the exact exposition of the transitions, conditions, or alternatives subject to which the judgment is true of reality By such exact exposition the content either becomes an articulated system, or at least reveals itself as fitted to take a place in such a system The former is the ideal of the disjunctive, the latter that of the hypothetical judgment Reality considered as absolutely known is of course *ex hypothesi* taken to be absolutely asserted But Reality is not by any sane person considered to be absolutely, i e completely and precisely known Degrees of certainty in apodeictic affirmation arise from the consciousness—a logical consciousness made explicit in the structure of judgments—that the individual's knowledge is but imperfectly identified with the ideal judgment which would qualify Reality by the complete content of Reality

The Hypothetical Judgment

1 In the hypothetical judgment, as we have seen, though important elements are made explicit, yet the connection which is affirmed implies an underlying reality which is not expressed in the content of the judgment Therefore the hypothetical judgment is subject to two elements of uncertainty, viz its own reference to the limited reality the affirmation of which it implies, a reference which may be partial or confused, and further the relation of that affirmed real system to the content of Reality as a whole 'The British Parliament is able to alter a statute affecting its own duration' This is rather a generic than a hypothetical judgment, but for the present purpose these two types must rank as very closely akin About such a judgment there is first the question whether it really follows, or how necessarily it follows, from the facts which we are prepared to affirm of the reality known as the British Constitution, and secondly, what opening these facts themselves, as compared with the greater reality of our entire experience on the ground of which *they* in turn are

affirmed, leave for error or for unseen modification Any disjunctive judgment, as compared [1] with a given hypothetical judgment of which it may be regarded as a development, makes this underlying real system explicit, and therefore has no source of uncertainty but that of a failure in the necessity by which this system itself is attached to Reality as a whole But of course the disjunctions which we commonly use are for the most part systems within known systems, and therefore stand on the same logical level of certainty with hypothetical judgments Such e g is the disjunction which expresses the number and nature of the conic sections, resting as it does on the ultimate real system which we take to be the nature of space Only disjunctions that embody a complete and coherent sphere of knowledge, such e g as the nature of space, have the character which has just been ascribed to the ideal disjunction But even with such a disjunction we still have the difficulty in the background, 'Does the real system which we affirm'—in this example the nature of space—'*really* emanate as a necessity of knowledge from the whole reality which is forced upon us by experience?' It appears to me to be quite idle to maintain that all judgments, or even that all necessary or apodeictic judgments, are on a level in this respect Ultimately, we may imagine, nothing can be rightly known without knowing all else rightly, so that every isolated fact and principle of knowledge would be implied in, say, the existence of morality or in the existence of an intellectual world But as knowledge is in fact constituted its parts are fragmentary and incoherent, and there is much that we affirm upon only a partial or limited necessity, while much again is so incorporated with the whole fabric of our real world that we feel bound to maintain the former if we would not fall into hopeless contradiction with

[1] It is absolutely necessary, when we attempt to compare judgment forms in respect of their essential import, to select instances which belong to one and the same progression A hypothetical judgment drawn from an advanced phase of science has at once more content and more precision than a disjunctive judgment drawn from everyday experience To judge the capacities of the two forms by instances so selected would be like judging the powers of civilisation and savagery by comparing a civilised infant without allowance for age to an adult savage

the latter[1] The assertiveness of affirmation is not indeed measurable outside the calculus, but it is capable of being perceived and to some extent compared. It is not a mere feeling, but an insight into connections. It is absurd to maintain that in affirming the whereabouts of a friend, or the harmonious or other effect of a combination of colours, or the continuity or non-continuity of matter or of space I am pledging my intellectual existence to the same degree as when I affirm the relations of the multiplication table, or (subject to the requisite interpretations) the law of causation, or the existence of moral purposes.

The uncertainty which may attach to apodeictic judgments arises then from the same cause as the uncertainty of problematic judgments, but the cause is operative in a different mode and in a slighter degree. We always *feel* certain when we judge, for all judgments involve the same psychological identification with self-feeling But we *know* that this certainty is conditional on our expressing Reality with precision and completeness, and this we are well aware that we never do. Of course, to begin with, the apodeictic form of judgment is no guarantee, any more than any other, against falsehood or frivolity There may be no underlying real system at all, or that which is taken to be referred to may in no way justify the sequence erected on it Any false generalisation is an instance of this. Or again, the necessary transition may be quite incompletely set forth, so that the judgment sinks *ipso facto* into a problematic judgment in spite of its apodeictic form. Such are judgments in which a remote cause or consequence taken as a sign is substituted for the ground of the sequence, so that the judgment though not untrue in fact has the appearance of a riddle. 'If the stick will beat the dog the old woman will get home to-night' Here we have no indication of the real arrangement on which the sequence rests, and the condition, so far as can be seen without copious supplementation from ulterior knowledge, is irrelevant and equivalent

[1] This must be read subject to reservation as regards the actual language in which abstract principles are expressed It is a recognisable function of the body of knowledge, not a limited set of stereotyped ideas, which we may be justified in holding indispensably necessary to our reason See below, Book II, chap vii

to an unknown condition, implying as it does a number of unknown conditions. This is the explanation of the examples to which Sigwart appeals as showing that apodeictic judgments are not in fact made with any peculiar certainty of conviction. 'There is a common idea that the apodeictic judgment stands for something higher than the assertorical It is believed that if we start from the problematic judgment and ascend to the apodeictic we steadily increase the certainty of our knowledge, and add to the worth and dignity of our assertions This idea must be relinquished All mediate certainty must stand in the end on immediate knowledge, the ultimate premises of every proof cannot be proved. The usages of life stand in comic discrepancy with the emphasis we lay upon apodeictic certainty The sayings "It must be so", "It must have so happened" are judgments apodeictic, but the confidence they express has most modest limits'[1]

This is not the place to criticise the fundamental view which Sigwart here expresses, but to which he is, happily, not faithful throughout In treating explicitly of the nature and bases of inference we shall see that the distinction between mediate and immediate knowledge coincides with the distinction between what is known and what is only on the way to be known If Sigwart meant what he said in this place, he would have cut himself off from all possibility of believing in science As to the examples which he adduces, they fall into their place, according to what has been said, as problematic judgments 'It must have so happened' is an inference from reality under a condition, to reality without an expressed condition, and therefore is problematic

The degrees of certainty belonging to the apodeictic judgment itself are as we saw of the same kind as those which characterise the problematic judgment The whole of the reality on which the sequence is intended to be based may not have been brought to bear upon the sequence, and even if it has, its own relation to the reality which is the ideal of knowledge may be so disproportionately trifling as to make the judgment an especially inadequate embodiment of the ideal Reality which alone is certain

[1] Sigwart, Logik, i 238, E Tr 183, quoted in Bradley's Logic, p 186

I may illustrate this conception by our present knowledge of Hypnotic and kindred phenomena [1] The curative action of Hypnotism, or the beneficent anaesthesia which may be produced in some forms of the Hypnotic state, are now as it seems matters capable of embodiment in empirical generalisations. These rules or laws can be exhibited, like all rules or laws, in hypothetical or disjunctive form, that is to say, as consequents following upon conditions, or as alternatives arising within a certain identical content. On a certain degree and kind of hypnotic trance a certain anaesthesia is consequent A hypnotised subject is capable of some three or four recognisable alternative states In certain forms of nervous derangement acquired, or, as it seems, congenital, repeated hypnotic treatment exercises a sanative influence

Now although I personally entertain no doubt that there is, as the phrase goes, 'something in' all these conceptions, and though I am prepared to affirm them, i e to judge them true of reality, as embodiments of an experienced content which must be affirmed somehow and which I cannot affirm otherwise, yet so far as I understand myself I do not stake my intellectual existence upon them as I do on the existence of causation, or morality, or beauty On what I *mean to say* in them, on the experienced content from which they are drawn, I do stake my intellectual existence, but this content, apart from my explicit judgment, is to me an x, a thing-in-itself, a nothing. I must stand or fall by my judgment as it is, not merely by my ultimate intention to embody reality, which is the common and formal feature of all affirmation And I know perfectly well that by my explicit judgment on such matters as these I am very likely to fall The reason of this is not that the cases on which I rely are few in number, compared, e g with ordinary cases of the operation of digitalis on the heart or of mercury on the liver One case is enough, as we all know,

[1] I believe myself justified, when writing for purely logical purposes, in treating the actual phenomena known to science as freely as may be necessary to give them sharp outlines, and to avoid a mass of reservations and qualifications that would be only an encumbrance for my present purpose I on my side hope to keep clear of wilful distortion, but the reader on his side must not accept my illustrations as citations from a biological textbook

where the framework of knowledge stands ready to receive it with the grip of necessity. The reason is rather that the organised system of reality within which the sequences in question have their force, lies or has hitherto lain outside the great fabric even of biological science, not to speak either of deductive reasoning on the one hand or of philosophical construction on the other. The principles of abnormal psychical phenomena have not been as a whole identifiable either with the principles of physical causation or with those of normal psychical development. The judgment is therefore obstructed by the want of contact or necessary relation between the system of such abnormal phenomena, which it must in some way refer to reality, and the entire *cosmos* of normal evolution. Supposing, however (I speak merely by way of illustration), that the abnormal states in question, even those which present the apparent puzzle of a morbid origin combined with a curative effect, could be exhibited as cases under the known principles of evolution, the whole ground and certainty of the judgments relating to them would be put upon a new foundation. It is an old idea [1] that many states and susceptibilities of the soul, which are commonly treated as mere freaks of nature or capricious results of disease, may really have their place among the phenomena of evolution no less than sleep and waking, or the oneness, expressing itself through heredity, of parent and child. If peculiar forms of sensitiveness and peculiar 'morbid' states or transitions to states could be brought under such heads as survival, reversion, or analogous development, their underlying reality would be grafted on the main stem of the organised real world, and the necessity with which they were affirmed would become more determinate and more concrete.

The Disjunctive Judgment

11. Disjunction by its form aims at the standard of a complete and therefore of a real system. 'Real' because it points to nothing beyond itself as an implied ground of truth. We have seen sufficiently above that no objection can be raised against the reality of the content of any judgment by reason of its being extended in space or time. No judgment confines its reference within an atomic 'now', and no reality can display itself as existent within an atomic now. It is grammatically

[1] See Hegel, Encyclopädie (Anthropologie), sect. 404.

possible however for a disjunctive proposition to express a judgment which is hypothetically disjunctive 'A man who would act so must be a knave or a fool' This shares the character of all hypothetical judgments in implying an unexpressed real system as the basis of its truth, and its certainty must be judged as the certainty of a hypothetical judgment But the disjunction according to the ideal prescribed by its form is in itself an exposition of the reality that determines its parts, and therefore is not a sequence within a presupposed system, but is itself the content of a real system It therefore properly ranks with the generic judgment, to which it is affiliated, as quasi categorical, and has only the imperfection of certainty which arises from the comparatively minute range of reality that is comprehended in any such simple system

It is obvious from what has been said above that the degrees of certainty here discussed are not numerically estimable, because they are not reducible to ratios of equal alternatives It may therefore be justifiably complained that the phrase 'degree of certainty' is misplaced, and should be transformed into 'stage of logical necessity' I have no objection to some such transformation provided that it is distinctly understood that modality affects the assertiveness of assertion, that this assertiveness is a matter of content and not of the formal copula or reference to self-feeling, but that if we extend the notion of the copula to include the material or logical grasp by which a complex content is fitted on to a complex Reality then we may say that Modality is a matter of the Copula In any case, the progressive incorporation of a content with the understanding,[1] that is, with the organised ideal system by which the understanding permanently qualifies the Real, is the same thing as the progressive participation of that content in the certainty that could only be complete in a judgment that should exhaust Reality.

[1] Cp Whewell's account (see Mill's Logic, I, p. 279) of coming to perceive the necessity of a principle which he had before accepted as fact. This is merely the acquisition of a precise and coherent insight into its dependence on reality Cp Book II, chap. VI

A NOTE ON ERROR.

In the first edition of this work there was no treatment of Error *eo nomine* The reason of this was that it appeared to me to be substantially dealt with in the general account of the conditions of truth, in the theory of negation and possibility, and in the account of the rectification of judgment by scientific process

It will be well, however, to sum up shortly the results of these views in their bearing on Error *eo nomine* I acknowledge indebtedness to Professor Stout for the distinct form which he has given to the question (Aristotelian Proceedings, 1910–11, 'The Object of Thought and Real Being'), and for his precise statement, amplified in private correspondence with me, of the nature of 'relative possibility' as affecting conceivability Yet I believe that I am advancing nothing which does not fairly follow from the theory of this work

The fundamental character of Error is always the assertion of a merely possible alternative to be *the* alternative true of reality, i e the assertion of something conditionally true, without regard to its condition (I should say 'unconditional assertion' were it not the case that in error there is rather a tendency to assertion under a false condition)

I distinguish two main cases

(1) There is what may be called 'Logical Error' This is when the condition, under which alone the affirmed alternative is realised, is omitted or replaced by another, but not excluded by an incompatible condition A simple case is when we are drawn on to affirm the consequent of a hypothetical judgment without thinking of its antecedent. 'If that one little link were made out, I should be of royal descent,' passes in cases of bias or failing mind into 'I am of royal descent' Here there is not necessarily factual error The alternative affirmed as actual is one relatively possible, i e conceivable from a certain point of view, and may be in addition actually true, i e conceivable from a complete point of view But the limitation suggested by the incomplete point of view is not attended to and expressed, while, as in ordinary everyday judgment, other more or less indifferent qualifications may be expressed Such judgments may be what we call true in fact (as the affirmed consequent of the hypothetical above *may* be true in fact), but as in every judgment there is a presumption of necessary connection (see vol 1, pp 134–5), the indifferent qualifications produce a false nexus and therefore a Logical Error All judgments except those, if any, which express absolute truth have some degree of 'logical error'. What they assert does not 'hang together', but it may be a fact, and indeed all fact is asserted in this way

(2) I call it 'Factual Error' when the necessary condition for the affirmed alternative being actual, not only is not expressed, but is replaced by a condition or character belonging to an actual alternative

which exclude the one affirmed 'The lady in the garden, wearing a pink dress, is Mrs A (when the wearer of the pink dress is Mrs. B, and Mrs A, really has on a blue dress) For the judging mind, this case is much like the last The status of the condition, which should attach the alternative to reality is unnoticed And in this case it is a status not merely of obscurity or irrelevance, but of hostility But the point of view of partial truth is preserved, and what is asserted is a conditional element of reality taken as true without regard to its real condition The alternative affirmed is conceivable or possible from a certain limited point of view (i e as lady in the garden), but is affirmed as actual, i e as conceivable from a complete point of view, or, without allowance for the limitation suggested by the point of view from which it is possible In factual error, the complete point of view involves a hostile or excluding condition, a condition peculiar to a different alternative, so that the alternative affirmed is no longer conceivable nor possible

We may say, therefore, that the alternative affirmed in factual error 'belongs to reality (Green, Prolegomena, sect 23, above, 1 327), but not *as* affirmed For the condition through which it is identified with actual reality is a condition which, S and P being fully determined, excludes it Therefore from a complete point of view it is inconceivable, and does not belong to reality The judging mind has not fully determined the content of its own affirmation, but when this is determined, the affirmation becomes a contradiction in terms

END OF VOL I

LOGIC
BASTARD BOSANQUET

LOGIC

OR THE MORPHOLOGY OF KNOWLEDGE

BY

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ABBREVIATIONS

Bradley's Appearance = Bradley's Appearance and Reality, ed 2. Sonnenschein, 1897.

Bradley's Logic = Bradley's Principles of Logic Kegan Paul, French & Co, 1883

Keynes = Keynes's Formal Logic, ed 4. Macmillan, 1906.

Prichard = H A Prichard's Kant's Theory of Knowledge. Clarendon Press, 1909.

Essentials = Bosanquet's Essentials of Logic. Macmillan, 1895.

Sigwart = Logik, von Dr Christoph Sigwart 2nd edition. Freiburg, 1889.

BOOK II

OF INFERENCE

CHAPTER I

THE NATURE OF INFERENCE

1. INFERENCE shares the essence of Judgment, but, at least *qua* explicit Inference, has in addition a *differentia* of its own. The essence of Judgment is the reference of an ideal content to Reality, the *differentia* of Inference affects the mode of this reference, and consists in Mediation. Inference then is the *mediate* reference of an ideal content to Reality. If I affirm that I spoke to you in the street yesterday simply because I find it in my memory that I did so speak to you, that is, apart from refinements of analysis, simply a judgment If, as against your denial of the fact, I corroborate my recollection by pointing out that I *must* have spoken to you, because you afterwards acted upon something that I then told you, then I am reasserting the content of my original judgment, but with an addition and modification that turns it into an Inference I then refer an ideal content to Reality, not as directly given in memory or in perception, but on the strength of a content distinguishable from the former content, bearing a certain relation to it, and itself referred directly to reality. **The Essence of Inference**

By speaking of 'mediate' reference to reality we have mentioned the *differentia* of Inference, but have not explained it Direct affirmation appears to explain itself, but mediate affirmation is even at first sight somewhat mysterious. We are at once met with the old question, 'How are synthetic judgments *a priori* possible?' The qualification *a priori* adds nothing to the qualification 'true' which is claimed by all judgment as such The question therefore is in plain

English, 'How can one content claim to be true of Reality on the strength of another content distinct from the first?' 'How can any synthetic judgment *qua* synthetic—i e going *from* content *to* content and not simply accepting either a mere occurrence or a mere conjunction—how can such a judgment conceivably be justified?' The answer to this difficulty, like all answers in philosophy, is at first sight a mere restatement of it Whether such a restatement is an explanation depends on its congruity and coherence with reason and with experience It is possible—so the answer must run—to proceed in knowledge from content to content, because the world as known consists of universals exhibited in differences, and the contents from which and to which we proceed are not shut up within their respective selves, but depend on a pervading identical character or universal of which they are the differences '*Of which* they are the differences'[a]—for here is the objection which meets us on the threshold. Suppose that I find in a room a hundred different objects—books, guns, china—all marked with the same label, say with the owner's name. Well then, it may be said, here is your 'identical character' or 'universal', but what can you infer from it beyond itself? It tells you nothing of the object to which it is attached You may go on for fifty cases affirming that *a* having the label *x* is a book, *b* having the label *x* is a book and so on, but you cannot tell in the least what the fifty-first object that has the label will be, whether a sporting rifle or a china teapot There is an identity throughout all the objects, but they are not, or seem not to be, *its* differences. They simply contain it, and are in no way leavened by it You cannot in any way determine their predicates on the basis furnished by this pervading identity

The whole of our previous and subsequent discussion really deals with this radical difficulty Logic is little more than an account of the forms and modes in which a universal does or does not affect the differences through which it persists I can only point out that all turns on the distinction between the abstract or powerless and the concrete or dominant universal To interpret the latter by the former,

[a] Cf vol 1, p 45 note

to reduce all universals to marks, i e to the level of the example just mentioned, is a fatal tendency of popular logic A very elementary example of a relatively concrete universal may be found in the nature of a geometrical figure, say of the circle or the triangle Given an arc of a circle, we have the radius and centre, and can lay down the whole circumference The given arc is not simply repeated, it is continued according to a universal nature which controls its parts, and with a result which though involved in the given arc is yet outwardly and as an actual content distinct from it This is clearer if instead of a circle we take an ellipse, in which the given fragment of the curve cannot in any sense be said to be simply repeated without change in constructing the remainder There is something in the curve as given which is capable of dictating a continuation and completion of its outline distinguishable from the given arc or fragment itself. Just so with a triangle—given two sides and an angle, we can find the third side and remaining two angles

And we can now see that in the first example, which seemed so hopeless, the same relation would be traceable assuming the label to have any meaning at all A *mere* mark, which conveys nothing, is not even a mark, for what is it a mark of? But supposing the label to indicate A's ownership of the things, then we could infer all sorts of legal consequences about them from this ownership, and these consequences would not be the same for all the objects, but would be modified by their nature, e g it is probable that some of the things would be liable to seizure by a judgment creditor and some would not. Thus here too the universal would be an identity pervading different manifestations

The universal in its differences is then the basis of mediate judgment or inference But it is also the basis, as we have amply seen, of judgment as such, i e what would usually be called immediate judgment The above examples, however, furnish the further distinction to which we shall find it convenient on the whole to adhere Mediate judgment or inference is the indirect reference to reality of differences within a universal by means of the exhibition of this universal in differences directly referred to reality The differences

indirectly referred to reality may fall outside, or include, or even consist exclusively of, the differences directly referred to reality Immediate judgment, according to its idea, would be the mere reference to reality of differences as united within the identity or universal It might be more intelligible if we were to substitute 'parts' and 'whole' for 'differences' and 'universal', but then it would have to be borne in mind that we are not speaking of quantitative parts, i.e that the kind of whole in question is not necessarily the sum of its parts Subject to this reservation, I have no objection to defining Inference as the indirect reference to reality of parts within a whole on the strength of the nature of that whole as revealed in parts directly referred to Reality And the definition of Judgment would bear a corresponding modification Of course I do not mean that all the forms which have already been discussed under the head of Judgment are *substantially* confined within the definition of Judgment and excluded from that of Inference But for the present, in order to obtain a clear view *prima facie*, we are considering only explicit Inference and excluding all that takes the outward shape of mere Judgment

Some Accidents of Inference

2 The above account of the essence of Inference will be best illustrated by considering some accidents of inference which I have endeavoured to exclude from the definition

Mental Transition in Time

1 First among these comes the attribute of mental transition in time, with which that of an advance from known to unknown may in one sense be identified

The account given above [1] of the Judgment in time applies also to Inference as a mental process in Time The first and most fatal error as regards both Judgment and Inference is to introduce the idea of an actual and instantaneous transition from content to content This idea combines the error of denying that inference, as a mental process, has duration in time, with that of denying that as intellectual insight its parts are inward to each other and exempt from temporal succession The universal itself, or intellectual synthesis of differences, is not a fact in time, and throughout the interval which inference occupies as a psychical process the operation

[1] Bk I chap 1

of the universal as a growing insight is traceable in every point of time, but is not shut up within any atomic moment.

But apart from the idea of instantaneous transition, there is an idea of advance in time which has great appearance of truth and which is indeed in one sense true It is unquestionable that in inference we start from data, from facts thrown down before us, it may be in chaotic disorder and with no suggestion of a result [1] We go to work upon these facts, and after the labour of hours, of days, or of a lifetime,[2] we light upon a conclusion which issues from them and to which they are related as premises, i e which exhibits them as differences in a universal How is it possible to deny that we have here an advance in time from data without conclusion, at any rate to data *plus* conclusion, and, if we go by the old syllogism, to conclusion *minus* a large part of the data? The difficulty which I find in stating the above antithesis is a first indication of its fallaciousness It was impossible to write simply 'from data without conclusion to conclusion without data' A conclusion without data is an obvious contradiction in terms, and if even part of the data are dropped (as the middle term in the syllogism) the conclusion sinks *pro tanto* into a ὅτι rather than a διότι—a fact instead of an inference No doubt we are apt to pluck off our conclusion like a fruit from a plant and carry it away for consumption Practical life requires this procedure. But we must remember that from the moment of severance death has begun, and that the intellectual product can bear isolation far less than the material The idea of an actual transition *from* data *to* result, so far as it is founded on this habit, is in science simply a pernicious blunder The case in which the result is a systematic insight that includes the premises in a transmuted form does not of course fall under this censure But this case is not as a rule contemplated by the traditional forms of inference

[1] I put the case at its extreme against myself We must however remember that we can only see in the facts what we are ready to see there, what we bring with us So however disorderly in fact, the data are really from the beginning theorised upon by our apprehension, because it can only apprehend them on the strength of its own existing content [2] See the famous preface to the 'Origin of Species'.

And of course it might be correlatively maintained that facts are not data, except by virtue of a result, or if this is not true of data but only of premises, then that inference does not start from data but only from premises[1] At first sight such a contention seems to blink the difficulty The conclusion, it seems, may be removed by a week's work from the data or premises, and granting that they are not rightly called data or premises till the week's end when the conclusion is won, still the facts concerned were present and active at the beginning of the week and certainly entered into the advance that has been made

Two things are here to be distinguished, viz mere reproduction and inferential reproduction The universal active in the mind is not apparent as a whole within the mere psychical facts of the transition Its operation is extended throughout a series of the fugitive psychical facts or ideas, and although in *logical* thinking its operation is conscious, i e. selects and modifies within the content of these ideas, yet it is not in itself necessarily a conscious activity. It acts in consciousness, but need not be conscious of its own principle of action In rudimentary reproduction we see a man far off, and a name comes into our mind, apparently as a detached fact, without any reason that we can assign or think of trying to assign It is only later, when we *clearly* recognise the man, that we become aware either *that* we recognise him, or how and why we recognise him. Logical thinking consists of making this process conscious, but essentially and fundamentally the intellectual tendency which controls reproduction need not be present as a distinct content operating in reproduction So far the psychical process might in theory begin with data alone and then go on leaving them behind to result alone, *not* carrying throughout the transition any conscious unity or continuity of content.

But in *explicit* inference, at any rate, this is not the case. The essence of inference is to drag into consciousness the operation of the active universal as a pervading unity of content on which inference depends The conscious operation of the active universal in inference is what we have to dis-

[1] But then it would seem the data must be premises of the premises

tinguish from the mere implicit action of the universal in rudimentary reproduction Therefore when we speak not of mere reproduction, but of inferential reproduction or passage from data to conclusion, then it is true that you can no more have data or premises without conclusion than conclusion without data or premises. The appearance to the contrary, which I have admitted to exist, arises from our fixing our attention exclusively on the conclusion *par excellence* in which and not before it we happen to rest This conclusion, though it may close a stage of science or set at rest a longing of our hearts, is in no way different in logical character from the first steps of preparatory activity with which reason penetrates the facts laid before it We meet the data with a judgment when they are laid before us, and we modify this judgment continuously throughout our inference. Any section taken, so to speak, across the interval of intellectual activity which elapses from first data to ultimate conclusion would lay bare the *whole* structure of an inference, just as a section across the time during which a chord is sounding on the organ would exhibit the *whole* harmonic structure of the compound tone Of course we may draw partial conclusions, discard their data, and work with these conclusions as with fresh data, and so far we make an advance from content to content, discarding the old in favour of the new. But this process, though necessary in practice, comes under the observation made above upon a conclusion severed from its proof, and is not typical of inference, but of its limitations

Transition in time from content to content as between data and conclusion in inference is not really possible But it is true that the process of inference, though continuous and bound together by a conscious unity, is extended in time and includes considerable modifications of the judgments from which it starts The appearance of a transition arises from discarding data, which is unjustifiable, from transforming data, which is right but is no transition, or from comparing ultimate conclusions and primary data while neglecting the intermediate phases which constitute the continuous and—not *transitional* but—*inclusive* inferential evolution. It is in

this character of transition without conscious continuity that rudimentary reproduction differs from explicit Inference

Thus our definition of Inference did not say that the differences which form the data are *previously* affirmed of reality, although it is only from them as affirmed of reality that the reality of the inferred differences can flow

Discovery or Novelty

ii Secondly, it follows from the above considerations that *discovery* is an accident and not an essential of inference Inference is not essentially passage in time whether instantaneous or extended Therefore it is not the case that a conclusion ceases to be an inference the moment that it becomes familiar, the moment, that is, that it ceases to be a discovery On the contrary, discovery without proof is conjecture, an element of proof is needed to constitute inference, and indeed to constitute discovery The activity of inference cannot be identified with the perception of something new It is quite a normal occurrence that the elements which are indirectly referred to reality should *also* be directly referred to reality Whenever, indeed, as the ideal of inference requires, the original data themselves are transformed and freshly elucidated, this happens as a matter of course When the working of a machine is about to be explained we see a wheel or piston to be there as a fact, and we ask what it does The answer tells us why there must be such a wheel or piston, and this is not superfluous though we knew beforehand that it was there The part in question then becomes to us an element or difference in the pervading identity or universal which is the working of the machine And if we live fifty years and see the machine every day, understanding it thoroughly, still the use of any one of its parts, considered as necessitated by the nature of any other actual part or set of parts combined with the working of the whole machine, remains to us an inference and never becomes a mere fact. Thus novelty or discovery is an accident of Inference

Omission of relevant matter

iii Thirdly, it follows from the above considerations that omission in the conclusion of contents employed in the premises is an accident of inference Inference is confined neither to what is novel nor to what happens to interest us The appearance of a necessity of omission arises from various

causes The transformation of data is an ideal which is likely to be misunderstood It resumes the old data in a new expression Again, it is usual and in complicated matters unavoidable to confine ourselves in dealing with any universal to some aspect determined by context or by curiosity And the habitual omission of the middle term in the Aristotelian syllogism is perhaps due in part to the above causes, and in part to the vicious habit of severing the conclusion from the premises which the rhetorical associations of early logic tended to foster [1] But the entire content of the universal, so far as recognised in the necessity that unites its differences, is the true content of every inference, and there is no logical reason for neglecting to make explicit any portion of truth which our perception of it generates

A question arises on the margin of this subject of omission with reference to the systematic realities of which we spoke under the head of the hypothetical and disjunctive judgments. If I directly affirm the reality of a complex system, such as a railway, or a government, or a mind, and include in my affirmation a mention of many parts and properties as systematically interrelated, am I in doing so rightly said to be inferring? It rather seems here as if the absence of omission destroyed all semblance of a *conclusion*, and how can there be inference without a conclusion? 'Cannon Street railway station has interlocking points and signals' To any one who understands the subject it is unnecessary to complete this by the further explanation (which even if inserted may of course be a mere qualification, not a fresh judgment), 'such that opening any one line *ipso facto* needs the signals to be at danger for all the lines that cross it' In such a judgment, regarding it, as we have every right to do, in the light of a single affirmation, I am obviously embodying matter which has an inferential character But whether I am actually inferring or not depends—*not*, as one might be tempted to say, on the novelty of the conclusion implied, but—on the degree of insight with which the judgment is made It is in

[1] Because the rhetorician only wants to prove, not to understand If he reaches his conclusion, the steps by which he reached it cease to interest him or his audience

short either an inference, or the allegation as a fact of relations that must have formed the conclusion of an inference, although the inferential connection *may* now be lost If however we go into such detail as proves that we have an insight into the why and wherefore of the system, then we have no choice but to say that we are inferring but are stating our inference confusedly The confusion is in not distinguishing data from conclusion—how much is immediately affirmed of reality, and how much is mediate The judgment is a mediate judgment simulating an immediate character [1]

An ordinary hypothetical judgment is really an analogous case to the above (which might be represented by a disjunctive judgment) Inferential matter, a relation or nexus, seems to be affirmed of reality, but yet the omission, which would leave as affirmed what seems to be the conclusion, cannot be made The ground *per se* is not affirmed of Reality, and so the consequent *per se* is not affirmed to be true The moment that 'If' passes into 'Because' you can omit the ground and affirm the consequent *per se* But retaining the 'If' we cannot affirm the consequent We cannot affirm upon mere supposition, nor can we infer without affirming Yet certainly, as in the last paragraph, we seem to have before us an inferential activity 'In four-dimensional space (i e supposing such space) a knot can be tied in a string whose ends are held' In this judgment the nexus between four-dimensional space and tying a knot is undoubtedly inferential. The moment we affirm the reality of the ground, we also affirm the real possibility of the consequent But the hypothetical judgment as such affirms neither the one nor the other. The entire judgment is no doubt itself a consequence of an underlying reality, the affirmation of which it implies, and on the ground of which—a ground asserted to be real—the *whole* complex content of the hypothetical judgment is asserted to be real, subject to its own inherent reservation

[1] The existence of these ambiguous inferences—half inference and half memory or authority—favours in appearance the restriction of inference to what has novelty When an inference is just made, then, if ever, it has definite data When an inference is familiar, it is too likely to rest on the ground that it is remembered to have been approved of Nevertheless, not novelty, but systematic necessity, is the true differentia

embodied in its own further ground But this does not help us to determine the presence or absence of inference within the hypothetical judgment itself

The fact seems to be that hypothetical affirmation is a contradiction in terms, and so too is hypothetical inference. The whole process, apart from any categorical meaning which it may make explicit, which is a matter of degree, is a mere make-believe You choose to treat as real in one sense what you do not affirm to be really real,[1] and you record the groove of necessity which manifests itself when the artificial reality is considered as though forming part of the real reality. Of the differences within the universal which determine the remaining differences (in this case the consequent) part (the hypothetical ground) are only sham reality, and therefore although we seem to exert inferential activity, yet we cannot affirm the conclusion of the inference. Here then we have the two degrees of impropriety in omission. Even when the ground of inference is affirmed, as with ordinary premises, the reality of the conclusion is restricted to the precise sense imposed by that ground, and it is therefore theoretically unsafe to affirm the conclusion apart from the ground But when part of the ground of inference is not affirmed, then we have really the case of the problematic judgment, and if the consequent is affirmed with omission of the supposed ground it can only be affirmed problematically, i e cannot be affirmed as true, or in the proper sense affirmed at all The reference to reality is then incompletely mediated But on a pure supposition no inference can be erected The element of supposed reality is the element of reservation, and the element of real reality is the element of affirmation. A pure supposition would be all reservation and no affirmation If I suppose that over a certain spot of ground gravity ceases to operate, I can form some kind

[1] This process has many degrees The content of supposition may be real all but some very minute relation The conclusion from the supposition can then be affirmed subject to a very minute reservation The *main* content of the conclusion may depend on what is absolute fact 'If that picture were ½ in to the left it would hang symmetrically with that other picture' gives the conclusion The picture A hangs symmetrically to ½ in with the picture B

of conception of the consequence I affirm the present state of things with reservation for the modification introduced by the limited absence of gravity But if I suppose that there is to be no gravity at all in the world, the reservation gets the upper hand, and nothing, I presume, is left for me to affirm —not even, as when part of the conditions only is affirmed, a problematic conclusion

Omission in inference is thus accidental and hazardous in various degrees It is, as we saw, in one form (as omission of the condition or reservation in judgment) the mark of problematic judging Omission however as here discussed is omission of matter relevant to the inference, or, which is the same thing, falling within the conclusion In this sense, as within the relevant content, selection is the converse of omission, and is equally an accident of Inference If you select, you omit, and if you omit, you select But you need not do either

Omission of irrelevant matter

iv On the other hand, actual data must always present irrelevancies, and must be exhibited as transformed—not necessarily *be* transformed *de novo* by an act of discovery—in the inferential operation Otherwise there could not be the circuit through the universal which we have taken to be the differentia of inference It is natural therefore to think of Omission and Selection with reference to the *actual* data on which, as referred to reality in their crude form, a given inference depends Within these actual data Selection is of the essence of Inference, but Omission depends on the existence of irrelevancies in the data, and although in theory these disguises and superfluities cannot be wanting, yet they may be reduced to an almost inappreciable margin, and the element of omission in that case becomes inappreciable also There is then, apparently, selection without omission, there can never be omission without selection

Three terms.

v. It has been maintained that the presence of three 'terms', as required by the Aristotelian syllogism, is an accident of inference and does not touch its essence I must leave this question, which is largely verbal, to decide itself in detail from our whole account of the subject Here I will merely indicate the distinction on which in my opinion the

answer to it depends Of course an inference is constantly drawn by the combination of very numerous facts and conceptions Aristotle's [1] or Lotze's Inductive syllogism, or a Sorites, or a Euclidean construction, are familiar examples of inference so drawn The only question is whether these numerous facts or conceptions correspond to phases or elements in the logical act of Inference, and whether, if they do not, they can claim the title of logical 'terms'. Must there not always be (i) differences or parts directly referred to reality, (ii) the universal nature or continuous identity which binds these differences or parts into a whole, pregnant with a capacity of accepting and arranging further differences or parts, and (iii) further differences, identified as parts within the pregnant whole which controls the inference, and, on the strength of this identity, referred to reality? It may be observed that this last 'term', moment or element of inference, may and ought to include the two former But it contains them in another sense than that in which they appear as isolated elements of inference, and therefore is not superfluous nor tautologous We may have a thousand observations of the places of a moving heavenly body, but these thousand data are not a thousand terms The thousand observed places fuse into the law of the orbit, and the law of the orbit dictates the remaining places which form the path that the body traverses Or, as the above instance really verges upon geometrical construction, we may take an example more cognate to what is commonly meant by Induction, though it is hard to find a good example of a process which does not exist If typhoid fever attends a certain milk-supply through a large portion of its ins and outs, including many dozens of cases, then we shall no doubt be apt to suspect that danger attaches to that milk supply as a whole, and consequently menaces any localities as yet unexamined to

[1] τὸ καθ' ἕκαστον is a term in both premises of the Aristotelian Inductive Syllogism Obviously this means that each premise would be a conjunction of judgments, or a conjunctive judgment Such a group of judgments would correspond to the so-called premises in Lotze's Inductive Syllogism, which 'premises' are really only a single premise, out of which Lotze takes his conclusion *per saltum* If the other premise were filled in, his inference would show three terms

which this same supply extends its operations Here again the three elements of Inference are conspicuous, though, as we shall see, they are in any such statement exceedingly ill-defined and their connection ill-warranted

The lower limit of Inference

3 I have thus far been speaking of explicit Inference, that is to say, of inference in which three or more 'terms' or intellectual elements are consciously distinguished and combined And it is true, as I said above, that the nature and phases of implicit Inference must really be gathered from the whole theory of judgment which I have stated in Book I to the best of my power. But a few remarks and a few examples may be useful in throwing light on the modifications which have to be traced.

The function of which I have attempted, in the theory of judgment, to write the later history, is the activity of the universal in the mind, or in other words, of the mind as the universal

The reproduction of Ideas

1 I have at present neither space nor competence to enter upon psychological controversy with reference to the so-called Association of Ideas. But it is necessary to define my position by explaining that in as far as any doctrine of Association involves the hypothesis of reproduction by other ideas of ideas as separate particular units, i e the denial of real identity or of the active universal, I am unable to reconcile such a doctrine with logical phenomena And logical phenomena, if we include in them the judgment from its very beginning, take in by far the larger part of the known phenomena of mind I cannot suppose a discontinuity—in my opinion moreover wholly unmotived by experience—between distinctly logical phenomena and the quasi-intellectual activities of primitive and animal soul-life And therefore I shall treat the fundamental activity of thought as the same throughout and as always consisting in the reproduction by a universal or a real identity, presented in a content, of contents distinguishable from the presented content, which also are differences of the same universal

It may be that in early soul-life this reproduction is unconscious, and that its results, the images which it brings before the mind, are not used as ideas, i e. are not distinguished

from fact or known to be symbolic of a content other than themselves. The results of experience may be made available for the guidance of action in an animal through suggestion effected by reproduction, but not distinguished *as* suggestion from any presented reality In this process we have something that does the work of judgment and inference, and that has the same fundamental nature with them But it is not judgment, because the images which it causes to succeed one another in the mind, not being distinguished from any reality, as a mere meaning necessarily is,[1] cannot be affirmed in qualification of reality [a] And *a fortiori* such early thought is not inference, because it is not judgment Inference as we saw involves assertion

To begin with, then, we may set down the lower limit of inference as at any rate not prior to the beginnings of judgment. Yet even this *prima facie* boundary is drawn subject to a large reservation on account of the primitive reproduction or redintegration to which I have just alluded The unconscious extension of a sensation by reproduction fulfils some functions of inference

General necessity of Judgment

11 And when we come to judgment in the strict sense, the task of drawing a line between implicit inference and what is not inference at all becomes an impossible one Fortunately it is also, in this rigid form, an idle one What we have to say upon it amounts to this All Judgment, we are told [2] with emphasis, claims necessity That is to say, every one who makes an assertion,[3] though of course he has, as a rule, never heard of logic or of a ground, yet believes that he cannot think otherwise than as he asserts In full-blown Inference he backs up this belief by a distinct allegation of separate but connected matters which he takes to justify his conviction. In implicit Inference we must distinguish the feeling that there

[1] When psychical images come to be employed for the sake of a meaning which they convey, *they ex hypothesi* are not treated as fact And their meaning is not itself a psychical fact, but is an intellectual activity which can only enter into fact by being used to qualify reality

[a] On the nature of an implicit idea, which is a mere qualification of sensation or perception, see Bradley, Appearance, ed 2, 606

[2] Sigwart, vol 1 p 237 E Trans 1 182.

[3] A conscious lie is only a sham assertion, except in as far as the hearer is induced to judge it true

is a justification from the incipient selection of definite matters as forming the justification It is probable that, as Sigwart implies, the feeling of justification is in one form or another essential to judgment. An uneducated man or a child, if his perception or his memory is doubted, will sometimes merely reiterate his assertion. This reiteration implies on the one hand that he cannot formulate any inferential support for his original judgment , he does not know how to travel outside the content of his assertion in order to invoke external aid Such a phase of the judging activity is well illustrated by the impersonal judgment, in which the place of the significant subject which develops into the pregnant genus-idea, or ground, or condition, is devoid of all content Yet on the other hand such reiteration implies an effort and failure on the part of the speaker to get beyond the original content, and a consequent return to that content, which is the germ of the *motived* inability to think otherwise that constitutes the necessity of inference In such a mind, we may suppose, imagination and conception do not fall apart, and his thinking satisfies the criterion of necessary truth which Mill criticised as defectively explained by Whewell, in that he cannot even *imagine* (not to speak of *conceiving*) the matter to be otherwise than as he asserts it to be

An educated man makes a similar justification explicit when he tells us that he relies on the evidence of his senses The phrase is perhaps primarily intended to be ironical, as implying that the senses give the fact and not mere evidence of the fact, but its irony fails because it is strictly true Sense, though it is a fact, cannot give *the fact*, and is strictly, as the supposed speaker calls it, evidence—circumstantial evidence or datum, not 'testimony'[1] which implies assertion The phrase 'evidence of the senses' then, if taken seriously, conveys the consciousness that sense-perception has an inferential character, and rests on a necessity arising out of combinations of elements among which sensation is but a part or datum When this consciousness, which experi-

[1] Evidence in this application may have originally meant obviousness or intuitiveness, 'Evidentia,' 'Evidenz,' and would then have no close connection with the common meaning of 'evidence' as = 'testimony'

ence of illusions soon forces on reflecting men, is thoroughly attained, then the perceptive judgment is known to need justification, but it is not known in what this justification consists nor that it may lie in a connection of content apparently going beyond the observed conjunction. Attentive observation and precise interrogation of the memory are the engines which suggest themselves as securing the necessity of judgment at this stage Of course these processes imply a reliance on certain principles But the inference is so far formal and general, not material and specific It is rather a general conviction that perception can be relied on, than an individual inference that this particular perception is rightly construed to give this particular content. And therefore the inference falls apart from the judgment as such, and cannot be taken as an element within it When we pass this point, we come to something much more like Inference proper

111 Prior to Judgment, as we saw reason to suppose, the operation of the universal or the real identity which governs reproduction is unconscious Of course it has a result in consciousness, but the mind is not aware of the limits and pervading ground of the process from which this result emanates. I cannot say on what definite stimulus my friend's name rises to my mind when I see him at a distance, not being yet aware that I have recognised him, nor what is the operative content which makes a certain room recall a long past incident which occurred elsewhere In the phase of judgment which has just been alluded to this real identity emerges into consciousness as the meaning of sentences and as the active guide of perception and memory. In this capacity it is attended by a necessity at first actual and then perceived, which at least reveals itself (when men talk of the evidence of their senses) as a partly intellectual necessity. But up to this point the real identity or meaning has simply been suggested and affirmed, as this or that ideal content, to be true of reality, and has not within itself displayed any articulated or selective character It has shown no systematic organisation to which thought could appeal as a definite individual compulsion prescribing the nature of the ideal content which it reproduced. The matters affirmed have simply

Specific necessity of Judgment

been conjoined within a unity or identity, as philosophy and self-conceit may be conjoined in the same man They have not been shown to cohere as parts in an intelligible whole, not, that is, as the third angle of a triangle coheres with two given angles and a given side, or as personal liberty in England coheres with the supremacy of law [1]

But a further principle makes its appearance, as we saw, with the judgments of individual character, of ideal measurement or of ideal enumeration Here the universal takes on the character of a system, which governs its parts on the basis of its pervading nature From this point onwards we have in fact the full essentials of Inference, and it is very much a question of convenience whether the inference takes implicit or explicit form So long as we retain the form of direct synthesis our definition forbids us the title of explicit inference For the identification of the subject-idea with reality is presupposed and not affirmed, and the qualification of reality by the predicated content is therefore direct in form, though indirect to a large extent in substance 'To a large extent' only, for according to the view which I take of judgment the affirmation in all the more genuine and natural forms of assertion is both direct and indirect even in substance If I affirm 'The Czar of Russia can throw Europe into a blaze by lifting his finger' I am judging both categorically of the historical individual, and necessarily or inferentially of the wielder of enormous forces And the same holds good in some degree if I speak of the British Constitution, or of the force of gravity. The educated mind sees an argument in judgments dealing with these matters even without the help of vocal accent and inflexion which can be used to drive home the inference From the individual judgment then, through the generic, as far as the pure hypothetical which has already been discussed in this context, we have implicit inference which verges upon explicitness in proportion as the operative ground or reason is more clearly set out in the subject-idea

[1] See Dicey on the Law of the Constitution I may venture to remark that works of this class are a valuable study for logicians, because they illustrate forms of necessary connection which are not dependent on geometrical perception

The generic judgment shows the union of the two types in its fullest significance The individual content here claims to be a presupposed qualification of reality, and therefore, as reality, has the predicated content *directly* identified with it, while the very same self-complete organisation which entitles the subject-content to be taken as real, also enables it to demand the predicated content as a necessary consequence, and to act as a middle term attaching this content *indirectly* to reality 'Poetry is a form of art which employs ideas as the medium of representation' Here we are at once qualifying a reality and drawing an inference

iv. These principles may advantageously be elucidated by the example of what might be called the true immediate inferences, which may properly be mentioned here on the threshold of Inference I refer to such processes as Recognition, Abstraction, Comparison, Identification, Discrimination. All these titles are obviously drawn from characteristics which in a certain sense no judgment or inference is without, and which reciprocally imply one another[1] But they also can be and are used as names of processes, of cases of the judging activity, in which one or other of its aspects asserts itself *par excellence* They are cross divisions to the progressive stages of judgment which were described in Book I, and might be spoken of, though not with equal appropriateness, as present in all these stages short of disjunction. They are separated from one another and from other forms of judgment rather by practical and methodological than by strictly logical distinctions

The true immediate inferences

a. Comparison is a good example The Comparative Judgment, as described in Book I, fills an important place in logical evolution. The variations of a common quality between more and less are the simplest explicit case of identity in difference But the reflective comparison of common life both stops short of and goes beyond what I have called the comparative judgment. Comparison in the ordinary sense is a name applied to the intentional cross-reference of two or more given contents, in order to establish, between those contents *as given*, a general or special identity, difference,

Comparison.

[1] See Introduction, sect 5

or partial identity (likeness) And with the establishment of a relation of equality, or of quantitative difference which implies equality ($a > b$ implies $a = b + x$), popular comparison *diverges* into equation, in which the cross-reference is retained throughout The equation is essentially *comparative* You cannot say 'a is equal' any more than you can say 'a is the same'[1] In Comparison, identity &c is stated as a result, or else very strongly implied, in an abstract form If it were made concrete and definite the cross-reference to the contents *as given* would be superfluous or impossible, other and profounder standards being introduced and the contents having no longer their original shape The result required in comparison is such as 'A is like B' If we say 'A and B are both red', this too is comparison in virtue of the cross-reference implied in 'both' But if we say 'A is red and B is green' we are passing out of the process popularly called comparison into ordinary investigation, aimed not at a particular cross-reference, but at developing the facts which may come to hand And if we go to 'All a (including a and b) are coloured surfaces', the original data have disappeared, and comparison in the popular sense has become impossible When the process has justified in the concrete the abstract idea which guided it, it has put an end to its own *raison d'être* and passes into the normal operations of knowledge

Abstraction

β Abstraction, again, affords an example worth considering Abstraction in general is the necessary consequence of definite thought, and indeed of all definite activity. All activity has its restrictions and limitations, selects and omits, and is so far abstract. But though all thought is abstract, yet all thought need not be abstraction as a special process Abstraction in this sense is a methodic activity guided by a special reflective idea, the idea of obtaining the part out of the whole by omission of other parts The whole is theoretically always, and practically often, more knowable than the part. It is easier to say that $99 \times 5 =$ five hundreds *minus* five units, i e 495, than to multiply out ninety-nine by five Subtraction may be regarded as the specific term for abstraction when the latter deals with the parts of a homogeneous or

[1] Cf Sigwart, 1 p 85, E Trans. 1 p 69

quantitative whole The hydrostatic explanation of the cup which retains the juice in a fruit-tart is an example of abstraction [1] which obtains knowledge of one aspect of a heterogeneous whole by omitting all the rest But the examinee who added that for the atmosphere to sustain the liquid within the cup it was necessary that the cup should not be more than thirty feet high had passed from mere abstraction within the given whole to independent consideration of the hydrostatic relation involved in the example

Abstraction, then, like Comparison, when considered as a method *par excellence*, is one of the processes by which Reason, armed with reflective ideas, breaks into concrete data in search of the unity of the universal The reflective idea which guides it is the equivalent in general knowledge of the mathematical axiom that if equals are taken from equals the remainders are equal Withdraw a known relation from a known system of relations, and the relations which remain are known It is plain that if the whole and its internal relations are really known so as to justify such a process, the withdrawal is a mere intellectual or ideal distinction This is so even in mathematics To know the difference between two quantities is the same as to know the greater as the sum of the lesser and the difference An algebraical sum treats subtraction and addition as on a level Abstraction would thus seem primarily to restrict itself to instances where, as in mere numerical conceptions, the withdrawal of a part leaves the other parts unaffected But as this is never within any real whole theoretically the case, although by compensation or in loose-knit wholes it may seem to be so, the instances envisaged by abstraction occupy in truth no separate region from those which form the matter of all definite knowledge Thus the guiding idea of abstraction is only a provisional idea It amounts to no more than this, that within known wholes

[1] In saying this, I do not mean that the system of laws which an investigation, beginning with such an abstraction, ultimately brings to light, must be more abstract than the example which is the datum The semi-logical and almost arbitrary character of these methodic processes as popularly limited is illustrated by the fact that abstraction, as in the case before us, so easily slides into systematic construction which leaves the example behind

known changes may appear to leave remainders known *as unchanged*. For to say, as we said above, simply 'known remainders', really lets in all that positive knowledge can tell us of the positive effects produced by the change on what remains. In this we go beyond abstraction. The supposed unchanged remainder, then, is predicated of the whole as modified by the withdrawal of some parts.

But really of course the abstraction is not what operates. Neither real nor ideal abstraction can help except by conferring or illustrating knowledge of the real whole in question. 'The Parnellites are chief men in Ireland, and were Ireland separated from England would be chief men still.' But *would* they? The abstraction puts the question, but does not answer it. The answer depends on our knowledge of Ireland. 'He has lost his wife and yet goes on much as before, therefore her loss has made no great change in him.' But perhaps in removing one motive to his habitual acts the loss supplied another. The inference even from this *actual* abstraction is utterly baseless except as a conclusion from our knowledge of the whole man, to which of course the new fact created by the actual abstraction must contribute. But had we had such knowledge before, we could have gone to the conclusion without the actual abstraction, and apart from such knowledge we cannot go to the conclusion on the basis of the actual abstraction.

From the difficulty of bearing in mind the necessity, often extremely obscure, of this circuitous route through the nature of the whole, and the inapplicability of mere subtraction in the complicated relations of non-mathematical reality, abstraction is perhaps the most fruitful in mistakes of all methods of knowledge. Knowledge in fact is one, and any method which consists in the exaggeration of a mere characteristic of knowledge is *ipso facto* hazardous.

Recognition

γ. *Recognition*[1] is another of these curiously limited pro-

[1] I restrict recognition to the elementary meaning of knowing *again*. The 'recognition' of a right or a principle, i.e. the admission of it, has interesting connections with the former case both in Logic and in Philology, and illustrates the ease with which these 'processes' pass beyond their normal sphere into knowledge in general. But it is truer to usage to regard this latter import as metaphorical.

cesses In its complete form it appears to be reflective reproduction under the influence of an idea of identity, followed by comparison and identification of the content reproducing with the content reproduced Recognition differs from Perception and from Inference as such both by dealing with a reproduced content, and by always ending in a direct comparison of contents We do not speak of recognition either where there is no reproductive process, or where the process, though it may establish identity, does not end in direct comparison When we meet a friend whom we see every day, there is no *process* of reproduction , the extension of the sensations is given along with them and the apprehension of his identity is a datum of perception For true recognition to take place, it would be necessary that the first datum should create a second, on which two data the further process would operate But in Inference the two data may just as well be given , and this is also the case of course with mere Comparison On the other hand, if I ask for a tune of which I know the name, but fail to recognise it when it is played, then I have inferential identification without recognition. For of course I know, supposing that I am confident in my recollection of the name and in the pianist's knowledge, that it *is* the same tune which I asked for , but, when played, it fails to reproduce the desired effect in my mind, and either there is nothing to compare, or if I compare the tune I hear with my idea of the tune I wanted, the result is distinction and not identification Thus recognition is absent, though *inferential* identification is present. Inferential identification, however, though ever so circuitous, *may* set up a direct comparison ending in identification, and if so, then we have recognition This is too common an experience to need illustration.

The reflective influence of the idea of identity may be active in recognition to very different degrees, and the idea itself may be suggested in very various ways Probably these ways may all be included under imperfect reproduction. An interest in identification is necessary to make the idea work , but an interest can only operate in logical thought by attaching to a suggested content Our interest in recollecting a man's name operates through the natural but unsuccessful efforts at

reproduction, in which a prominent syllable of the name, or the like, occurs to us And like the rest of these methodic processes, recognition loses its differentia when the abstract relation between the special contents in question ceases to interest It is recognition to say 'That is the man who was with me in the train yesterday' It may or may not be recognition to say 'That is Professor Huxley', for this is a matter of fact which I may infer otherwise than by direct comparison, and which may not at all be meant to indicate an identity with a special content reproduced in my mind And when I go deeper into knowledge and say 'Professor Huxley is one of the leaders of scientific thought in Europe' I have altogether got beyond recognition pure and simple, because the interest is no longer that of mere identification but of concrete description

Discrimination, &c.

δ Discrimination and Identification, and many other methods or processes, might be analysed in the same way All of them are in one sense characteristics of Inference or Judgment as such, and therefore enter into each other and into the various processes which have just been described. But each of them may also be regarded as a special though transitional method, guided by a more or less reflective idea of the result to be obtained, and subsuming under this reflective idea all matters in the content which are favourable to its purpose Discrimination or Distinction is present in all judgment, in all inference, in all comparison, and in all recognition But it would be pedantry to deny that we constantly set to work upon a presented content or two contents as yet unexamined, with the clearly envisaged purpose of making out a contrast or difference which we expect to find between them Two Acts of Parliament on the same subject ought to deal with different aspects of it, and we may fairly set ourselves to distinguish the purpose and provisions of the one from the purpose and provisions of the other What we have to keep clearly in mind is that the name Distinction is a title drawn from a merely dominant and not exclusive characteristic, that it therefore is not a desirable basis of logical discussion, and that the process of Distinction itself is transitory, because it can only continue as such so long as the result is abstract, and

so long as our interest attaches rather to this abstraction than to concrete and material content

Inference in above processes.

ε There are thus two principles which limit the inferential and non-inferential character of the practical processes which we have been considering. As processes guided by reflective ideas, they must necessarily involve grounded selection [1] resting either on presupposed subsumption [2] or on general connection of content And when we have grounded selection, we have, as we saw, the essentials of inference—we have at least a suggested distinction between direct and indirect reference to Reality On the other hand, the abstract and therefore accidental character of the controlling ideas renders it impossible that explicit inference should form the essence of these processes The moment we really found our argument on an explicit ground going deep into the nature of the subject we get a conclusion that must go beyond mere identity, likeness, or distinctness, which with one modification or another, but always in more or less abstract form, are the guiding ideas and interests of these subordinate methods of knowledge

Comparative Science.

ζ If the above processes, including Comparison, are arbitrary and vanishing phases of knowledge, how do we come to speak of Comparative science ?

The Comparative sciences are the sciences of organic and intellectual evolution through its varied series and ramifications Their data are thus, in the first place, *actual*, independently of the operations of the science, and in the second place are essentially types relative to definite functions, and

[1] For the guiding idea operates through a selection within the content.

[2] When I recall a man's name on seeing him, this recognition is not based on a necessary connection of content But the accepted identification or subsumption of the man under his name in which I rest when the name is reproduced is as good *ad hoc* as such a connection The reflective idea of identity guides me to select characteristic marks in the presented content, which I subsume under that idea 'That gait, voice, gesture, is surely a help to his identity' Then if I succeed in reproducing anything not present, this reproduced content goes up to fill up the idea of identity

'That face of his I do remember well,
But when I saw it last it was besmeared
As black as Vulcan in the smoke of war,'—

and then a whole history comes up and mere identity gives place to description of character

so not as a rule capable of being illustrated by the results of *direct*[1] interference. Hence it follows that the sciences in question (i) begin with cross-references between their actual data—the method of ordinary comparison—and (ii) retain their data untransformed in these cross-references—a leading peculiarity of ordinary comparison.

On the other hand, the abstract ideas of identity, difference, &c which guide ordinary comparison could not form the content of any science, and the comparative sciences go beyond 'comparison' by seeking for definite concrete principles of evolution and affiliation between the types with which they deal.

All science, of course, compares, but chemistry, for example, is not 'comparative' in the above sense. It does not begin by cross-references of mercury to carbon and of carbon to gold, as philology does with Latin and Greek, and Greek and Sanskrit Chemistry has to create its regular series of phenomena by experiment before it can lay down principles that connect them, and each series at first concerns the nature of a single group of substances only The data, as data of science, are not actual And chemistry does not in its generalisations retain its data untransformed The underlying principle, the molecular or atomic hypothesis, is the essence, the element of rationality and of interest In its results, as at its starting-point, it would be sheer distortion to call chemistry a comparative science of elements and their compounds It is an analytic enquiry into the fabric and behaviour of matter The elements and their compounds have no individual or characteristic value like that of a language, or a polity, or a group of myths In short, in the sciences which are analytic *par excellence* the rationality and interest are on the side of the underlying principle, while in comparative science the underlying principle serves rather to connect and illustrate realities which have independent functional importance Science is one, and these distinctions are matters of degree. But even should chemistry ever succeed in representing its data as evolutionary products of an intelligible process and

[1] Variations of animals under domestication are hardly for this purpose to be set down to *direct* interference

so as thenceforward challenging comparison *ab initio*, still this will be an ultimate achievement and not a method pursued throughout Geometry, as we saw in treating of the quasi-generic judgment, mimics evolutionary procedure with some success But its data in their pure form are really made, not given as realities of independent significance

4 I will now attempt to exhibit in their true light some species of Inference, each of which has in turn been erroneously identified with its principle

Species of Inference which have been treated as its principle

1 Induction in Mill's sense of the term, i e Induction by incomplete enumeration, or inference from particulars to particulars, is obviously to be identified with the species of inference in which a confused or implicit universal, indicated by a common name, is the ground in mediate assertion respecting concrete things or events I do not mean to examine here the case of Induction by complete enumeration, which has in fact been sufficiently illustrated by the analysis of enumeration in Book I[1] It is enough to remark that if this Induction really relies on the completeness of its enumeration, it ceases *ex hypothesi* to be Inference If, again, it relies on some discovery made during the enumeration, then the *completeness* of the process is without influence on the result.

From particulars to particulars

In the Induction by incomplete enumeration, or inference from particulars to particulars, in which Mill finds the fundamental process of inference as such, there is apt to be at first sight nothing at all which binds these particulars together The pervading identity or universal, which we affirm to be the operative power in inference, often appears in popular practice as in Mill's theory, to be simply non-existent That is to say, it either creeps in under the shelter of a mere common name, or may even be absolutely ignored in the expression of our inference, because the common name which would express it is presupposed, or perhaps is not known or does not occur to us 'Why do you think A likely to be a good scholar?' 'Well, because B and C and D are good scholars' Here it is plain that something known to both speakers is presupposed and not expressed, perhaps for example the fact that A, B, C, and D were educated at the

[1] See Bk I chap iv

same school But often the operative identity is left unexpressed not because it is clearly understood, but because we do not *think* it definitely at all 'Why do you think that picture is by Mantegna?' 'Because it reminds me of some pictures of his in the National Gallery' Here the words 'reminds me' appeal to a merely psychical fact,[a] and express in doing so my inability to produce a distinct formulation of the ground on which I have gone

Thus we are presented with something like an antinomy Identity is necessary to Inference, but some Inference takes place without Identity

The explanation of this contradiction, as distinguished from the logical justification of Inference from particulars to particulars, is afforded by what has been said about the ultimate nature of Inference The 'particulars' are not particulars, but differences in a universal The universal, however, which in elementary reproduction operates unconsciously, may in elementary inference be very far from explicit in thought, or, and this is by far the commoner case, there may be an obvious deep-seated identity in the nature of the concrete instances, which is not in its entirety relevant to the attribute about which we draw our conclusion Then, in accordance with the principle of analogy, we follow the dominating identity, and come to a result the *precise* or relevant ground of which we are unable to ascertain The conception of inference from particulars to particulars is thus an illusion arising from the activity in inference of presupposed, superficial, or unanalysed universals

Subsumption. 11 Subsumption is the complement of inference from particulars to particulars. I speak here as above of the natural and normal process, and not of the process by completed enumeration, which is devoid of inferential character Subsumption is based on the conjunction of attributes in the actual concrete nature of a subject or subjects The identity of nature which is implicit in inference 'from particulars to particulars' is here made explicit in the content of an individual or indi-

[a] I suppose it may be either a psychical fact or an indefinite meaning —either a mood of mind, or a something in the actual painting which I see but cannot express in words

viduality But this identity, though seldom wholly destitute of inferential significance, is in respect of the conjunction of attributes within it a confused and not a scientific concrete The connection of the attributes is proved by it not as a principle but only as a fact Of course, however, an inference which is really matter of principle may borrow the shape of subsumption, and in doing so may or may not continue to imply a principle that really goes beyond subsumption The relation between inference from particulars to particulars and its complement subsumption is thoroughly illustrated by Mill's discussion of the subsumptive syllogism Putting aside the notion of a *petitio principii*, which only applies when the major premise in Barbara is regarded as a complete enumeration, we find that the major premise consists in an explicit enunciation of the common nature which really warrants the conclusion Mill regards this enunciation only in the light of a summary of particulars, and as we have seen, the facts of rudimentary reproduction and even of elementary inference bear him out in the view that it is not indispensable The point however is, that though the enunciation itself is not indispensable, yet the operation of that identity which the enunciation formulates is indispensable It is this which, in the form of an ideal content considered as a subject qualified by attributes, is the point of union in subsumptive inference. Here again the nature of the active universal determines the inferential form

Calculation and Equation.

III. Calculation is a divergent form of subsumption, in which, by passing through the stage of complete enumeration, the universal operative in the inference has been transformed from an ideal content existing *in* individuals to a totality where parts are units The concrete individuality, i e the common generic nature of the individuals, has faded away by abstraction into a mere denomination of units, and the attributes which were conjoined within the generic content have also become denominations of the numerical wholes These numerical wholes which have arisen out of the 'extension' of the ideal content by means of enumeration are related to each other as measurable parts and wholes in the system of number Thus the subsumptive syllogism, 'All Englishmen

are Europeans, all Londoners are Englishmen, therefore all Londoners are Europeans,' may be seriously taken in the aspect of extension, which through the affinity between the individual and the unit is always closely allied to the aspect of number. But to carry this aspect to a genuine result we must not simply say 'Englishmen = English Europeans', &c &c, for the insertion of 'English' in the predicate makes the sign =, which implies restriction to the aspect of number, superfluous and meaningless And if we do not use =, but retain the copula 'are', then the repetition in the predicate goes a long way to destroy the meaning of the judgment by reducing it to a tautology.

Calculation proper

a If we seriously intend to draw a conclusion from the relation of individuals as units, i e apart from their content except in so far as it distinguishes them into groups, we must first constitute each of our wholes into a numerical whole by complete enumeration, and then refer these wholes to one another in respect of their *measurable* identity or want of identity, i e equality or inequality, which latter, as we saw above, being assignable as an exact difference, involves the former Then we should get something like Englishmen = $\frac{1}{20}$ Europeans, Londoners = $\frac{1}{4}$ Englishmen That is to say, Londoners, *numerically considered*, are a part that repeats itself four times in the numerical whole of Englishmen, and Englishmen numerically considered are a part that repeats itself twenty times in the numerical whole of Europeans Thus the same numerical whole presents itself as thoroughly identical with itself in its differences or different relations, whether as the number of Englishmen, obtained by simple enumeration, or as four times the number of Londoners, a relation obtained by comparison of enumerations and analysis of a sum into its parts, or as a twentieth part of the number of Europeans, a relation obtained in the same way as the last mentioned The quantitative universal, which is the same in kind throughout all its different aspects—not distinguishable as a subject controlling attributes and as attributes that severally do not exhaust the subject—is thus an embodiment of the rule, 'Things which are equal to the same thing are equal to each other' Calculation is mediate judgment, in

which, from the nature of the whole that operates, there is no distinction between subject and predicate

It is obvious therefore that Calculation cannot be applied to wholes that consist of heterogeneous or non-quantitative parts, or at least, if so applied, can only deal with them in their aspect, probably a subordinate one, of homogeneity. If the number of lines in Macbeth is $\frac{1}{30}$th of the number of lines contained in all Shakespeare's plays together, what then? The relation of Macbeth to the other works of Shakespeare's mind must be expressed by other universals than this There have always been logicians whose gaze has been fascinated by the simplicity and certainty of calculative processes, but it is idle to place the ideal of argument in a type which depends on the relations of identical units The tendency to acquiesce in this ideal has no doubt been strengthened by the absolute reciprocity of the equational judgment, which has been explained above This reciprocity anticipates, though at a long interval, a prominent attribute of notional definition or of any complete and concrete knowledge, and this anticipation of a characteristic which is rightly ascribed to the logical ideal has increased the attractiveness of computative or quasi-computative processes as types of logical method

Equational Logic

β The importance of the Equational logic is so great that I will return for a moment to its principle, which has been cursorily alluded to on the previous page The point of logical interest in regard to it is that it is not at first hand a calculus at all, though it is a calculus, and a very effective one, at second hand In this respect it is of the same grade as the calculus of chances, with which it is closely allied By saying that the logical calculus is not a calculus at first hand, I mean that the judgments with which is deals are not judgments that embody numerical or quantitative relations as such, and therefore, as was shown above, have no fair claim to the sign = as copula This is absolutely clear of the judgments which Jevons calls Partial Identities, in which the employment of the sign = is not in accordance with usage. It is not intelligible to say[1] that Iron = a metal, or that

[1] Principles of Science, p 40

Diatomaceae = a class of plants, or that Mammalia = a class of vertebrates These judgments are obviously subsumptive judgments, intended to express the conjunction of certain attributes in certain individuals, or else the identity of certain sets of individuals under certain different aspects or descriptions But the sign = does not express this conjunction of attributes or identity of individuals, except as the attributes or identity of a quantitative whole, and as on the other hand the judgment suggests no obvious quantitative aspect, weight or number or value, in which identity can be asserted, it follows that the form of the judgment simply contradicts its content, i e the judgment is not intelligible

With Simple Identities the case is somewhat different It is not indeed intelligible to say that 'Lord Salisbury = the Prime Minister of England', or that 'St Mary's Church at Oxford = the University Church' [1] But it is intelligible —I now take Jevons' examples of simple identities—that 'The smell of a rotten egg = that of hydrogen sulphide,' and that 'The colour of the Pacific Ocean = the colour of the Atlantic Ocean' And the reason why it is intelligible is this It is possible to interpret these judgments as establishing identity of degree [2] in a quality capable of variations, i e quantitative identity or an equation of colours or of smells. And by a metaphorical usage based on this fact identical *conceptions*, though not strictly quantitative, are sometimes said to be *equivalent*, and this equivalence is rudely symbolised by the sign of equality But the point to note is that equivalence cannot be affirmed on the ground of individual identity It means equal amount and kind *of intension*, and does not

[1] Unless we meant to affirm that Lord Salisbury when Prime Minister retains, for example, his normal height and weight, or that St Mary s when considered as the University Church suffers no diminution of size. It might be urged that this is worth considering, for of Merton Chapel, if I remember right, a parallel assertion would be untrue, the choir being a college chapel, and the church as a whole a parish church

[2] Jevons does not in the least distinguish true intensive equation, as in a colour match, from identification of individuals or classes of things in a sense quite beyond quantity He fails to distinguish Quality from Kind or Individuality Hence 'Deal = Landing place of Caesar' is to him a 'simple Identity, no less than 'Colour of Pacific Ocean = Colour of Atlantic Ocean

mean identity of component individuals Thus it is simply false that 'Exogens = Dicotyledons', for the meanings are not equivalent, though the individuals designated by the names are, *exceptis excipiendis*, the same.

In the first instance, therefore, the judgment as formulated by the Equational logic is not an equation, because it does not restrict itself to a quantitative aspect, but predicates individual identity Although you say that Diatomaceae = a class of plants, you may not go on to say that this class of plants = another class x (meaning that the two are equal in number), and that therefore the Diatomaceae = the class x. The original judgment might indeed *happen* to justify this calculation, but in passing through it would have entirely lost its peculiar import

The office of computation in the Calculus is not to compare quantitative attributes of objects, but to secure complete enumeration of possible judgments In this office of secondary import it somewhat resembles the translation of a material disjunctive judgment into a numerical statement of chances. But the statement of chances gives a numerical result, whereas the logical calculus, after protecting itself by a computation of combinations, returns to a result in the shape of identification or identifications Working as it does solely by the identification of individuals under different aspects, i e. as conjoining in themselves different attributes, the equational logic is obviously a species of subsumption, and rests ultimately on the subsumptive principle that attributes conjoined in the same individual are conjoined with each other. Thus in regard to the nature of the universal which is operative in inference through the calculus, there is little to say beyond what has already been said in dealing with subsumption. The only peculiarity of this species of subsumption is that in it the undefined capacities of subsumption as such for conveying connections of principles are cut down to the most abstract expression of individual unity, without being allowed to develope in the direction which such abstract unity naturally suggests, viz that of participation in a numerical whole.

IV 'Construction' [a] is a term frequently applied to a method

[a] Cp II 319 note below on the relation of this passage to vol I, p 42 ff

Construction Physical

or element of inference. *a* It primarily indicates an auxiliary process employed in geometrical or mechanical reasoning. This process consists in making accessible to perception a geometrical or mechanical complex of relations which embodies a problem or theorem that is under consideration The actual physical construction—a diagram on paper or a model or experimental machine [1]—though not ideal, but an object of sense, is nevertheless abstract in its sensuousness, being purposely cleared of the irrelevancies which encumber our ordinary perceptions And further, the nature of space is so closely related to sense-perception, that ideal spatial relations can be adequately symbolised by actual figures presented to perception, although the ideal relation underlying a theorem is always both *more* and *less* than the visible or tangible lines and points. 'More,' because the visible lines can be but a case of the ideal relation; 'less,' because the perceptible lines, though reduced to black on white, still include errors and irrelevancies which the mind in working with them disregards The case of a working model or machine made to experiment with is at first sight different from that of a mere diagram. But the difference is only in degree. Both the diagram and the machine are really *in pari materia* with that which they represent to the mind, the diagram with ordinary perception, the machine with ordinary physical causation But it is harder to say where the line is to be drawn between fact and representative of fact in the case of the machine, which shades off gradually into the ordinary operations of nature Such a thing however as a working model [2] that illustrates the lever action of the limbs shows how the machine as such stands between natural process and abstract mechanics It is the abstract *physical* expression for

[1] An excellent example of construction in this sense, the solution of a problem in actual physical embodiment, is the machine for drawing sound curves of any shape by actual pendular oscillations There is such a machine, I think, with actual pendulums, which thus does not merely *mimic* the curves (as the machine with cog-wheels may be said to do), but actually constructs them in terms of the theory

[2] A complete model of the ear or eye may seem not to be abstract, but only magnified, it is however abstract by its isolation from other parts, its fixity, and its capability of being taken to pieces

a natural activity, and paves the way for its abstract ideal expression

Construction in this first and simplest sense is not even a case or species of Inference The production of a visible material figure or object does not even, strictly speaking, *enter into* the essence of the inferential process. It is however a peculiar auxiliary method which depends upon and throws into relief the characteristic nature of the universal—the abstraction of sense—with which Inference is concerned in Geometry or abstract mechanics The lines drawn on paper, though peculiarly adequate symbols, are nevertheless only symbols of the lines with which geometry actually works And of course we do not draw the lines on paper at random, and they would be of no use if we did. We only draw them in accordance with the requirements of the universal operative in the inference, so far as these requirements are already known and can guide us from moment to moment The aim is to aid intellectual reflection in fixing and following connections which suggest themselves within that universal, and as we have seen, the material lines or even cords, pulleys, levers, &c. *represent* an intellectual work already partly accomplished in the exclusion of irrelevant elements But the material or physical construction *is* not the active element in the accomplishment of this work This construction in the sense of actual drawing or model-making is a process *characteristic* of geometrical or mechanical inference, but not identical therewith.

Imaginative.

β. But this brings us to a secondary sense of construction —secondary not in logical value, but, as I think, in usage and the growth of meaning Of course actual lines on paper are not essential to simple geometrical inference in which we can 'carry the figure in our heads', and if so, the visible external figure cannot in theory be essential to *any* such inference Professor Clifford, it would appear, was almost independent of external aids to realisation in considering geometrical or physical problems But when we work with ideas of lines, and combine spatial elements *in our imagination*, and not on paper, is not this still construction, and yet is it not now of the essence of inference?

I fear that at this point a further refinement cannot be

avoided. It seems to me obvious that the imaginative experiment is a different thing from the intellectual perception of unity. The imaginative experiment may be misdirected and void of result, as a line drawn on paper in addition to an actual diagram may be irrelevant and meaningless The imaginary line projected on imaginary paper, though necessary perhaps to the intellectual apprehension, is it seems to me purely on a level with the external line perceived through sense But when any line either given on paper or suggested in imagination has been intellectually grasped as symbolic of a relation relevant to the universal which we are developing, then it is taken up into the inference and has passed from a step in physical or imaginative construction to an element in necessary apprehension In looking for that point in an ellipse in which the sun had to be placed in order that the ellipse might represent a planetary orbit according to the theory of gravitation it is easy either in imagination or on paper to select the middle point of the longer axis The question as to the nature of the process is not whether I draw or only imagine, but whether in doing either one or the other I am guided by insight into the connection of the data and into the conditions of the problem. If I seem to myself to have such an insight, but am, as in the case above-mentioned, mistaken in the connection which I fancy that I detect, then I infer, but wrongly. If however, or in as far as, I put the sun in the wrong place or in the right place by an isolated act of sensuous fancy, then I may be constructing, but I am not inferring

Intellectual, in Geometry and Mechanics

γ. The organised or articulated intellectual perception itself, in contrast with the physical or imaginative experiment—or rather taken as including this experiment, for the sensuous ideas cannot be dispensed with—affords a third meaning which may be put upon construction. Of course this perception has the effect of a synthesis, of a putting together as well as of a distinguishing. And especially when the relations thus put together as having their unity in one universal are spatial relations, readily symbolised by imaginative pictures, there is a plausibility in translating the Greek term 'synthesis', which in technical logic expresses the unity of differences inherent in all intelligible judgment, into the

term of Latin origin 'construction', which may be held to express this same idea with the additional implication of *intentional* operation *in time*, by which elements of intellectual perception are put together like the parts of a machine.

Construction in this third sense, a sense largely insisted on by Kant, is a hybrid idea. I cannot doubt that it designates a true species of inference—inference dealing with the abstractions of sense—by a title actually drawn from and implying an accident of inference, viz *intentional* combination, successive in time, of visible or imagined forms Now the *intention*, except in as far as it is a general intention to infer correctly, falls outside inference In inference nothing can guide us but inferential necessity, and a special intention, such as that of drawing a line on a slate or on paper, or of constructing a model of a joint, arises in some motive external to the inference proper. And as with the intention, so with the fact of material construction It is a mere accident of inference. Therefore the intentional combination of perceptible or imaginable elements falls outside inference proper except in so far as it is guided by inferential grounds But it is true that, especially wherever we have novelty or discovery in inference,[1] imaginative or perceptive construction is an indispensable auxiliary to thought

δ And finally, by insisting on the ideas of synthesis, of intention, and of the value of imagination in inference, we arrive at a fourth meaning of Construction,[2] in which it is alleged to be an essential element or even the essence of *all* inference, as an *intellectual* combinative process, *not* confined to the region of geometrical or mechanical abstractions. Thus understood, Construction becomes a convenient expression for the distinct realisation in inference even of universals which are not numerical or geometrical, if they are more than subsumptive. Transferring a spatial metaphor to such universals, it appears to explain the definite necessity which unites their differences, by reference to the precise and unambiguous coherence which belongs to geometrical relations. But we have seen that even as applied to geometrical inferences

Intellectual, without limitation to Geometry, &c.

[1] See p 8, supra

[2] Bradley's Principles of Logic, p 235, and passim

construction is to a great extent a metaphor drawn from an accident of those inferences, and when we deal with universals which are not at all sensuous the simplicity of the expression becomes actually deceptive It is easy to say, for example, that we intellectually 'construct' such a whole as the British Constitution out of isolated facts, principles, and accepted ideas, and then proceed to perceive its nature But it is plain that anything valuable in the 'construction' is coincident with and guided by the growing insight into the nature of the content before us which constitutes the inference itself We must distinguish from this usage that by which we speak of constructing a Utopia Here of course we are ostensibly not inferring, though we have really to use inference We are playing, employing the imagination according to arbitrary interests and motives, and not under intellectual guidance, until we begin to draw consequences from what we have said. And the root of the whole usage which we have examined is that in all inference, so far as it has novelty or is a process modifying itself in time, there is an arbitrary and external element which supplies guidance to the attention until the true principle and the relevant details have been disentangled, and operates throughout the inference by the side of the true principle which gradually displaces and finally ousts it. This arbitrary element may indeed be regarded as the universal itself in an imperfect form, but for this very reason it contains much in the way of suggestion or experiment [1]—much tentative synthesis—that has to be dropped and erased before the inference can assume its true and final form And the name construction depends largely on the elements which, having been intentionally and therefore tentatively inserted, are ultimately dropped

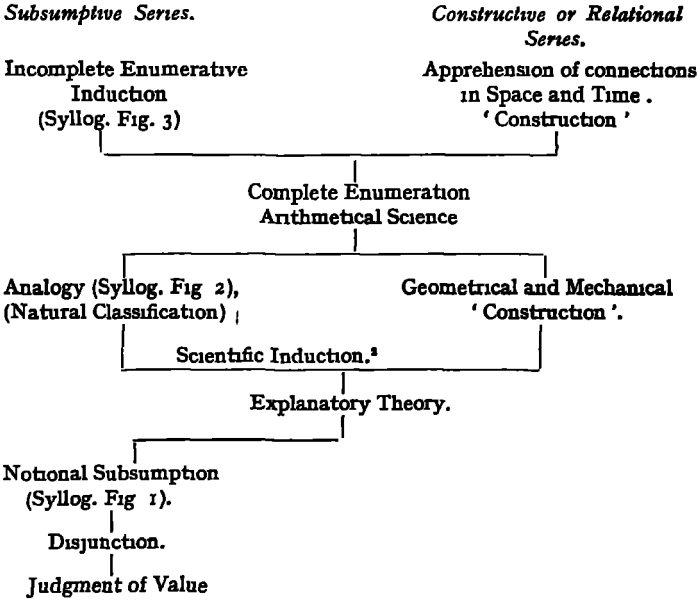
The outcome of our account of Construction then is this. Construction is a term drawn from moral, physical, and psychological adjuncts of inference In the case of geometrical and mechanical inference these adjuncts are so far akin to the matter of the universal, that the term 'Construction' drawn from them may be held a fair designation of such

[1] Cp the case above cited in which the focus of the ellipse was the point required, but the middle of the longer axis was the first point selected

inferential processes, e g of Kant's account of matter on the basis of attraction and repulsion. In the case of other and less sensuous universals this is not so, and Construction as applied to them is a mere metaphor and not even a case of Inference. Thus Construction should never be assigned as the essence or as an essential element of Inferential activity.

The above account of four main cases[1] of Inference may serve as an anticipatory sketch of the course which our discussion will pursue in the following chapters. (See scheme annexed)

SCHEME ILLUSTRATING AFFILIATION OF ARGUMENTS AS DESCRIBED IN BOOK II.



[1] Or five, counting among them the process which Construction in its fourth sense is abusively applied to designate.

[2] Scientific Induction is treated as a transition by which Explanatory Theory may be reached from the side of Analogy The scheme is intended to represent each type of inference as a combination of that vertically above it with that with which it is connected by a horizontal line This does not apply to the relation between Explanatory Theory and Complete Enumeration, which stand where they do merely as being intermediate forms

APPENDIX TO CHAPTER I

On the Relation of Symbolic Logic to the Theory of the Present Work.

I cannot pretend to give an adequate account of Symbolic Logic in its present development But there is a matter connected with it which readers of this work may rightly demand some attempt to explain, and on which, moreover, the co-operation of students working from the side of philosophy (or, if this is held a question-begging phrase, from the side of a monistic philosophy) and from the side of Formal Logic respectively is highly desirable I mean the ascertainment of the precise point of divergence at which Formal Logic, construed as including pure Mathematics, parts company from the sort of Philosophical Logic that is aimed at by the present work, attempting to follow the tradition that is one on the whole with that of Aristotle and Plato

The problem thus arising in the first place (1) on a certain point puts the theory of the present work on its defence, while in the second place (2) as it involves the conditions under which the divergence comes to pass, the answer to it would determine the reservation or no-reservation subject to which Formal Logic can be held from the monistic point of view to be true

Formal dist Philosophic Logic

1. I will first say something about the theory of the present work as it concerns the point of divergence of Formal Logic (in Mr Russell's sense indicating Symbolic Logic and pure Mathematics) from Philosophical Logic in my sense of the term

Point of divergence implied in ed 1

a The point of divergence in question, according to my account given above, is where quality passes into quantity by the transformation of the judgment into the equation Here begins, according to that account, the transformation of the wide and plastic logical idea of system and members,[1] into the narrow and rigid idea of quantitative whole and parts, passing through which it reaches the last abstraction in the shape of denomination and numerical units. The denomination appeared to be a last abstraction of the whole or system,

[1] Not yet explicit at the point in question—becoming in fact explicit by contrast with the formal framework then brought to light

the units, of the parts or members The numerical series seemed to be an ideal scheme of the relations of members within systems [a] from a certain limited point of view, that is, when distinguished only by a logical order of progression assigning every unit a place in relation to every other.[b]

Mathematics, then, seemed to be especially connected with quantity, and to be characterised accordingly by the homogeneousness of the wholes—space, time, and number—with which especially it appeared to deal, their actual characteristics being idealised to a point which made the nexus of their relations a matter for almost purely hypothetical judgments. Absolutely pure hypothetical judgment, an assertion of implications following upon a supposition which is in no way attached to an underlying real system, I do not believe to have a meaning [c]

The Logic which had the same origin as Mathematics when thus conceived, was then familiar as Equational Logic, founded upon the same transformation of the Judgment which gave rise to Mathematics itself. This transformation, when applied to Logic in general, was open to serious criticism because of the inadequacy of the equational form, which ought to indicate a purely quantitative relation, to the meaning of a true logical judgment It seemed to be neither Logic nor Mathematics [d]

[a] e g I 168.

[b] I think I am entitled to repudiate any suggestion (if really directed against my doctrine, see Russell, Principles, 114) that my view was satisfied with the psychological conception of counting the acts of counting Number was for me always a relation of ordered series, which we became aware of by counting Cp I 168, 170 And I should be wanting in courage were I not to admit, in spite of the authority against me, that though every class has a number, yet the connotation of every number seems to me to be determined by relations to a whole independent of the class which has the number

[c] See Russell, Principles, p 14 This observation affects, I should suppose, Mr Russell's extreme use of the hypothetical proposition in illustrating the meaning of implications You cannot say, I should urge, 'If a donkey is Plato, it is a great philosopher' The hypothesis scatters your underlying reality to the winds, and what I should call the basis of implication is gone In a real system, S qualifies P as well as P, S

[d] Cp Bradley's Logic, p 24 ff with Mr Keynes's defence and reinterpretation of the equational form, pp 189 90

Point of divergence on Mr Russell's view.

β. The conception of pure Mathematics has now, as I understand, freed itself from all special relation to quantity, and the view is affirmed that the science to which it applies consists wholly in logical deduction from logical principles. Thus its point of divergence from what is here treated as Philosophical Logic is not that assigned in the account that was summarised in the previous paragraph, but is rather that to which my description finally traced the abstraction It deals, I gather, not at all with existents, actual space, actual time, or actual characters of totalities represented by number, but with the formal essence of propositions—not with facts, but with the mutual implications of propositions, and these propositions containing none but logical constants. It is, in strict principle, to be defined as one with Symbolic Logic, which again is one with Formal Logic [a]

The path which its conception traverses in attaining this point may be illustrated by a quotation from Mr. Russell [b]

'*Whenever two sets of terms have mutual relations of the same type, the same forms of deduction will apply to both.*[c] For example, the mutual relations of points in a Euclidean plane are of the same type as those of the complex numbers; hence plane geometry, considered as a type of pure mathematics, ought not to decide whether its variables are points or complex numbers or some other set of entities having the same type of mutual relations Speaking generally, we ought to deal, in every branch of mathematics, with any class of entities whose mutual relations are of a specified type, thus the class, as well as the particular term considered, becomes a variable, and the only true constants are the types of relations and what they involve Now a *type*[d] of relations *is to mean*,[e] in this discussion, *a class of relations characterised by the above formal identity of the deductions*[e] possible in regard to the various members of the class, and hence a type of relations, as will appear more fully hereafter, if not already evident, is always a class definable in terms of logical constants[f] We may therefore define a type of relations as a class of relations defined by some property definable in terms of logical relations alone'

[a] Russell, Principles, p 9 [b] Ibid, pp 7-8 [c] My italics.
[d] Mr Russell's italics [e] My italics.
[f] [Mr. Russell's note] 'One-one, many-one, transitive, symmetrical, are instances of types of relations with which we shall be often concerned'.

Am I presumptuous in observing that we see here the progress towards homogeneousness of the subject-matter and pure logical implication much as it was set out in the account to which the previous paragraph referred? If two sets of terms have mutual relations of the same type, the same forms of deduction will apply to both; but to be of the same type, if I understand the passages rightly, has no other meaning than to be such that the same forms of deduction will apply. So, am I wrong in suggesting that very great and serious differences between two sets of terms may be neglected (such as differences between points in a Euclidean plane and complex numbers) if only a common set of formal deductions can be found which apply to them? This is the course that was described above, in pointing out the increasing homogeneousness of the subject-matter of Mathematics as the characteristic differences e g of space and time come to be neglected The science has a right, I should say, to define the same type of relations as it finds convenient. But in its claim to truth the amount of difference which it has omitted will have to be taken into consideration

I gather, then, that any reference to quantity or to any form of actual existence is now to be taken as excluded, and that pure Mathematics is to consist, as we said, in logical deductions from logical principles. And the province of Symbolic Logic is practically the same. It claims, as I understand, to have thrown off the limitation which offended us in the Equational Logic, and is now prepared to deal with any type of relation whatever, in no way implying a preference for statements framed in quantitative form And it is noticeable that the Symbolic shape is treated as accidental and unessential. This is more important than it seems. For the idea of a perfectly and purely Formal Logic is necessarily, I presume, as we see in the simple case of conversion, to deal with its terms as if they were units unmodifiable except in position, expressions to be combined or transposed as wholes, and without intrinsic modification[a] But the present symbolism is in itself more

[a] I am aware that in the most formal logic, when expressed in ordinary language, grammar will hardly permit this rule to be carried out But there is no doubt, I think, that purely formal procedure

adaptable, as is shown for instance by its admitting a difference of 'sense'[a] between a relation and its converse; and also, as we saw, the symbolism as a whole, which must always be a check on the change induced in terms by change of combination and inter-connection,[b] is considered unessential. Therefore the intrinsic modification of terms, demanded by a sound theory of inference, though discouraged, as I think, by the habitual use of a conventional symbolism, is not in principle excluded.

Thus the point of divergence between Symbolic Logic, including pure Mathematics, on the one hand, and what I call Philosophical Logic on the other, is, if the views just stated are to be held decisive, at the end and not at the beginning of the quantitative development of the judgment The ultimate divergence would appear to take place at the point where the theory of the hypothetical judgment has been explicitly laid down, exhibiting for the first time an embodiment of implication as distinct from mere subsumptive conjunction[c] And it would consist in isolating the relation of implication in such a judgment from the limitation imposed on it and the extensions ascribed to it owing to the doctrine that every hypothetical presupposes and depends upon a systematic whole taken to be real. Thus the limitation on illegitimate hypothesis, which forbids such an implication as 'if the number 2 were Socrates, it would be a great philosopher'[d] is swept away; and with it the suggestion that in a pure hypothetical the implication is reciprocal I give this latter

becomes uncertain in so far as a term is modified by its place in a context. The preference for predication by 'is' with a noun, over a concrete verb, is a simple case You cannot readily 'convert' a verbal predication

[a] '*sens*', direction

[b] Cp my remarks, p 202, on the changes inevitable within a systematic argument, where the meaning of the term develops I therefore deny that 'the syllogism in all its figures belongs to Symbolic Logic' Principles, p 10 In my view it contains a principle of life and growth which cannot be symbolically represented

[c] In my scheme the meeting-point of abstract relation of Space, Time, and Number, with the pure Hypothetical Judgment, and the development of the latter as an expression of sheer relativity or implication See vol 1, pp 86 and 233–4

[d] Principles, p 14

merely as an instance of a loss which seems to me to arise from a non-philosophical interpretation of the proposition in accordance with colloquial usage [a]

This then is the acknowledged point of divergence, not the establishment of the quantitative judgment, but the admitted emergence of implication as a relation between propositions If this view is to be taken as decisive, the account of quantitative judgment in the present work shows first the steps antecedent to the divergence, and next the full nature of the divergence, rather than traces the divergence forward from its critical point. To do this on the present basis would mean to give an account of Symbolic Logic, which I admit myself unable to offer.

The divergence restated.

2. In what, then, precisely does the divergence consist? Surely in this, that Symbolic Logic [b] is interested in the laws of mutual implication between propositions—the modes of formal Deduction—while Philosophical Logic is interested in the conditions of logical stability [c]

Logical stability explained.

a. I will first say what I mean by this latter conception. I understand Truth to be that characteristic of a system of propositions which makes it free from self-contradiction and from contradiction with the rest of experience This characteristic (which I take to be what Plato means by *ἀλήθεια* 'trueness') may be technically described as logical stability [d] (Plato's *βεβαιότης*). It involves the conception that every judgment is at once determined as to its meaning, and criticised as to its non-liability to contradiction, by its place

[a] Even if the proposition is taken as itself non-mental, the assignment of its import may be, and in Mr Russell's case I think is, drawn from colloquial usage. See below, p 48 I do not feel sure whether the statement that only judgment or belief, involving mind, can be true or false, is an abandonment of the above position, or a concession to popular phraseology Cp Principles, p ix, and Philosophical Essays, pp 172-3 I think the difference is unimportant for my point, which lies in the isolation of that, whatever it may be, which is true or false.

[b] I omit henceforward, for brevity's sake, to add 'and pure mathematics', which I understand to be implied throughout

[c] The sharp ruling out of Inductive Inference from the subject-matter of Symbolic Logic is very significant (see Russell, Principles, p. 11) To me, for example, it means that the ultimate principles which underlie alike Formal and Informal Inferences, are ignored

[d] See e g Rep. V and VI and notes in my Companion

in the whole system of judgments which represents our organised experience Its degree of non-liability to contradiction, internal or external, is its degree of logical stability, and I take it to be the main interest of philosophical logic, as I indicated from the beginning,[a] to analyse the forms which are generated by the impulse towards such stability or satisfactoriness, with reference to the various degrees in which they embody its achievement. The rules of inference come in rather as the modes of passing from grade to grade of logical stability than as expressive of relations between this and that or these and those propositions How and under what reservations premisses yield a conclusion, for example, is the study of the formation of a whole of relatively higher stability than its isolated data, not a mere playing a game under given rules.

Now this conception at once brings the consequence that there can be no unproved premisses or prior foundations [b] of knowledge. For the proof of every judgment is ultimately in the system as a whole; and there can be no indefinables nor colloquial interpretation of propositions in any sense of the latter term, for every proposition and every judgment takes its meaning as well as its certainty from the system. And there can be no implication as a relation between isolated propositions, for the only implication is that which at any point is necessary to the avoidance of contradiction in the system. If we intend to set out without reservations or presuppositions, and investigate what is meant by the presence of trueness or satisfactoriness in the forms of thought, this, I suggest, is the only way to go about it. We start, in this case, not from unproved premisses, but from minima of experience, which are found to lead both in truth and certainty beyond themselves, and possess, until they have so led, no more than a provisional meaning and certainty.

β. Symbolic Logic diverges from Logic as thus conceived by omitting the conception of logical stability. It takes the proposition primarily as an isolated unit—not mental—but recognises the mutual implication of propositions. Its interest lies in the study of the various general types of deduction, or 'the general rules by which inferences are

[a] Vol 1, p. 3 [b] Below, p. 266.

made', that is, I presume, the rules by which you can assert or deny a proposition of one general type when assuming one or more propositions of the same or some other type It is recognised that true and false propositions respectively have different relations of implication as regards all propositions; but I do not gather that these implications are capable of degree, so that a proposition could be more or less true because of being more or less supported by other propositions Its truth or falsehood I understand to be self-contained, a private affair of its own, not lying in its coherence or incoherence with a system.[a]

In a word, the resulting difference is twofold. As the foundation of the science you get a number of indefinables and of unproved premisses, assumed really for reasons of convenience—that is to say, with a view to leading proofs from them in the simplest and neatest way.[b] And as the goal of the science you get an enormous command over the forms of deductions applicable to propositions which themselves, though always general, are unchangeable units. You have no such phenomenon as a system operating within propositions [c] so that their ideal shape and certainty is always ahead of their given form, and makes itself felt as an impulse to self-transformation and completion in other forms. On Mr. Russell's philosophy, I presume that a proposition would not be an entity capable of embodying such an impulse (not being in any way mental) But, as I admitted just now, I do not know whether Mr Russell's philosophy is generally held essential to the logical doctrine which he shares with others.[d] In any case, whether the proposition is mental or not, and whether or no we subscribe to pluralism, the proposition for Formal Logic is a fixture, in relation with others,

[a] I regret that, owing to want of acquaintance with the general literature of Symbolic Logic, I am unable to say whether Mr Russell's philosophy is accepted by the principal authorities who agree with him about Logic His logic and his philosophy appear very precisely adapted to each other, and he thinks them essentially connected (p viii) It is certainly not easy to discuss them apart But I presume that his logical doctrine might be held on logical ground only, by way of abstraction and not as final truth

[b] See e g Russell, Principles, p 17 [c] See p 266 below.

[d] He holds that it is so Principles, pp viii and 24

but alike in truth and in import[a] independent of them. Thus there is no idea of developing a system, or indeed anything whatever, from a single point. The logician takes, as it seems to me, whatever he wants in the way of first principles without considering their relation to any single system, and he makes usage and convenience, not systematic necessity, govern the interpretation of single propositions For example, 'by help of the first nine principles out of ten axioms,[b] we can prove the law of contradiction.' And again, he does not raise, as a matter of principle, such questions, dependent for us on the ideal of judgment, as the reciprocity of the hypothetical or the exclusiveness of the disjunction, and he appears to me wholly to leave out of account the problem of a categorical basis for the hypothetical and the disjunctive[c] In other words, he takes implication as a bare and ultimate fact, an indefinable.[d]

I am not arguing against all this. It seems to me to be natural, if we either deny, or choose to abstract from, the idea of systematic coherence, about which I shall argue below.[e] And I do not see why, if we find these assumptions work in a special science, we should not make use of them as a matter of 'taste and convenience' (I extend the application of Mr Russell's phrase on p. 17 but I judge that it does represent what with him is a great influence in the shaping of his science)—whatever our doctrine of Reality may be.

Only, to those who hold a different philosophy, the assumption that you may go forward without the idea of system will always appear a reservation or special postulate, affecting the truth or ultimate logical stability of sciences which need it for their foundation.

This is the point at which controversy might arise It would

[a] We must not say 'in meaning', for Mr Russell's proposition does not have the meaning, but is it [b] Principles, pp. 16–17

[c] See above, p 273, on illegitimate hypothesis.

[d] Principles, p 14 Cp Mr Russell's explanation, Mind, 75, 375 My only point here is that implication is taken as a relation between two propositions, just as it is felt or found We are not allowed, for instance, to argue, from the nature of the system involved, on the question whether if p really implies q, q must imply p. I think Mr Russell would accept this statement [e] Cp c IX.

be foolish and hopeless for an outsider to criticise the procedure of these sciences on their own ground What he not only has a right to do, but is bound to do, is to attempt to indicate the difference of attitude and presuppositions between them and what he conceives to be Philosophical Logic. And this attempt, however modest, must in the end lead to a different opinion, or an opinion differently formulated, about the *truth* of Formal Logic in the large sense here in question.

I say, an opinion differently formulated For though Mr Russell holds that it is a merit of his philosophy to maintain the complete truth of mathematics, yet he is too widely experienced in all regions of culture not to admit the immense *prima facie* difference between the kind of truth aimed at by mathematics and that aimed at, say, by biology or by political philosophy[a] or the philosophy of religion. He must, I imagine, have his own account to give of the relation between a science which stretches so far as his general science of inference in one way, and sciences which include so very much more in another way. I should call it the difference between a very highly conditional system of propositions, dependent for their truth on enormous reservations, and systems of propositions which approximate very much more to categorical truth about reality But however this may be, every student must see the problem, and it is not for me to conjecture the solution which other students may propose.

[a] I hope the reader will treat it as irrelevant that I have myself attempted to write about political philosophy. I am referring to the place of the science as such in a hierarchy of sciences like that sketched by Plato Plato's reservations on the truth of mathematics are noteworthy.

CHAPTER II

Enumerative Induction and Mathematical Reasoning

In tracing the evolution and affiliation of the forms of explicit inference, we begin with a more developed phase of consciousness than that to which we went back in describing the rudiments of judgment The judgment-form in which the universal first reveals itself as the simple unity of differences must have given place to the articulate perception of things, events, and relations, before we can grasp it as a system whose parts are in necessary connection. In other words, we have no longer to deal with the simplest judgments of perception—the impersonal and the elementary comparative judgment—but may go at once to the world of things with their characteristic identity, and to the abstract relations of number, space and time in which that world is constructed by the mind.

Enumerative Induction

1 On the basis of a world of things and occurrences, which involves a language, i e a more or less systematised nomenclature,[1] the articulate universal presents itself in the form of individual objects having characteristic identity, in which identity general attributes coexist Language forces such coexistence on our attention through its natural classifications, and the first suggestions of inductive enumeration issue from language—which has in fact gained its significance by the same process conducted naturally and without explicit intention Such suggestions are operative, as we saw, in the so-called inference from Particulars to Particulars, of which Incomplete Enumerative Induction is a form that has been made superficially rational.

Syllogism in Fig 3

a. Throwing this inference into the shape naturally prescribed by the tendency of the concrete individual thing to

[1] Cp Introduction, sect 4

become the subject in judgment,[1] and making use, as Aristotle also did, of the conjunctive judgment as representing a single step in inference, we obtain an inference analogous to the Aristotelian syllogism in the third figure —

a, b, c, d, are rational,
a, b, c, d, are men,
∴ Are all men rational? or, Men may be rational

The ground of argument being the characteristic unity of the unanalysed individual object or event, naturally takes the place of the subject in judgment—of the concrete individual which is taken as real—and therefore gives rise to that syllogistic form in which the middle term is the subject of both premises

Experience bears out the view that some such type as this represents the simplest operation of mediate inference All that is said for example by Stuart Mill about inference from Particulars to Particulars must really be taken as applying to inference of this type, for it is impossible to state an inference in a shape that will even appear to be convincing, unless we supply by a second premise the element of unity between the particulars, always operative in the mind, which is necessary to bind the particular differences into the differences *of* a universal The use of symbolic letters brings to light this formal necessity, which significant names satisfy without acknowledging From '*a, b, c, d* are good books' to '*e* is a good book' no sort or kind of inference holds or is in any way suggested The gulf between mere *d* and mere *e* is beyond the leaping powers of the boldest conjecture. These symbols are true particulars, and we can see from them how far true particulars will carry us in inference But from 'Ivanhoe, Waverley, and Rob Roy are good books' to 'Guy Mannering is a good book' there is a self-evident passage by means of the identity of authorship, which is too obvious to be expressed, but which would form a premise in any explicit statement of the inference That this same ground would justify the conclusion 'The Surgeon's Daughter is a good book' is an illustration of Bacon's remark that enumerative Induction

[1] See on categorical and hypothetical elements in judging, Bk I, chap 1

'precarie concludit, et periculo exponitur ab instantia contradictoria'.

Speaking generally, the coincidence of several attributes in one or more objects, or of several relations in one or more events, is the usual starting-point of conscious conjecture and investigation And this starting-point is all that the present form of inference embodies Conjecture or pure 'discovery' differs only in degree from proof. Thus the conclusion may be fairly represented by a question or a modal affirmation. A, B, and C are great lawyers *and* had a classical education ∴ Has a classical education something to do with making great lawyers?[1] or 'A classical education may have something to do with making great lawyers' In the case of events, which are designated more analytically, i e more with a view to an intended inference, than individuals can be, a single judgment is the natural form for elementary arguments of this kind 'He killed his bird every shot,' i e 'Such and such times he fired, all those times he killed. When he fires, he is likely to kill' But in careful reasoning or experiment 'such and such' becomes an elaborate identification by marks

Divergent tendencies

β In this argument by incomplete Enumeration we find a union of divergent tendencies Plainly, it has no major premise, but no less plainly, the principle on which it primarily relies is the principle of subsumption Its conclusion is therefore particular or modal, and affirms that in instances or under conditions which we are not in a position to assign explicitly, the attributes x and y are or would be united. For the premises neither express precise conditions nor furnish the basis of a complete inference by subsumption In order to assigning precise conditions, the form of subsumption, i e of identification in unanalysed subjects, must be surrendered. In order to furnish a complete subsumptive inference, the universal must appear in the form of Allness, i e in the form of a totality of examples enumerated up to a limit which its nature prescribes Such an enumeration, or an allegation equivalent to such an enumeration—the allegation might be

[1] The disjunctive import of this question, as is natural with a rudimentary type of inference, is simply Yes or No

of mediate origin—would be the major premise in a pure subsumptive syllogism In the present argument as it stands we simply formulate the first impression made by this discovery of an articulate[1] identity in difference, and the first suggestion towards completing the articulation of the universal thus indicated This suggestion is most naturally to be taken, in accordance with the type of the inference, as quite abstract and superficial, consisting as it does in a single pervading attribute, chosen, so far as we are told, at random

Individual Judgment in induction

γ. It is obvious that the study of inference must retraverse, in part at least, the path taken by the study of judgment. In the present form of inference the mind is at the same critical point at which it stands when in the Proportional Judgment it has become familiar with the identity of things beneath their attributes. Only in the study of Inference we see the actual process by which the judgment-forms pass into one another—in this case, in the sphere of enumeration, by the natural pressure of their own recurrence Incomplete Enumerative Induction is an obvious result of recurrent individual Judgments, which accentuate the common distinguishable aspects prevalent in things and in their individualities—i e in their characteristic natures

In Lotze's representation of an argument akin to this, he has set out the recurrent individual judgments as so many premises of the syllogism ,[2] and, rightly recognising that the number of *such* premises is not limited, he has thereby cut the knot of the question whether inference essentially proceeds through three terms It is not worth while to dispute about a matter of symbolic representation So I have only to explain the relation of his scheme of the argument to that here given, by pointing out that his *entire set* of premises corresponds to either *one* of the two which I lay down, while the

[1] See distinction between explicit and implicit inference, in last chapter

[2] Lotze, Logik, Eng Tr , p 100 'Now it often happens that the same predicate occurs or does not occur not only in two but very many different subjects, P, S, T, V, W, and the question is what consequence can be drawn from the premisses PM, SM, TM, VM, which belong in form to the second figure of Aristotle Our conclusion runs as follows, "All Σ are M" '

relation of the individuals P, S, T, &c to the common attribute Σ forms the other of the two premises essential to the inference The use of a conjunctive judgment in inference of this type is in accordance with the principles laid down in Book I [1] as regards the unity of the judgment, and with Aristotle's practice in his Inductive Syllogism, though not in the ordinary Syllogism in figure 3 which I have treated as the inductive syllogism. By using the individuals, not in a conjunctive judgment, but as subjects to several premises, Lotze has forced his inference into the second instead of the third syllogistic figure, the only common term being the common *predicate*, which therefore must formally serve as middle term though it does not operate as such His transition from P, S, T, to Σ is effected without an explicit judgment, and the whole process is more akin to the colligation of a conjunctive judgment into a singular or generic judgment as described in Book I,[2] than to a process of Mediate Inference It is obviously more convenient to use the third syllogistic figure, in which a common *subject* is the middle term, for Induction, and to reserve the second, in which a common *predicate* is middle term, for Analogy This was the scheme followed by Hegel, and I venture to think that the deviation from this scheme,[3] like other modifications which Lotze has introduced into a logic largely resting on Hegel's ideas, is far from being an improvement

The critical point which is involved in the inference before us is the point of divergence between the concrete and the abstract forms of the universal This is fundamentally one with the divergence between subsumptive inference and the inference which depends on the systematic necessity of abstract relations That these two types of inference unite again in the systematic and definite concretes of the higher sciences and of philosophy is obvious from the interconnection of the hypothetical and the disjunctive judgment But for the greater part of their evolution they are distinct, though not fundamentally discrepant

[1] See Bk I, chap 1, sect 1 iii [2] I c

[3] See Hegel's Wissenschaft der Logik, II, pp 131, 148-9 Hegel, following the order which he finds convenient, calls Aristotle's figure 3 'figure 2', and Aristotle's figure 2 'figure 3' Is it possible that Lotze was misled by this ?

The abstract universal operates in all systems or totalities that can be regarded as aggregates of homogeneous parts, although this very word homogeneous indicates that the whole has a nature which is also the nature of the parts All strictly mechanical science—all science, that is, which regards its objects in the light of number, space, matter and motion, is due to the operation of the abstract universal. And in a certain sense, as we shall see, there is nothing which does not in some degree correspond to these abstract relations, nothing, at all events, which in its analysis presents features discrepant with their abstract necessity

The concrete universal follows the track of the individual totality, and displays itself, first, imperfectly, in analogical inference, and then in the teleological conceptions which govern the higher evolutionary sciences, especially in those sciences which have for their object-matter the achievements and the intellect of man

Mathematical Reasoning

2 To assign the directions of this divergence in terms of traditional logic is not so easy as to describe their real nature. Logic has been compelled to adapt its types of inference to the false directions in which it has looked for them. Thus in diverging towards the abstract universal we are leaving the track of true subsumption, but yet we are following the path on which Formal and Quantifying Logic have taught us to seek the perfectly regular subsumptive syllogism Whereas by following the fortunes of true subsumption we lose all hope of attaining the genuine syllogism of Allness, and yet we go forward through syllogistic types—the second and third figures—which we have been taught to regard as only demonstrable by reference to that syllogism.

Number and Analogy—Divergence

1 The lines of advance *really* suggested by Incomplete Enumerative Induction lead to Analogy on the one hand and to complete Enumeration on the other In the remainder of the present chapter I shall follow out the latter with its affiliated types of inference, and return in the following chapter with the discussion of Analogy to the central line of concrete inference which will take us to the end of our subject.

Complete Enumeration

a. It is unnecessary to repeat the analysis of the Enumerative process which was given in Book I. It is sufficient if

as false ideal.

we bear in mind that complete Enumeration is the establishment of the universal as a numerical totality or aggregate of homogeneous parts, and therefore necessarily depresses the pervading nature or identity of the universal into a *denomination*, and its differences into *units*. By an extreme of abstraction all connection between the parts, beyond the fact that they are units in an aggregate, is done away with, so far the numerical point of view prevails and the universal takes on the uniform attributes and modes of synthesis which belong to a numerical totality as such. It has been said by Mr Ruskin[1] that two and two do not in fact necessarily make four, but more often make five. So of course they often may and do, but not by the process of enumeration, nor by calculation, which is a mere abridgment of enumeration. Nearly all fallacies and paradoxes depend upon a confusion of categories. It is well to be reminded by a man of genius that there are other spheres of knowledge besides calculation, but it would be wrong to take the paradox for a truth, and to impute to the system of number what is a simple omission of our own. Calculation is quite equal to the task of equating 2 + 2 with 5, if it is allowed to indicate the generation of an additional unit somewhere among the 2 and 2.

Complete Enumeration has been operative as a false ideal both in the doctrine of Induction and in the doctrine of the syllogism. Incomplete Enumeration naturally suggests an extension of itself up to Complete Enumeration. It is readily seen indeed that in dealing inductively with the ordinary objects of perception completeness of the process can never be attained, because the universal nature of an object is not comparable with nor reducible to an enumeration of individuals. Therefore such enumeration must fall into the Infinite series. And it is no less plain that if we interpret the universality of the syllogistic major premise in Barbara as depending on a complete enumeration, the inference is at once reduced to a *petitio principii* by the direct affirmation of the conclusion in the major premise. Yet though both these shortcomings are obvious, still the mere aggregation of instances always

[1] I quote from memory, merely for the sake of illustration. I have no serious quarrel with the statement.

tends to obtrude itself as a feature of certainty in Induction, and syllogistic reasoning always tends to assume the shape in which mere extension, i e mere identity of individual units, is the bond of union between the predicated attributes. Instances of this tendency are to be found in the diagrammatic representations of inferences whether by circles included within larger circles, or by straight lines of varying thickness,[1] in the quantification of the predicate, in De Morgan's numerically definite syllogism, and in the logical calculus All of these treatments are founded on a view of reasoning which diverges from concrete determination by attributes, but stops short —except in the case of the numerically definite syllogism, which is simply calculation—of arithmetical inference by true numerical relations All of these processes work well up to a certain point, being, technically, examples of quasi-subsumption—subsumption introduced into a sphere in which its concreteness is lost All of them, finally, are cases of the tendency, so fatal in popular science, to accentuate at the expense of everything else any aspect of any content which affords the slightest prospect of reduction to a mechanical, i e to a calculative procedure For calculation goes by fixed rules and according to regular series, and is *in that sense*[2] an easy process, whereas concrete enquiry into actual and material conditions or connections is inventive and creative —the very travail of the mind

The real ground on which number of instances may be a source of certainty in Induction will appear when we treat of that process in its scientific aspect No doubt the influence of Complete Enumeration as an ideal has operated in part through association with the Calculus of Probabilities. This calculus however is not the true warrant of Induction, and indeed

[1] See Sir William Hamilton's Lectures on Logic, vol II, Appendix

[2] I am not so silly as to maintain that abstruse calculation is easy in the ordinary sense of the word But I take it that its difficulties, though insuperable to untrained minds, are not of the same order as those presented by original investigation of actual conditions, in which the intellect is thrown entirely on its own guidance, and can gain but little aid from general rules And it is not merely the logician but also the physicist who may complain of calculation from assumptions being substituted for investigation into them

in the case of an infinite series, which for the above-mentioned reason must always be the ultimate nature of mere enumeration of instances, the calculus can have no application.

Enumeration as Arithmetical computation

β. But Complete Enumeration in its strict and proper sense leads up to Arithmetical Computation, and in due course to the generalised or symbolic forms of computation which are founded upon arithmetic. The judgment which corresponds to this form of argument is, as we saw in Book I,[1] the equation, a type of judgment in which the predominance of the whole as determining the parts relatively to itself is no longer visible, conformably to the fact that we have left the field of subsumption, and are now dealing with combinations of connections devoid of subsumptive character. The nature of inference, which is common to such combinations and to subsumptive reasoning, has been explained in general terms in discussing the Essence of Inference, and will be more particularly examined when we have looked carefully at the type of Inference now before us.

In addition to what was said in the last chapter on the nature of Calculation, some more special remarks may be added here. Though Calculation may take the most varied forms, yet it must always depend in the last resort on the conception of a whole which is the sum of its parts. Enumeration is the synthesis of this sum out of, or its analysis into, the homogeneous parts or units themselves, through the correlative and all but identical processes of addition and subtraction. It is represented by such an equation as $3 = 1 + 1 + 1$. The changes of sides and signs in an equation exhibit the true relation of addition and subtraction. Multiplication and division are similarly correlative, and represent synthesis or analysis not by help of the ordinary unit, but by help of an artificial unit. The equation $100 = 10 \times 10$ represents multiplication and division alike, being simply an analysis or synthesis by means of a compound unit. Thus multiplication and division are more powerful than addition and subtraction, but less widely applicable, because the compound unit must be uniform. If you have ninety fives, the numerical system gives you the total in a moment

[1] Bk I, chap. iv.

as 450, but if you have such a succession of figures as 4, 5, 3, 7, 9, you have no identical compound unit, and must therefore proceed by the simple one, i e by addition

In multiplying and dividing powers by help of their indices, the procedure (which governs I presume the use of logarithms) reverts to the form of addition and subtraction, that is, to the *apparent* enumeration of simple units instead of compound units. Thus the relation of 8 to 32 comes to be represented not by the *ratio* 1 4 (eight taken once compared with eight taken four times), but by the *difference* between 2^3 and 2^5 Multiplication proper was the construction of a quantity out of or its analysis into an identical compound unit repeated so many times Involution is the construction of a quantity out of, or its analysis into, a simple enumeration of the employments of a certain compound unit in multiplication by itself, that is to say in repetition of itself, or of a quantity generated by repetition of itself (the given compound unit), its own number of times It is plain that as each step enumerated is a multiplication, or an employment of a factor in multiplication,[1] in order to multiply or unmultiply (divide) one power by another we do not multiply or divide index by index, but simply count on or count off the number of acts of multiplication designated by one of the indices Thus in dividing 2^6 by 2^3 we do not take 2^2 as the result, but 2^3—the *difference* of the two indices In dropping from 2^6 to 2^3 we have counted off three multiplications by 2, and have thus reduced 64 successively to 32, 16, and 8 So far we are dealing with simple enumeration applied to a complex process

It is possible of course to trace the same development further, and did the author's knowledge admit of his attempting the task, an interesting scheme of continuity in calculative processes might be obtained When we come to powers of powers and roots of powers we are dealing with complex enumeration applied to steps consisting of complex

[1] To say 'each step is a multiplication' would *prima facie* mean that 4 should = 2^1 instead of 2^2 But yet it is not wholly false, for of course the difference between, say, 2^1 and 2^7 consists of *two* acts of multiplication by two, not of only one In short, multiplication involves two factors, and would not be represented by enumerating only one to start with In every further step the previous result is one factor in the process.

processes. If the index of 2^6 is divided by the index of 2^3, the result 2^2 is obviously the cube root of 2^6, and if the index of 2^6 is multiplied by the index of 2^3, the product 2^{18} is obviously the cube of 2^6. Here we are constructing by multiplication (complex enumeration) a quantity (18), the compound units 6 and 3 employed in the multiplication being themselves representative of simple enumerations of repeated processes of multiplication (complex enumeration).

Simple enumeration may of course itself be represented as the chronicle of a process, i.e. as consisting of units equal in number to the number of times the process has been repeated. Only in it, in arithmetical progression, the process, though applied to its own results, does not obtain the power of creating a progressive difference.

Calculation compared with argument

γ. All arithmetical calculation, and therefore in the last resort all calculation whatever, may thus be reduced, I imagine, to enumeration, or, in some form, to enumeration of enumerations. And thus the entire method rests on the conception of the whole which is the sum of its parts—the universal whose differences, though distinguishable, are taken as equal and homogeneous. From the nature of this universal, in which the whole does not present itself as a concrete system, it is almost futile to enquire into the types and shapes which it assumes in inference. Is an Equation correlative to Judgment or to explicit inference? Is a combination of Equations necessary to explicit calculation, as a combination of Judgments is necessary to explicit Inference?

The equation, it must be remembered, is a comparison of numerable relations in the *abstract*, and therefore corresponds not to any form of Singular or Perceptive Judgment, which are correlative to the simple Judgments of Enumeration, but only to a universal Judgment, and more especially to the pure Hypothetical. This is enough to show that the Equation is essentially of a synthetic or inferential character. And there is also a special reason why this character is more emphatically marked in an Equation than in a generic or hypothetical judgment. Every *judgment*—using the expression in the narrower sense in which it excludes equation—is liable if driven home to be accused of a fallacy *a dicto secundum*

quid ad dictum simpliciter, for the concrete significance of the subject dwarfs and renders trivial the conditions under which alone the attribute can really attach, and some at least of these conditions are habitually omitted, or, if we prefer to say so, presupposed But in the equation the whole content is homogeneous, and no one part can dwarf any other. We may not say that 99,999 = 100,000 We must put in the condition, however trivial in real life, represented by + 1 on the left of this equation, or − 1 on the right Therefore every equation, even the simplest, is not only hypothetical, but it is hypothetical on the basis of an explicit intellectual process or synthesis of differences There is in this sphere no such thing as massing facts together, and leaving you to choose how you infer, or whether you really and conscientiously infer at all To simply equate the whole with itself as a whole, as true Judgment, dealing with differences of attributes, may appear to do, must give tautology, and so nonsense, in Equation Being debarred from even the appearance of such judgments as 'All Exogens are Dicotyledons', the equation must always have on the one side or on the other an explicit synthesis of differences It is therefore in itself a step nearer to explicit inference than the hypothetical judgment

The Equation then exhibits an inferential connection more clearly than an ordinary hypothetical judgment. In respect however of not being a categorical assertion it is on the same level with that judgment, and only qualifies reality in virtue of the real element which underlies its hypothesis; i.e. pure arithmetical computation qualifies reality in as far as it expresses the properties of the system of number. 7 + 5 = 12 means *If* five is added to seven we get twelve, and is categorical in as far as it involves the assertion, 'The system of number is such that "if five is added to seven", &c' And as all prerogatives of a subject are absent (as is also the case with *pure* hypothetical judgments) no difference is more especially referred to Reality than the others All are referred indirectly (i e through the system of number), and without priority

The combination of equations bears the same relation to the single equation as the combination of judgments in

explicit inference to the single judgment In each case it is impossible to draw the line between the single act and the combination An equation may be taken as involving any number of equational steps, just as a judgment may be taken as involving any number of intermediate judgments In short, an equation, like a judgment, is the form of conclusion as well as the form of premise, and in ultimate analysis always partakes of both characters But for this very reason there is no difference of principle between the single form and the combination, and it is sufficient in discussing inference to treat of the latter which has the advantage of being explicit. It may here be pointed out that as the equation is non-subsumptive, so the varieties and peculiarities of syllogistic figure disappear in the combination of equations. In every equated term or expression the whole is present in its entirety and no form of it has any such peculiarity as we understand by the Universal, Particular or Individual, —the subjects and predicates of the syllogism This indifference corresponds to the nature of the numerical whole and renders arrangement and, apparently, number, of terms, in calculation a mere matter of practical convenience But in every system of equations, if *bona fide* treated as a single inference, the three terms may be detected Our insight develops along the chain of equation, and does not simply drop one term out and replace it by another

Applications of calculation

II Lotze has treated of equational inference, i e of calculation, under the titles of substitutive and proportional inference, and of inference from the constitutive equation. I will comment briefly on the first of these forms at present, and will return to the others when something has been said of the matter to which they apply

Substitutive Inference.

a The point of substitutive inference—which is described as a species of syllogism and as possessed of a major premise —consists in substituting in the conclusion for the middle term M the developed content of M as assigned in the major premise, under the operation of a condition *s* representing the peculiar modification attaching to the minor term S The argument is thus written in symbolic form [1]—

[1] Lotze, Logik, Ε Tr , sect 109

Major Premise $M = a \pm bx \pm cx^2$
Minor Premise $S = s\ M$.
Conclusion $S = s\ (a \pm bx \pm cx^2\ .)$.

This argument, in which *a*, *b*, *c* &c represent any marks within a concept, becomes efficient, as Lotze observes, only when reducible to quantitative terms, because in other cases the particular change effected by *s* in *a* or *b* is simply taken from experience and is not really drawn from the form of the argument, which might therefore just as well have been thrown into an ordinary syllogism But on the other hand it is worth observing that if, in constructing such an inference, we remain within the sphere of the *quantitative* universal, then the relation of subsumption and the prerogative of the major premise necessarily disappear We have then simply two equational connections, related to an identical whole, and therefore capable of giving rise to a further connection. But M is not in that case generic, nor is S specific, nor is the connection of S with *s* ($a \pm bx$ &c) known through their conjunction in and subordination to a concrete individuality M. M is no doubt here the assigned meeting-point of the relations, a form of the quantitative universal which pervades the equational connection before us, but S is no more a case of M than M of S And indeed, having once been led to observe the connection of S with its development *s* (*a* &c), we no longer judge this true on the mere ground of conjunction in M, but on the ground of a systematic necessity revealed through M. S or *s* M, if it is or has a true quantitative relation to M, cannot be *bona fide* a *case under* M, an element in a concrete individuality or case of a generic nature M, and with this relation the whole idea of subsumption vanishes

And the favourite and catching phrase 'substitution' must be similarly treated Substitution is a consequence and not a principle of inferential relation It arises from the identity of the whole with itself in all its forms, the discernment of which identity is the task of calculation We may infer, to take a very elementary example, from $24 = 12 \times 2$, and $8 = \frac{24}{3}$, that $8 = \frac{12 \times 2}{3}$ We here 'substitute' 12×2 for 24, because we possess the connection which tells us that the former is a

synthesis identical with the whole 24 To 'substitute' is simply to treat a whole as identical with itself

This 'substitutive' inference then, in its mathematical shape, has no syllogistic character, no major premise, and no real dependence on a principle of substitution It might fairly be spoken of as an inference by equational identification. It must include, so far as I can see, the entire range of strictly arithmetical computation, whether in algebraical or in arithmetical form, as contrasted with computation applied to geometry, mechanics, and physics It establishes a ratio, for, logically at least, ratio is a genus of which equality is a species, but it does not ostensibly compare ratios, and therefore does not explicitly challenge problems in concurrent but heterogeneous series But we must remember that *any* calculation which does not merely develop the properties of number depends for its meaning on *some* differences of real aspect correlative to the differences of numerical aspect. 'The shelf-space M of this shelf = the shelf-space required by thirty octavos' 'I can have shelf-space which = 100 M, I can have shelf-space which = the space needed by 100 × 30 octavos' Here no doubt we are dealing with homogeneous quantity—feet and inches—all through; but the wholes which are compared are differently motived, and these differences of motive—books compared with shelves, and one shelf with a library—are what give the inference its point. The next step is that these differences of motive affect the actual denominations of the units themselves We come to deal in short not with simple equality—identity of number of the same units—but with equality of ratios, i e. with *identity of the ratio* between the several quantities of a set, with that between the several quantities of another set or other sets. The unit of enumeration, in this case, must not be identical

Connections in Space and Time

β. We must now break off somewhat abruptly to consider the apprehension of Connections in space and time, which must be treated for our present purpose as an independent root of knowledge, and must be investigated before we can proceed further with the analysis of calculation Primarily no doubt the apprehension of these connections is an offshoot

of the rudimentary judgment, which as we saw in Book I[1] must construct its world of Things largely under the influence of growing spatial and temporal discrimination But starting as we have done in explicit inference with the developed concrete perception of the world of things, we have no choice but to assume also the developed abstract perception of relations in space and time We reason from these relations or connections before we have subjected their elements to accurate enumeration or measurement, and it is the nature or non-numerical inferences from such relations that I now propose to consider.[2] 'A is to the right of B, B is to the right of C, ∴ A is to the right of C,' 'A is prior to B in time, and B to C, therefore A to C' In such inferences as these, Mr. Bradley has told us, we first construct, and then perceive. I have expressed at length in another work, and briefly in the preceding chapter, my objections to employing the term construction as if it were a self-explaining account of an intellectual process. But I have conceded that when restricted to the sense of intellectual construction, neither imaginative nor physical, it affords an apt description of the peculiar work of inferential apprehension in the field of space, time, and motion[a] At all events we must, I think, agree with Mr. Bradley that in examples like those just given there is no *bonâ fide* major premise, and therefore no syllogism. The form 'A is to right of B, B to right of C, A to right of C' is so obviously the natural shape of the inference thus expressed, that we may be surprised at being reminded that, *qua* syllogism, it has the defect of four terms To fulfil the syllogistic requirements we must set down as a major premise 'What is to the right of B is to the right of C', or even 'What is to the right of B is to the right of that which B is to the right of'. In the latter case the entire content of the argument recurs in the minor premise 'A is to the right of B and B to the right of C'. Plainly this minor premise would carry the conclusion without a major

It was the author's experience, when engaged in teaching

[1] Bk I, chap II

[2] See Bradley's Principles of Logic, p 225 ff, which on this point I have followed very closely

[a] See p 33 above, note a

elementary logic, that pupils had an invincible tendency to construct 'syllogisms' in one of these two types, the former of which is defective as a syllogism, and the latter unreal and ineffectual In using the former, 'A to right of B, B to right of C, therefore &c,' their instinct was just, so far as the argument to be expressed was not truly subsumptive An extension of this just practice to subsumptive inference in which it becomes erroneous was no doubt the cause of Dr. Thomas Brown's remarkable view as described by Mill in an interesting passage [1] The complementary error, to which also pupils are prone, is, in constructing a syllogism, to surrender the whole task of articulating the steps of the argument by simply putting as the major premise some syllogistic canon, in analogical argument some principle of analogy, or in 'constructive' argument such a general principle of construction as 'Whatever is to the right of B is to the right of that which B is to the right of'. It thus becomes necessary to mass the whole content of the particular inference in hand within the minor premise. Wherever a syllogistic canon is taken as the ultimate major premise of inference, this error is committed. The error consists in taking out the active form of the inference —the intellectual function which the syllogistic or constructive arrangement expresses—and making this a mere portion of the content from which the inference is drawn. In drawing the inference the intellectual function is inevitably active, and the principle expressed in the major is no justification of the activity of this function, but merely a content on which it operates as it would on any other content, so that the explicit major really adds nothing to the argument [2] *This* case, of subsumption under the principle of subsumption, needs itself to be subsumed under the same principle or another, and so on ad infinitum. Take 'A mark of a mark is a mark of the thing marked, growth is a mark of organic nature which is a mark of life, · growth is a mark of life'. Here we have an act of subsumption under a principle of subsumption, which

[1] Brown thought the major premise *always* superfluous See Mill's Logic, i, p 225, sixth edition

[2] See Bradley's Principles of Logic, p 475, Hegel, Wiss der Logik, ii, p 151, and the Author's Knowledge and Reality, p 275

act itself falls outside the principle itself and needs justification in a prior syllogism, if the minor premise and conclusion needed justification in this

On the other hand, subsumption does not allow the subject to be merely treated as a point of attachment, as is the case with abstract series in space or in time. If we tried to infer thus in concrete matter we should get something like Jevons' Added Determinants, which is an excellent example of inference that is almost necessarily fallacious, owing to the disproportioned effect of the same added determinant on two concrete conceptions 'A child is a human being, A. B. is a big child, ∴ A. B. is a big human being;' 'Pericles rules Athens, Aspasia rules Pericles, Aspasia rules Athens' The concrete subject in subsumption takes up the determinants into itself and transforms them in a way which we cannot predict. A woman's rule over her lover is not *in pari materia* with a statesman's rule over a commonwealth, and a big child may be a very small human being We must not try to read off conclusions from series in subsumptive matter as we can in space, time or number Construction or abstract connection, on the other hand, deals with relations which bear on each other with systematic necessity, and which are not affected, or are assumed to be affected only within known limits, by the idiosyncrasies of the points of attachment. In 'A to right' &c we take A to be a point or body in space; if not, the inference is unmeaning Therefore in this sphere no premise has a prerogative, and the reciprocal modification of relations is constructed in the argument and not presupposed in the nature of the Subject. This characteristic of 'construction' applies to number, space and time, and to the mechanics of abstract matter.

The apprehension of relations in time needs no separate treatment in logic from that of relations in space. Apart from measurement, connections in time are not capable of any great complexity, seeing that they contain nothing that corresponds to a curve or angle in space. Mere succession is the relation of one set of changes to a permanent subject; but in this there is implicit from the first the idea of duration, which involves the elementary comparison by the permanent subject of two

distinguishable sets of changes, each of which is *primâ facie* the measure of the other There is no science of time in the sense in which geometry is the science of space. From this point of view it has been said that time is one degree more ideal than space, i e its essential character of successiveness falls outside the actual contact of events with sensation and exists for the most part in the ideal medium of memory It is true however that the present has duration, and does not exclude succession But in thus possessing duration, in spite of the fugitive character of actual sensation, the present of course displays an ideal nature which makes it continuous with temporal succession, a succession which pervades even the present itself

The intelligence in bringing order into the feelings begins by apprehending space or time and constructing its perceived world in these forms By developing them, however, into scientific characteristics of this material world, it further proceeds to idealise them into laws and proportions, and thus to take away their immediate and perceived character The first step in this idealisation is the conception of absolute, standard, or uniform space and time, a conception which guides the process of measurement, but which is strictly speaking, in its common-sense form, an abstraction to which no real thing nor process corresponds Constancy of ratio throughout the perceptible world is, as we saw,[1] all that measurement can give us, and is what we really mean to assume Uniform space or time, as embodied in a single series and not in a comparison, is a contradiction in terms

Calculation applied to Geometrical Reasoning

γ The apprehension of connections—even of explicitly mediate connections—in space and time does not necessarily involve the enumeration of parts with a view to precise measurement Such apprehension begins, as we saw in Book I,[2] with the perceptions indicated by 'Here' and 'There', 'right' and 'left', 'nearer' and 'further', 'now' and 'then', 'future' and 'past'.

But all these expressions involve a continuous series, and such a series is the beginning of measurement The spatial elements however, the straight line, angle and arc, are not

[1] Bk. I, chap iv

[2] See Bk I, chap. i

constructed by measurement but are given spatial relations, although the discovery of their properties cannot be effected apart from the comparison of quantities I am not prepared to say that the identification of corresponding geometrical relations might not (in spite of the etymology of the word 'geometry') be prior to intentional quantitative comparison Vertical angles, or right angles, or the triangles into which a rectangle is divided by the diagonal,[1] might be seen to be *the same*, before numerical comparison was applied to sides, angular distances, and areas

Apart however from definite history, nothing can tell us when an implicit character becomes explicit It is certain that all developed consciousness of spatial and temporal connection must operate through quantitative comparison

Geometrical and strictly mechanical reasoning is the endowment of the quantitative universal with control over the combination of homogeneous parts in space, or in space and time together In this type of reasoning, though essentially 'constructive', we are often reminded of subsumption by the peculiar working of the quasi-generic judgment

The square and the cube are elementary and striking instances of the power of enumeration, i e of arithmetical ideas, to represent or to describe generically a purely geometrical relation. The 'square' of a number *is* not a square surface, nor does it, so far as I can see, display in itself the difference between the superficial and the linear unit But such a number, or rather such a numerical relation, does indicate definite combinations of parts by which a square surface may be recognised or constructed, and therefore the numerical relation may for many purposes be regarded as equivalent to the surface The *purely* homogeneous universal, *qua* homogeneous, already at this stage fails to grasp the nature of the content as such, whose structure is to be represented by the differences of the

[1] In elementary schools it is now the practice to familiarise young children with such relations as this, e g by folding square pieces of paper, first along a line dividing them into equal rectangles, and then along the diagonal, thus exhibiting the equality of a right-angled triangle to a rectangle of the same height and half the base, both being halves of the same square This is an equation and so a measurement, but rudimentary in as far as unanalysed

universal, but succeeds in a way unknown outside mathematics in presenting an adequate key to this nature

I presume that the marvellous processes by which curves of all types have been subjected to the dominion of the equation must ultimately be regarded by logic in the same light as the relation of the square upon a line to the square of a number These processes do not, as I understand, exhaust the nature of the curves, but they exhaust a combination of directions and distances, referred to an assumed point, which can be given any required degree of accuracy in representing the curves Hence, though numerical relations have not in themselves the aesthetic or mechanical attributes of the particular curves which correspond to them, yet the nature of space permits a curve to be adequately and unerringly constructed by putting together homogeneous parts, viz distances, in accordance with these numerical relations It is obvious therefore that by experience, if not otherwise, both aesthetic and mechanical attributes may come to appear as if directly legible in certain numerical relations considered as controlling spatial elements. In the case of curves these numerical relations or attributes are expressed in *constitutive equations*, and form by far the most striking examples of a transition from and through the homogeneous to the heterogeneous. But strictly speaking all the simple proportions which characterise e g the triangle or the square are examples of the same principle

We spoke in Book I of this whole class of truths as ' quasi-generic judgments ', and insisted on the peculiar reversion which they display towards the type of concrete or organic totality. Never again in knowledge do we meet with such simple abstract and mechanical construction so unerringly analysing an individual and characteristic content. For this reason we are tempted to take the constitutive equation as the ideal of knowledge, and indeed *exceptis excipiendis* we are right in doing so But the *excipienda* are serious When we pass from abstractions like space, time and matter to the concrete evolution of the real world, to organisms, to political societies, and to human intelligences, not only is mechanical construction infinitely more difficult, but it is infinitely less adequate A curve after all is a line in space, though it is not a straight

line; so that it can be no such enormous leap to constitute a curve out of spatial elements. But a plant is hardly in the strict sense a phenomenon in space at all, and although a mechanical view of it, in a sense to be explained below, must certainly be aimed at by science, yet there will remain in it much to be *understood* which cannot be *constructed*—not a part of its actions, but the whole result.

If the constitutive equation is the type of quasi-generic judgment, it follows that Inference from the constitutive equation is the type of quasi-subsumptive inference. Such inference has two aspects. On the one hand it is pure calculation or combination of connections, like any mediate equation which has no meaning beyond the numerical system. It combines with a proportion between two changing quantities some particular values of these quantities, and follows out the modifications which result from this combination. This process is in itself, though more complex, yet not a whit more subsumptive or less purely calculative than to equate $2a$ with $3x$, and supplying any number as the value of a to fill in the number demanded by the proportion as the value of x. In such an expression a and x are merely generalised numbers, or numerical relations, or, if we like, names for a problem There is no true major premise, but in its place there is the generalised description of an identical numerical whole in the two cases of being constructed with a factor 2 and with a factor 3, and the inference consists in exhibiting the construction of any such whole on the basis of these factors respectively.

If a and x represent distances in space defined by some further relations (e g perpendicular to each other or the like), and the whole expression has the effect of characterising a definite figure in space, then we have the germ of what I have called quasi-subsumption. The inference is still constructive in the narrow sense, it proceeds by enumeration of indifferent parts as in the case first considered But it combines with this aspect another and a different one It exhibits a particular portion of a particular curve in the light of a characteristic modification of a generic type. The curve may be closed or open, quick or slow, symmetrical or unsymmetrical. And these attributes, although they result from the construction, are not

given within the construction The spatial nature of the type of curve in question is the quasi-generic content through which the subsumption is made It is not *pure* subsumption , for the construction would suffice in theory to draw out the *particular* curve before us and discover its properties, without ever giving it a generic name or observing that we are analysing the properties of something like a natural kind And in fact, I presume, this is the usual order of procedure Observations are obtained, upon which as data the curve is constructed, and its general nature is found only by means of this construction. But on the other hand, the mere fact that we know how to pass from an equational combination of numbers to a spatial figure shows that we are proceeding on a principle involved in the characteristic nature of such figures The scheme of the argument is—

A spatial figure constructed on such proportions has such and such an outline ;

This is a spatial figure constructed on *this* case of the above proportions ,

∴ This has *this* particular outline

The element of subsumption consists in the impossibility of passing from the ratio, which is only a generalised relation of numbers, to the markedly individual figure in space, except by identifying the subject in which the numerical relation and the characteristic curve-properties are conjoined as the nature of space, or, more closely, of a curve in space

It only remains to mention that when we take in the unit of time, and thereby are able to represent motion as a length, and when we further erect the abstractions force and mass as correlative points of view from which motion is regarded as affecting bodies, we have all the *organa* of what may be called pure mechanism, or abstract constructive science—a complex and elaborate system, founded ultimately on the combination of three abstractions, space, time, and number. This abstract mode of consideration is true in so far as it applies to the characteristics of real things, but its application is obviously limited. The human body, for instance, is of course a case in which the geometrical and mechanical laws of space and motion have reality, but to a large part of its activity these

laws in their purely abstract form have nothing to say. I proceed to discuss how and in what sense mechanism, and mediately, even *pure* mechanism, has a wider application

Calculation applicable to Disparates.

8. Lotze has spoken of inference from proportion as a limit of knowledge, and as assuming a type which from his description appears to be subsumptive, although he does not give it this name

Proportion.

Homogeneous Terms

(1) I am unable to see that we have in proportion either subsumptive inference or a limit of knowledge, so long as the four terms of the proportion are either of the nature of abstract number, or are *in pari materiâ*, i.e of the same denomination with one another. Numbers by themselves, it may be said, prove nothing, and computation in the abstract is not inference But it seems obvious that the properties of the numerical system as such are worth establishing for their own sake $2 \quad 4 \quad 3 \quad x \cdot x = 6$ is not a worthless *type* of inference, although the example is one which has no interest It depends on a relation within the numerical system, and is in itself, apart from its elementary character, as well worth establishing as any other consequence of an isolated abstract relation.

Or again, if we want to make a picture-frame of the same proportions with one that we have, but of different size, then all the numbers represent lengths, and we have, say, $16^{in.} : 24^{in} . \; x \; y , \; x = 32^{in} , \; \cdot \; y = 48$ In this example, however, the limit of which Lotze speaks is just beginning to make itself felt. All the numbers do no doubt in one sense stand for lengths in space; but they are lengths peculiarly related, in a way which falls outside the statement of proportion *In the object to be constructed*—this is the very reason for which we compare them—the one spatial length is to be at right angles to the other, or, in popular language, they are respectively length and breadth Thus 16 and 24, and their correlatives x and y, are in one sense of one denomination, but in another sense of different denominations, and it is because they are of different denominations—or dimensions—that we have an interest in comparing them. Here moreover, as in geometrical reasoning, we come upon a trace of subsumption, because the essential identity which is the foundation of the inference consists in the nature of an object which has length and breadth,

under which generic nature, as characterised by a certain ratio, we subsume a specific case of the same nature and of the same ratio. The meeting-point of the relations is a concrete and not an abstract, and this is the differentia of subsumption.

It would be easy to fill up many gradations of the appearance of heterogeneity in the terms of a proportion. First, the two *sets* compared cease to be measurable by any identical unit—whereas in the last examples both sides of the proportion *are* measurable as distances in space. Such are the proportions of times to distances and to areas, or of force to distance or to mass And secondly, the several terms, either *in one set* or in both, cease to be strictly measurable by identical units. While this is the case with one side only, some sort of proportion may be supposed to exist, especially as the other side may present variations of a quantitative character, though not exhaustible by repetition of an identical unit. But when neither side presents a true ratio, i e. when on neither side are the several terms measurable by an identical unit, then proportion is gone, and we are referred back or across to Analogy or Subsumption.

'*a* and *a* series.'

(2) Leaving the intermediate portion of these gradations for the reader to fill up, I pass at once to a class of proportions which prevail in a certain sense throughout the entire world of knowledge. I speak of related series of contents which might appropriately be designated '*a* and *a*' series. The examples which spring at once into the mind are the perceptions or sensations of light and sound, with their respective physical stimuli. It is true that in the connection of such series as these we come upon a 'limit of knowledge', but it admits of question how far even in the most favourable instances of them we can be said to have proportional inference [1] Perhaps

[1] I suppose that 'irrational' in mathematical language designates a relation which cannot be expressed as a ratio, viz as a relation between two numbers It would seem then that 'incommensurable ratios' must be a contradiction in terms The convenience of the expression in mathematics would not necessarily be interfered with by its self-contradictory character Many conceptions which involve a more or less latent contradiction are employed in special sciences—quantitative infinity,[a] or involuntary contracts, are well-known instances. It is interesting to notice that the reason of one science may be the unreason of another Hegel complains that mathematicians call everything

[a] See i. 163.

the relation of angle to arc, which Lotze gives as an example, is about the extreme instance of true quantitative proportion which, although quantitative, includes heterogeneous matter

When we look at the relation of musical sound to stimulus we find an ascertained parallelism between changes in the rapidity of periodic vibrations that reach the ear, and changes in pitch of the musical note which they generate in consciousness And further, the changes in pitch are a continuous variation of a pervading quality and therefore are quantitative, and these are commonly measured by certain intervals of change taken to be equal in the sense of sharing certain attributes and capacities which remain the same for corresponding intervals in all parts of the scale. If we attempt to make out a proportional statement of these relations we may get some such result as a vibrations in second $2a$ vibrations in second

note a its octave a_1, a relation which admits of the quasi-subsumptive inference $a = 16\frac{1}{2}$ · a = a certain note x, and a_1 a certain note y

But this inference is by no means purely mathematical or quantitative

First, we must notice the presupposed limitations, which do not exist in any purely mechanical or purely geometrical law. The vibrations must affect a hearing ear, and to affect a hearing ear they must be within certain limitations both of rapidity and amplitude, which from a mathematical point of view are wholly arbitrary

Secondly, we must observe that not only, as in true cases of proportion such as that between the angle and the arc, are the two sets of terms incommensurable with one another, but also it is very doubtful whether the two terms represented by a and a_1 can be strictly called commensurable In a sense no doubt the tones of the scale are units and serve as measures. But if we ask which note is 'twice' another, we are perplexed between the octave, which corresponds to twice the stimulus of the octave below, and the note, whatever it may be, which

irrational in which reason begins to intrude—i e in which systematic heterogeneity begins to appear, e g in the relation between straight line and curve Ratio is the reason of mathematics, and other systematic relations may be irrational *in this sphere*, just as ratio may be an irrational relation outside mathematics, e g in political science

is next above the lowest audible note For two of any identical unit ought to be twice one But just as the zero of the common thermometer is not a zero of heat,[1] so there is no sense in making the lowest pitch, or the point where pitch passes into separate sounds, a zero of pitch from which all its grades can be constructed by mere multiplication or superimposition We may say, if we like, that the quantity is intensive, i e that the grades by which the greater is separated from the lesser amount do not persist in a distinguishable form beside or within the greater amount when it is attained This, however, is only to say in other words that we are passing beyond a true quantitative relation All true measurable quantity is extensive In as far as it is *merely* intensive it is unanalysed, not referred to parts, and so not quantitative Here we are between the two, we have *degrees* which are not parts, and of which the whole is in no strict sense a multiple, but which not only exhibit an advance in themselves but *correspond* to parts of which *their* whole *is* a true multiple

Then, thirdly, we are confronted here with something like a genuine subsumption. The correspondence of 33 vibration per second to a note x rests on a concrete relation, which we find but cannot construct, between an impact on the living person through the ear and a reaction in the soul of that person. It is by subsumption under this characteristic individuality that we justify the conjunction, not otherwise necessary, of periodic vibration and musical sound

It should be noticed, too, as bearing on the concreteness of the operation which we are now considering, that loudness is a quantitative attribute of musical sound, and corresponds to a distinct feature of the sonorous vibrations, viz to their amplitude, which I presume must on the whole diminish with increasing pitch This ratio, and other analysable characteristics, fall within the same series of contents as that to which the correspondence between pitch and rapidity applies We are thus in such a case far from being able to obtain a simple proportion between contents as wholes

[1] The idea of an absolute zero of heat does not matter for the present purpose, which is merely to illustrate the nature of a series which is not in a true ratio See Mill's Logic, i, pp 441 and 446

When we come to the colour-sensations, the individuality or disparateness of the a terms increases, and the quantitative relation, still traceable in sound, heat or pressure, becomes much more obscure if it does not disappear Not to speak of the dark heat rays and the dark chemical rays, which mark differences in the action of the ether corresponding to no *saltus* in the ratio of vibrations, the transitions from colour to colour defy all attempt at quantitative expression No one could mean anything by saying that central violet is twice or three times or five times central red, except in so far as colours may be characterised by an intensity of illumination which is a different thing from the peculiarity of their hue It is needless to go into detail on this head It is plain that while the variations in the mechanical stimulus, the a series, are still strictly numerable, the terms of the a series are altogether ceasing to present commensurable and therefore numerable differences in respect of their distinctive individuality, although various numerable differences may be traceable within their content. Then proportion in the strict sense wholly ceases to apply, because there is a ratio on one side only, and not on the other; and there cannot be equality of ratios unless we have two sets of matters with a ratio between the members of each set

Nevertheless, there is in the colour series a uniform relation of a proportional character 'Wave-length x wave-length y · violet red' means not that red is such and such a multiple of violet, but yet that *in a series in which* wave-length x gives violet we can be sure that wave-length y will give red. And though these colours are not producible as multiples of one another, yet they are identifiable, on the assumption of correspondence to stimuli, by the process of colour-equation Given the means for producing true spectral red, a problematic red colour can be equated with it, and the difference between the two stated in terms of the stimulus

Proportion, Analogy, and Hypothetical Judgment

6. We have here, it might be said, passed from Proportion to Analogy, which abandoning its original meaning of proportion has come to signify an inexact comparison of relations But Analogy in the logical sense is not really an inexact form of proportion, but depends on other than quantitative con-

siderations, as we shall see in the next chapter. And though we are now passing out of the region where equality of ratios would grasp and penetrate the whole essence of the subject-matter, yet there is no need to leave behind us either precision in the designation of relations, or such numerical attributes as accompany though without exhausting the individuality of contents For these reasons it is better not to consider that we are here returning to Analogy, but rather that we are approaching, from the side of exact science, the hypothetical judgment, which forms the meeting-point between the concrete and abstract forms of inference And the hypothetical judgment, especially when retaining a partially quantitative content, represents what may be called the wider or the philosophical sense in which mechanism prevails throughout the knowable world In many regions it is not much that this view of things gives us, but it is always something

It is needless to pursue this hypothetical judgment through all the gradations in which it embodies the idea of proportion throughout the sciences Everywhere we have in the background the strictly numerable relations of space, time and mass. The exhibition of connected groups of contents as *a* and *a* series, in both of which the terms are as nearly as possible commensurable, is the ideal of natural science, or of *physical* science in the strict sense How far in detail chemistry or biology may be reducible in this sense—for there is no other sense in which they can be—to molecular physics, is a question of fact and practice At present it must often seem as if instead of *a* and *a* series we had rather *a* and *x* series, i e. correspondences in which even the more quantitative series has hardly a true ratio between its terms, while the less quantitative series has no ratio at all. I have in my mind such correspondences as those between changes of temperature and allotropic phenomena in chemical agents, or again between changes of temperature and degrees of irritability in organic beings. But the changes of temperature themselves, considered as an *a* series, have behind them as a true *a* series their mechanical equivalents in the way of mass and motion (footpounds), and thus the whole phenomena, even those of allotropism, have ultimately a true correspondence with a genuine *a* series Such

a correspondence would form the content of a hypothetical judgment, under which quasi-subsumptive inference would be possible

Consciousness and Conservation of Energy.

ζ. It is under this modification of '*a* and *a* series' that we ought to consider such a question as the relation of the physical changes in the human organism to the activities of consciousness I am not now dealing with the sheer question of fact, whether in the human or other organism, considered as a machine, the production of work can be experimentally proved to be limited by the supply of mechanical energy. We know too well from daily experience that the output of work has gradations and a limit—a limit related at any rate *inter alia* to the supply of food That this limit is prescribed by conditions precisely analogous to those which are embodied in the working of every machine, viz by the constancy of energy, seems to be the natural assumption so long as no other principle of gradation and of limit is convincingly demonstrated.

But we now come upon a second difficulty. We assume that molecular change and muscular contractions in the organism must be taken as theoretically subject to the constancy of energy, i e that life or the soul so far as operative in or through the organism cannot create energy out of nothing[1] These changes and contractions either are, or are reducible to, mechanical equivalents in motion and mass, and between them and the supply of energy a true *a* and *a* proportion might, if our knowledge permitted, be established

But between either these organic changes or the expenditure of energy, on the one hand, and the states of consciousness which sometimes attend these changes on the other, no possible proportion could hold good Here we have then a pair of series *a* and *x* (organic changes and consciousness) on the basis of a pair *a* and *a* (expenditure of energy and organic changes).

[1] I cannot think that Wundt, Logik, II, p 507, really means to deny this, though his expressions are bewildering, 'Hier (in the spiritual development) gilt vielmehr (as against the law of constancy of energy) ein Gesetz unbegrenzter Neuschopfung geistiger Energie' To judge from the Physiologische Psych II pp 461–3, he thinks that 'Geist' is operative throughout nature, and its 'creation of energy' must be a question merely of new *forms* of action But I cannot thoroughly understand his view

It does not matter for our purpose here whether consciousness is independent work performed by the organism, or is an unaccountable attendant upon such work which is not represented in its cost, or lastly, a consequence, or 'effect' if we like, of certain peculiar organic work, represented in the mechanical cost of such work, and inseparable from such work except by an utterly unmotived abstraction In all of these cases we have, whether directly or through organic activities known to accompany consciousness, what may be called an a and a relation, or more truly an a and x relation mediated through a, between mechanical work and consciousness But on the side of consciousness of course no ratio can be established, and therefore there is no proportion. We cannot safely say that to learn twenty lines of Virgil demands twice the expenditure of work that is needed to learn ten We cannot say that to write the same letter requires the same exertion on one day and on another We must however be on guard here Interruption, a different state of memory, obstacles in the content, impair the reliability of what might seem a unit of mental work But mere weariness of the organism which may lengthen the time taken and the apparent exertion demanded, seems only to affect the proportional and not the absolute work needed What we have under such circumstances is merely a case of a weak machine doing slowly what a strong one does at one blow. The weak machine may however be destroyed by application to work too heavy for it, and in this sense even an identical unit of work has not always the same total effect

If a unit cannot be obtained for the x or a series, what can our would-be proportion do for us? It enables us to say generally, though not precisely, that amount of even intellectual activity varies as expenditure of mechanical energy and is limited by the sources from which that energy is drawn, and further, to say precisely, though only as a hypothetical judgment of which the condition can never be entirely fulfilled, that if we could equate two amounts of intellectual activity, or if we could have the same intellectual activity repeated under the same conditions, we should find it had absorbed or had been accompanied by the same amount of mechanical work There is, that is to say, a formal and constant correspondence

between amount of mechanical energy and amount of intellectual work; but mechanical equivalence is so subordinate an aspect of such work that this constant relation tells us nothing by way of calculation, and only embodies in an abstract principle what we knew before—if he does not eat, neither can he think.

It may be objected that intellectual activity apart from molecular change, which latter is not intellectual, absorbs no work at all. But when any one can show us thought apart from an organism it will be time enough to speak of thought as an activity not dependent upon organic changes. *Prima facie* the complex molecular changes which accompany thought have thought for their natural outcome and consequence, and owe their high mechanical cost to this characteristic. That no mechanical expenditure goes directly to thought, but all goes to molecular change which is in some cases accompanied by thought, is only what we should naturally expect supposing thought to be conditioned by the activity of a physical organism. In any other case, i.e. supposing a contingent of energy to disappear into the thought-process and be unaccounted for in the balance-sheet of the body, we should have a proportionate amount of thought unaccompanied by material change, which is unknown in our entire experience and contrary to the whole analogy of that experience. All that we can say is that the *thinking being*, as we know him, is thus and thus conditioned. We cannot safely separate in theory what can never, to our experience, be separated in fact.

The Mechanical aspect of Knowledge.

iii. Mechanism in the widest or ultimate sense is thus compatible with the disappearance of the ratio in both of the corresponding series. The principle of 'mechanism' is by this curtailment reduced to the law of Sufficient Reason, and simply expresses the point of view from which the scientific understanding necessarily and inevitably regards the world. This point of view is not a tyranny to be avoided, but a claim which must be satisfied The Understanding is a necessary moment in the Reason. Instead of precise proportion, enabling us to construct or to predict by a formal process, we have in mechanical determination thus understood a conception which confronts us with a material problem. A man's character

deteriorates, or the prosperity of a nation decays The conception of mechanism or of sufficient reason entitles us to treat these phenomena as problems demanding explanation They must not be miraculous, i e not isolated, 'cut off with an axe' from the system of knowledge We express this demand by saying that every change—or more generally, every modification—has its reason It is only our tendency to illustrate the universal by the plural that makes us state the principle as if it necessarily applied to a number of examples in which the same ground has the same consequence What we really mean is that every content is a consequent, and that every consequent has a ground That the connection of ground and consequent is necessary, and therefore *if* the one is repeated without modifying circumstances *then* the other is repeated without modifying circumstances, is involved in the point of view from which we speak of ground and consequent at all But what we primarily mean is objectivity, not uniformity

Some uniformity, however, is for us a corollary from objectivity The ultimate fact of knowledge, on which the objective relation of ground and consequent depends, is the existence of *systematic* connections Now a system is a whole, a universal, an identity in difference From the very beginning this is what we meant by something objective, something by which intelligence could agree with itself in the world of meanings. But such a system or identity, however heterogeneous the parts that enter into it, of course pervades all these parts as a common character As we saw in the last chapter, if *a*, *b*, *c* are mere particulars, there is no bridge from the one to the other. A connecting quality there must be, although it need by no means take the shape of an immediate and simple quality. Thus in highly complicated matters we go back again to an earlier function of knowledge, and substitute equational comparison for exact enumeration But the comparison is not here the ground of inference, the contents have the partly quantitative relation, which admits of such comparison, as a consequence of their systematic connection, but the grounds of inference lie in the systematic connection itself Let the *a* series be the phases of the artistic individuality of a painter, and let the α series be a chronologically arranged series of

pictures in which these phases display their effects In such series we shall always have, among other processes of judgment and inference, a comparison by way of equation applied to pervading qualities as between phase and phase, and between picture and picture. No one would doubt that Turner's water-colour picture of Durham painted in 1836 has more of certain striking qualities commonly associated with Turner's art than that painted in 1802. But of course there would be no sense in saying that the one is twice or three times as 'Turneresque' as the other

In series like these we have subsumption or abstract hypothesis—which latter is the bridge from mechanical construction to concrete systematic inference—according to the nature of the subject-matter. In tracing the phases of an artist's genius we have properly subsumption, the unity which operates being concrete and self-contained But yet as regarded in the light of causal ideas the phase of mind may be distinguished from the picture produced, and the one regarded as the cause or ground from which, on the latent basis of a real individuality, the other necessarily issues. From this relative point of view the connection would be expressed by abstract hypothesis, 'If a_1, then a_2.' In this aspect everything in the universe may be referred to conditions outside itself, and nothing is free, complete, or self-contained For to regard things thus is simply a phase or moment of knowledge, the phase which consists in determining every x in terms of some correlated y. Cause and condition, reason and ground, are only species of the generic idea which presides over thought of this type. But in ground we have as we saw [1] a conception in which the correlated terms tend to fall together, and to pass from being parts related within a system to being the system itself. The whole conception which we are discussing—the conception of 'sufficient reason'—is simply a corollary of or aspect in the ultimate nature of the universal, which is in other words the ultimate nature of thought itself. We have constantly reiterated that every universal is an identity in difference, and it is only the converse of this to say that every difference has a distinct and assignable place among the differences of a universal.

[1] Cp. Book I, chap vi.

The statement of the principle of sufficient reason which was laid down above, 'There is no difference[1] without its reason,' may be illustrated by the formula in which Lotze[2] embodies it, A + B = C. This formula is intended to mean that a real subject A can only pass into a specific phase C under an assignable condition B. The 'proof' or analysis of the principle must be reserved for a later chapter. Here I am only dealing with its import And we see its import reduced to the most abstract type when we are in a stage beyond proportion and yet have not returned to true subsumption, i.e. when we are dealing with the pure hypothetical judgment. For we have at this stage a *de facto* correspondence of which the ultimate ground is more or less latent, between the terms a and α, b and β, and so forth. What then do we mean by correspondence? We could not say that a and α correspond if a and α were respectively isolated occurrences of their kind. For what could possibly justify us in picking them out of the myriad complications of reality and attaching them together to the exclusion of other events and relations? Correspondence involves the recognition of a universal which fixes the relations of the terms that correspond If the series a, b, c is to *correspond* to the series α, β, γ, then both series, simply in order to the possibility of selecting them, must be capable of being regarded as a_a, a_b, a_c and a_α, a_β, a_γ. If in a combination of musical sounds a slight harshness a is raised to a discord b, then the consequent feeling of pleasurable excitement α passes into a feeling of pain β. In virtue of being phases of the same combination the two sounds may be represented as a_a and a_b respectively, and the two phases of emotion, in virtue at any rate of relation to the same series of sounds, may be represented by a_α and a_β respectively. But neither sounds nor feelings, of course, are terms between which a ratio subsists. What we can infer is, by a process which is half-subsumption and half-hypothetical judgment, 'In a pair of series where the sound a_a causes the feeling a_α, the sound a_b will

[1] p. 82, above It is commonly worded 'there is no *change*', &c, but I have attempted to show in Book I, chap 1, that change is simply a case of difference and needs no separate logical treatment

[2] Logik, sect. 63.

cause the feeling $\alpha\beta$.' When we develope any such doctrine as that the source of pain is intermittent irritation of sense, of which musical discord is a species, we pass into subsumption of the higher kind, which has absorbed into itself what can be done by mechanical construction.

The point of view which we have been considering—that of reflection and relativity—is the point of view which has been distinguished by Kant and Hegel as the standpoint of the understanding It is not separable in kind from the mode of consideration which the same thinkers have designated by the term Reason. Nor is it an intelligible contention, even if favoured by the language of Kant in dealing with practical philosophy, that Reason could be actual and operative otherwise than as completing and containing the understanding That the understanding must have its rights is one of the cardinal principles of Hegel, which Lotze has but laboriously and ingeniously illustrated in his analysis of the pervading mechanical aspect of the world The real prophet of the understanding, however, was Schopenhauer. His treatment of the principle of sufficient reason as at once the fundamental axiom of human science and the innate source of its illusions, forms an ultimate and irreversible criticism on the aspect of intelligence which consists, to sum up its nature in a popular but not inaccurate phrase, in explaining everything by something else—a process which taken by itself is necessarily unending and unsatisfying.

In returning from the consideration of abstract necessary relations to that of concrete real totalities, we must remark that ideally speaking every concrete real totality can be analysed into a complex of abstract necessary relations. Were this not so, as it is Wundt's and Lotze's great achievement to have shown in detail, teleology itself would vanish. For adaptation disappears if the end can dispense with means, and a universe which had no necessary connections between its parts could have no definite or significant structure as a whole. In the remainder of the present Book I shall attempt to put these relations in a clearer light

CHAPTER III

ANALOGY

WE now take up the thread from the end of sect I of the last chapter, and returning into the track of concrete inference, we have to ask ourselves how we go forward in inference from a simple enumerative Induction when we do *not* accept the task of completing the enumeration In this case we no longer count the examples but we weigh them. We turn the focus of attention upon the concrete content which as subject of both premises as a real thing or things, formed the middle term of Induction and endeavour to deepen it by observation, and to define its relations by analysis. The first effect of this procedure is to transform the content in question from a subject into a predicate, as no longer a qualification tacitly presupposed of reality but as an attribute explicitly referred to it and under process of definition and extension

Analogy and Enumerative Induction

1 As regards the relation between Analogy and Enumerative Induction we have to remember that Induction only gave us a problem or suggestion, and consequently we cannot arrange the two types of inference in a simple concatenation by taking the Inductive conclusion as the analogical premise Rather we have in Analogy to go back upon the suggestive process of Induction, and repeat it with the requisite difference Suppose that the Inductive Inference or grounded conjecture has been—

The poor people *a b c d* are pauperised *x*;

The poor people *a b c d* are constantly cared for by charitable persons *y*,

∴ Being cared for by charitable persons *y* may have to do with their being pauperised *x*

If now we desire, as we ought, further to examine this suggestion on the basis of the direct experience which generated it, we shall still make this experience—the cases *a, b, c, d*

—the middle term or ground of inference, but we shall divert our attention from the number of the examples to their nature, and shall therefore put their nature as an attribute in the place of a predicate common to both judgments. Thus we obtain an argument having a form akin to that of the Aristotelian second figure. And the premises are now no longer conjunctive individual judgments, but are passing into generic judgments.

In the pauperised type x, what strikes us on further examining the cases is the loss of independence (A as common content of $a\ b\ c\ d$),

In the charitably-cared-for type y, what strikes us on further examining the cases is the loss of independence (A as common content of $a\ b\ c\ d$);

$\therefore$ The type y has a fundamental feature A akin to the type x and the two are thus closely coherent.

We are apt to think that in analogy we must conclude from old instances to new instances. But analogy is essentially an argument about the significance of a type, or of what in botany are called characters. Of course however this inference, like all others, has the aspect of discovery as well as the aspect of proof The deeper theoretical need is to find the link and limit of connection between the characters x and y sometimes observed in conjunction, i e to *prove* the one of the other But the reality of the distinction between x and y which gives interest to the inference makes it practically certain that in some examples we shall be first struck by y and in some first by x, and that we shall often need to make the circuit through the identical nature A before we can even detect or have ground to believe in the remaining character x or y This is the aspect of discovery Proof overcomes logical disconnection, discovery overcomes apparent presented disconnection Thus our account of Analogy can really satisfy the popular idea that inference is, here as in Induction, from particulars to particulars

I subjoin one or two simple examples, in order to illustrate the connection of proof and discovery, and the working of the process

Enumerative Induction (*or* observation in single instance).

Almost all animals have some power of self-movement ;
Almost all animals have some degree of sentience ;
∴ Sentience may be intimately connected with self-movement

Analogical Enquiry.

Self-moving creatures are creatures in need of special nourishment and protection (*qua* living creatures) ,
Sentient creatures are in need of special nourishment and protection (*qua* living creatures) ,
∴ Sentience is connected (by the requirements of animal life) with self-movement

I may give another instance which shows the transition from Induction to Analogy just *not* made. Newton guessed[1] that a diamond was combustible, because of its high refractive index relatively to its density, a feature which he had observed in many combustible bodies

Enumerative Induction

Oil, Canada balsam, &c are combustible ,
Oil, Canada balsam, &c. have a high refractive index relatively to density ;
∴ High refractive index may be connected with combustibility

In the case of the diamond the combustibility had not been observed, so that when applied to it the conclusion was a prediction or discovery But the essential import of the conclusion would have been just the same if no new case had been in question

The next step would have been to say—

' Combustibility has to do with such and such attributes of oil, Canada balsam, &c.
High refractive index has to do with these same attributes
∴ High refractive index is fundamentally connected with combustibility '

But this step, which would have constituted an analogical inference, has not, as I understand from the passage in Mill, been taken

[1] See Mill's Logic, ii, p 88 I write merely on the faith of the place in Mill. The instances by which I illustrate Newton's guess are therefore of my own invention

We may now look at an instance drawn from the relation of natural kinds

Enumerative Induction.

The exotic Pelargonia have a peculiar herring-bone structure in the petals;

The exotic Pelargonia have the same kind of seed-vessels as our wild geraniums;

∴ In flowers with the peculiar seed-vessels of our wild geraniums it is worth while to look for the herring-bone structure in the petals.

Analogy

The herring-bone structure is conjoined in the Pelargonia with the characters of Geranieae,

The flowers with such seed-vessels as our wild geraniums have the characters of Geranieae;

∴ That these flowers, e g. our wild geraniums, should have the peculiar herring-bone structure [1] is exceedingly probable.

Botanical classification might almost be said to rest wholly on analogy. The above guess, like a thousand and one such guesses which every field botanist is continually making, is verified in fact The eye that can detect the dominant habit of a natural *genus* or order in an unfamiliar species is constantly inferring in analogical form, on the ground of generic identity, to hundreds of details, which as a rule confirm its diagnosis on more minute inspection And the term diagnosis, logically applicable, but not customarily applied, to botanical science, reminds us of another great province of knowledge in which analogical inference is our guide and counsellor As in the normal so in the abnormal activities—the diseases—of the organic world, it is analogy that is the chief clue to what is taking place and to what we must anticipate. Diagnosis is to symptoms what classification is to characters. And finally we may here mention the class of examples to which we shall shortly return as throwing the

[1] As a matter of fact, all the Geranieae which I have examined possess this structure I might have mentioned this verification immediately after the Induction, but this would really take us into mere enumeration of instances It is the coherence with the properties of a natural kind that alone gives any help towards a general conclusion

strongest light upon the true *rationale* of analogy, although or because they tend ultimately to pass beyond its province I mean our judgment of the actual use or even of the intentional object of mechanical adaptations of every kind, whether in nature or in the work of man Thus we may infer by Analogy,

Cutting-tools have edges, and places for handles,
These flints have edges and places for handles;
∴ These flints are cutting tools

Here we go at once to analogy, without passing through the observation of conjunctions *as a first suggestion.* It is not simply from seeing handles and edges conjoined in knives or chisels that we know a cutting-tool must have a handle and an edge We know this from extraneous considerations, especially from the texture and use of the human hand. But nevertheless we might never have discussed the coherence of these attributes if we had never seen them in conjunction, and further, in the new instance of the flints, we have had to go through a process of observation which told us that here too there were both edges and handles or places for handles Thus the present example illustrates at once the true nature of analogy, and the ground and degree of its dependence on Induction. The observation 'Flint tools have edges, flint tools have handles', does not linger in the stage of induction, simply because we are not dependent on the nature of flint tools to tell us the connection in use between a handle and an edge We leap at once to this notion of cutting-tools, and compare the flints with them in respect of the conjoined attributes which we try to deepen and define.

Logical criticism of analogy

2. The logical nature of analogy may be analysed as follows.

Undistributed middle in fig. 2

1 As a formal syllogism in the second figure, having an affirmative conclusion, the analogical argument has the fallacy of undistributed middle We need not indeed trouble ourselves at this stage with questions of distribution involving the extension of the judgment Nevertheless a fault in the extensional relations of an argument infallibly indicates something which is *prima facie* a fault in its connection of content In the present case the fault is this—that what is materially the ground of Knowledge, the content which

underlies and links together the two matters which demand explanation, is, *qua* predicate or characteristic, in its wrong place, the place of a *consequent* Now according to the ordinary interpretation of the judgment, of which we have frequently spoken and which holds good for our present level,[1] the same consequent may have any number of independent grounds. There is therefore no formal necessity whatever for the two grounds or antecedents which in this case possess the same consequent to have any connection with each other beyond the fact that they do possess it.

But on the same ordinary interpretation of the judgment if we deny the formal consequent of either of the grounds while affirming it of the other, we can then deny that the two grounds in question are connected through the universal suggested in the premises before us Thus the negative argument escapes the formal defect which attaches to the affirmative. About any further or other connection that they may or may not have the denial tells us nothing, and therefore it is really a denial not of all or any connection, but of a particular connection through a particular middle term. The customary omission in the conclusion to specify the excluded connection is a fallacy *a dicto secundum quid ad dictum simpliciter*. It may be said that the denial takes on the absoluteness of the assertion on which it rests. 'Every Tory is a good man, He is not a good man, ∴ He is not a Tory.' If the major is true without reserve the conclusion is true without reserve, and in logic we are to suppose our premises true But still we must consider what the judgment means, i.e what it really is And no judgment is absolutely meant. The conclusion does not really represent the inference as a concrete thought unless we repeat in the conclusion, 'He is not a Tory, so far as his not being a good man prevents his being one' This refinement has no exclusive relation to analogical argument or to fig 2, for the negative argument falls into fig 1 as readily as into fig 2, the ground and consequent in negation being reciprocal In order to give the criticism a peculiar relation to analogy we must say, 'He is

[1] For the judgment is not purged of irrelevancy till it has passed through *scientific* induction

not a Tory, so far as the peculiar way in which he is not a good man prevents his being one.' This positive way is variable.

Except then for the purpose of negative inference, which is not purely analogical, the form of analogical argument in fig. 2 is at variance with its matter, and represents no inferential necessity at all. Inferential necessity is either subsumptive or constructive In order to obtain subsumptive necessity one of the premises would have to be converted and become a major premise. And this conversion would have to be material, not merely formal; for a formal conversion of an affirmative generic judgment would destroy its generic character and make it incapable of standing as a major premise. The Analogical inference as it stands shares with enumerative Induction the peculiarity of being a subsumptive inference without a major premise—an argument from a concrete content without the assertion that this content is absolutely dominant for the purpose of the argument Again, in order to obtain constructive or abstract necessity the relations of the contents must be reduced into abstract and mechanical relations akin to the universals embodied in the pure hypothetical judgment or in the equation. The formal defect of analogy as it stands is expressed by the 'probably' inserted in the conclusion, which indicates a coherence under conditions not precisely known

Real value of Analogical Argument

11 Seeing that the form of analogical argument is *prima facie* inconclusive, on what does its value really depend? It represents, we said, the phase of thought in which we no longer count but weigh the examples It might be said therefore that analogy is a material and not a formal inference. This is so far true that the value of analogy depends not on a formal conjunction of attributes in a subject, but on the material governing power or essentiality of a predicate Everything turns on the 'importance' of the character which forms the common predicate, and this 'importance' is closely bound up with completeness of definition I will return directly to the question of the importance of characters, but it is necessary first to point out that an inference without assignable form is no inference at all, and that therefore it is

not correct to say that analogy is a material and not a formal inference. Every inference has a form, in the sense of a definite relation between the differences of the universal which the inference exhibits. I do not however mean to say that such a form can be laid down antecedently for every inference. It is this relation, as we have seen, which varies with the nature of the universal, and which by its variations dictates the main types both of judgment and of inference We need not here take refuge in the form of complete subsumption through the conversion of one judgment. When that is possible—and no doubt analogy is on the road to it—we are already beyond analogy. But the form of analogical argument is to be found in the neglected aspect of the ordinary judgment, its strong *implication* of a value in the predicate. If all judgments were taken as reciprocal, analogy would be *ipso facto* an argument from ground to consequent, *besides* being as it is now from consequent to ground 'Two grounds that have the same consequent *ought* to cohere,' is the form as it stands. 'Two grounds that have the same consequent are consequents of the same ground, therefore *must* cohere,' is the *implied* form, or, as we said above, the matter of the inference This form might be identified with fig. 3, and so take us back to Enumerative Induction But the content being changed from instances into their defined nature, we are rather taken forward into the hypothetical judgment as used in constructive inference, *or* to complete subsumption in fig. 1. With reciprocal judgments the syllogistic form becomes indifferent, and the premises of analogy are implicitly reciprocal.[1] That their reciprocity is implicit and not explicit is in accordance with the nature of analogy, which is as we have agreed only a method of problematic conclusion, not a method of absolute and precise determination.

[1] The explicit form is A is B; C is B, C is A — The implicit form is B is A; B is C; C is A

But the implicit form deals not with mere instances as in Induction, but with *that* B which was found to be essential in A and C. Therefore the whole implied argument is—

If B, then A (which is also a sign of B);
If B, then C (which is also a sign of B),
∴ If C, then B and therefore A.

I now return to the question of the material 'importance' of characters, the attribute on which the implicit form of analogical inference depends It is possible, and is verified in daily experience, that a character or group of characters from which the remaining properties of an object cannot at present be derived by mechanical analysis may either amount for inferential purposes to a ground, or at least may serve as an unerring index of the qualities of the object Such a character or group of characters, and I may add such a symptom or group of symptoms, has logical 'importance'.

I will commit myself at once to the opinion that this importance rests in every case on a presumption drawn from what I may call morphology, or from teleology, these two ideas being regarded as secondary and primary forms of the same conception. In all objects or institutions made for a purpose by man, at least while their nature corresponds to the intention embodied in their structure, there is true teleology. In all organisms, parts of organisms, objects or structures that live a life or have in any way a being that is to our eyes individual and distinctive, there is morphology or *de facto* teleology. I am aware that a *de facto* teleology is a contradiction in terms Purpose implies more than actual result. But as a description of a result in language borrowed from a result of another class—from human operation—the phrase though contradictory may pass, and it is in this sense alone that I employ it If there is a peculiar principle underneath this ambiguous class of results, it is at least not, within our knowledge, a principle of intentional adaptation by a foreseeing consciousness. But I repeat that every universal, every persistent identity in difference, just because it is a synthesis of differences in a universal, may be regarded as a concentration of means in a distinguishable result. How far such a view is 'subjective' or in what sense it renders a real aspect of the nature of things is a question to which I shall return

It is on this characteristic of all universals that anticipation by analogy rests. Where we have a constitutive equation, i. e. an absolute rule for the synthesis of the differences, we can construct without teleology and without analogy. If,

again, according to Bacon's dream, science could arrive at 'forms' or underlying qualities, capable of doing the work of constitutive equations for every natural attribute and every natural object, then in presence of such a form we should not need analogy Or where complete concrete subsumption prevails, where we have actual conscious teleology, e g. in the philosophical analysis of laws, institutions, opinions, logical activities, we should never need analogy but for the all-important fact that all these contents are determined by growth and history in a way of which those who make and use them are not aware, and which they cannot control. Hence all such matters have an organic and almost a mechanical side, and can be treated by comparative science—analogy, as well as by philosophy—analysis No actual law or institution or idea has its form exclusively determined by its explicit purpose. All of them are loaded with inherited matter which may in part be an incumbrance, but in large part serves purposes wider and not less essential than the purpose which consciousness is able to recognise And in human affairs there is a bridge between the unconscious and the conscious function, for the latent purpose—which as latent is not a purpose at all—is actually one side of the explicit purpose and is continually emerging into explicit consciousness, so that the *de facto* operation of human energy in one stage *characterises* the explicit purpose of that stage itself and *enters into* the explicit purpose of the next Thus philosophy can deal with even latent or unconscious significance partly as an aspect and partly as a condition precedent of conscious significance The history of religion or of any achievement of man's intelligence is inexhaustible in illustrations of this principle Precise knowledge, however, whether affirmative or negative, whether constructive or subsumptive, excludes anticipation by analogy, for analogy is a stage on the road to precise knowledge.[1] This condition satisfies the popular requirement [2] that in Analogy we must know neither necessary connection nor necessary exclusion

To explain the connection of Analogy with Teleology, I will at once take the strongest class of examples

[1] Compare however p. 217, supra [2] Mill's Logic, II, p. 88.

We are on the border between analogy and a higher form of inference when we argue from a presumed genuine teleology to the conjunction of qualities in the content that it governs. We are so far already outside analogy that the argument must consist largely of judgments upon actual mechanical adaptations, the *de facto* use of which is a matter of precise knowledge and not of presumption We are not wholly outside analogy, because the *de facto* nature of these adaptations is not enough, in the case supposed, to carry our conclusion, which needs the actual and intentional purpose. This, in inferences of the type supposed, can only rest upon presumption If we find, near a known seat of stone-age inhabitants, some flints of peculiar shape and make, it is a mere judgment on a matter of fact to say whether they *are* adapted for use as knives or as hatchets; but to say what they *were* meant for, and so actually used for, and therefore whether we may expect to find near them chips of wood or bones of animals, is a question for analogical inference based on the nature of the country, on the known or supposed habits of the people, and on any convergent indications in the adaptations of the flint tools themselves Any character in such a connected group of characters, that gives the key to the pervading purpose of the whole content under examination, is an 'important' character 'Importance' is relation to the purpose or pervading nature, the 'import', of any system. If both qualities to whose coherence we conclude are directly derivable from the presumed purpose, then we are so far beyond analogy, but the element of presumption which consists in ascribing true intention or purpose is still analogical. If one or both of the qualities to whose coherence we conclude are not derivable from the pervading purpose, then we are more completely or quite completely in the region of analogy, and the inference will simply be that identity of purpose is probably both a ground and also a consequent of uniform structure

In the former case, both properties being derivable, we have :—

The flint knives *are* adapted for cutting wood;

Cut logs and chips *are* connected with cutting wood;

∴ Cut logs and chips[1] will probably be found near the knives (i.e *if* the adaptation which we observe in the knives is a true index of the use for which they were really made, and to which—a further presumption—they were actually put.

Or again .—

A telescope with the eye-piece at one side of the tube is probably a reflector;

Lord Rosse's telescope is a reflector,

∴ Lord Rosse's telescope probably has the eye-piece at one side of the tube

In the latter case we have, one or both properties being not directly derivable from the presumed purpose,—

A clock with a watch-movement is a carriage-clock,

A clock with plate-glass sides is a carriage-clock

A clock with plate-glass sides is probably a clock with a watch-movement

There are pendulum clocks with plate-glass sides, so the analogy is not made out. There is a reason why a clock with a watch-movement should be a carriage clock, viz. that it is a mechanism which the motion of the carriage will not disturb. But there is no reason that I know of for the second premise, which rests on mere custom and turns out to be a precarious basis of argument

Or again .—

A horseshoe-stand is a common shape of French microscope-stands,

A very simple stage is common in French microscope-stands,

∴ With a very simple stage one may expect a horseshoe-stand

The horseshoe shape is not the best for securing steadiness, so that there is no direct connection between the two properties of which the conclusion alleges the conjunction, and the purpose indicated by the name microscope-stand. And I have purposely introduced a limitation referring to the character of the conditions under which the purpose is carried

[1] I omit, merely for brevity, to say anything about the possibility of the chips &c having been removed

out, '*French* microscope-stand.' in order to illustrate the boundary-line between genuine conscious teleology and mere characteristic individuality Probably in this example the two join hands, French makers must have, or have had, an idea that the horseshoe shape and the simple stage best secured the purposes of the microscope

The former set of examples, with both properties derivable from the purpose, illustrate the general type of analogical inference affecting attributes connected with self-preservation in the organic world—chief or fundamental attributes The latter, with one property or neither derivable, gives the general type of analogical inference affecting attributes which are not vital, but which have in heredity or otherwise their own degree of constancy. But these examples, though illustrative of organic relations, are by their connection with conscious teleology upon a higher level than those relations

A further class of inferences, to which the last example prepared a transition, is intermediate between conscious and merely organic teleology I allude to the enormous class of daily inferences relating to time or locality of origin or to authorship, in the case of literary, artistic, or mechanical productions Obviously the conception of the presumed period, place, or person, as significant of peculiar characteristics, may itself be the presumed ground of inference in virtue of which the coherence of certain properties is analogically expected, or the individual characteristics stated explicitly may be the ground, and the name may be inferred by analogy Supposing a single conjunction (Inductive in its nature) to have furnished the suggestion 'This design which is beautiful is by A B,'[1] then analogical enquiry will infer in the form 'The beauty of this design is drawn from nature yet original and full of thought, A B 's designs are drawn from nature yet original and full of thought; · The beauty of this design is probably the beauty of one of A B 's designs.' The reader can construct further variations of this very common type of argument for himself

The common analogical inferences which run throughout our treatment of organic and even of inorganic nature rest

[1] In the form 'This design is beautiful This design is by A B'

practically on the existence of natural kinds, that is to say, on morphology or on *de facto* teleology I may explain the distinction of degree which I attach to the two expressions by reference to the general conception of self-maintenance or self-preservation Where self-maintenance means simply any reaction of a distinguishable agent against or upon an influence approaching it from without, I should speak of the distinguishable individuality of that agent as morphological, i e as consisting simply of a formal or recognisable self whose unity is charged with no especial interest Where on the contrary there is a self maintained or preserved which exhibits the attributes of life, or, however partially, of consciousness, I should say that the facts, and not our own choice, demand that we should apply the paradoxical idea of an *actual purpose*, or *de facto* teleology It is obvious that the conception is one varying in degree and not limited by any despotic necessity to the range of the organic world After all, it is a plain fact that elements combine in processes, and the moment we single out part of a process as a result [a] we introduce the conception of *de facto* co-operation on the part of means towards an end Every distinguishable persistent content may thus formally be regarded as an end, without, of course, any implication whatever of an aim pre-existing in a foreseeing consciousness. That from a purely mechanical or analytic point of view such constituent relations are absolutely indifferent to the whole which they co-operate in constituting is a necessary consequence of regarding such relations in their isolation. But without the further aspect supplied by an interest in the whole as such, not merely ethical and aesthetic judgment, but scientific judgment itself, loses all power of discrimination, and therefore all objectivity Even to name, as we saw in the Introduction, is to select and to identify

Analogy then rests on the 'importance' or significance of attributes, an idea well illustrated by systems of true conscious

[a] This singling out a part as the 'end' is essential to teleology, but must ultimately be determined by the nature of the 'whole' Thus teleology strictly speaking is a partial conception, and less ultimate than the idea of the whole See e g McTaggart's Commentary on Hegel's Logic, sect 255 I hope to deal more fully with this point in a forthcoming work

teleology which happen to be but partially known to us, but really dominant throughout the various grades of actual self-maintenance and individuality presented by the organic and inorganic world. Analogy is never demonstration A thorough mechanical nexus and a subordination to a conscious purpose in an intelligent being or rational system both *pro tanto* exclude it

No ratio of Identities to Differences

iii The obvious truth that *ceteris paribus* the predicate with more meaning has a deeper grasp of the import of the reality which it qualifies, and so is the safer ground of inference respecting that reality, has introduced the fatal fascination of the ratio into the doctrine of analogy I quote from Mill[1] a complete account of the idea so generated 'Since the value of an analogical argument inferring one resemblance from other resemblances without any antecedent evidence of a connection between them depends on the extent of ascertained resemblance, compared first with the amount of ascertained difference and next with the extent of the unexplored region of unascertained properties, it follows that where the resemblance is very great, the ascertained difference very small, and our knowledge of the subject-matter tolerably extensive, the argument from analogy may approach in strength very near to a valid induction If, after much observation of B, we find that it agrees with A in nine out of ten of its known properties, we may conclude with a probability of nine to one that it will possess any given derivative property of A If we discover, for example, an unknown animal or plant, resembling closely some known one in the greater number of the properties we observe in it, but differing in some few, we may reasonably expect to find in the unobserved remainder of its properties a general agreement with those of the former, but also a difference corresponding proportionately to the amount of observed diversity'

This passage gives us the valuable suggestion of negative analogy, to which I shall return below But as to the idea of ratio, we must be faithful to our principle that in analogy the examples—or the properties, it matters not which—are to be weighed and not to be counted Mill's idea is in fact

[1] Mill's Logic II p 90

that by counting the properties you weigh the examples And every one must be struck by the verisimilitude of the view which the above passage propounds But on pressing the matter home we see that at least the form which it gives to the right idea of insisting on the depth of the common predicate is a wholly unreal form, and takes us into the wrong track. There is no ratio without a unit, and, to begin with, a 'resemblance' (a point of identity) is not as such a content that can be employed as a unit It is impossible to say what is a point of identity and what amounts to many such points. Identity is systematic through and through, and its 'points' derive their value from their relation to a system It is impossible to break up such a system into numerable parts and points without prejudging the very question—the question of their respective values as index-qualities—which the enumeration is supposed to be a straightforward method of solving

It is worth while to illustrate this point Suppose that we are asked to compare two given plants of different species in order to determine their botanical affinity on analogical evidence—on the evidence of observed resemblances or points of identity matched against observed differences In order to meet the retort that affinity in botany is what we like to make it, by the arbitrary value which we attach to the characters, I will assign to affinity the definite meaning of relationship by descent as indicated through the accepted natural classification Thus the actual fact to be discovered by analogy, put at its lowest value, is how the plants in question are classified in the accepted natural classification, and put at its highest value is how the two are related by descent. Let one of these two given plants be a shrub six feet high, with branches and stalked leaves, with its inflorescence in branching masses, without any 'bract' or small leaf at the base of each mass, with white flowers, with nearly black fruit, and when young having its leaves covered with silky hairs. Let the other plant be herbaceous, six inches high, not branched, with no stalks to its leaves, its flowers in heads which do not branch, but which have four conspicuous yellowish 'bracts' at the base; the flowers are purple, the

fruit red, and the leaves have only a few hairs on them Add to these differences that the general look and habit of the plants are very different Now set against the above differences such points of identity as number of parts of the flower, structure of the flower (polypetalous with inferior ovary and epigynous stamens and petals), the structure of the fruit, the partly identical growth of the head of flowers (an 'umbel' in the small plant, and a 'cyme' in the large one), and certain peculiarities of the leaf surface, such hairs as there are being closely appressed, and the nerves having a peculiar prominence Perhaps it is rather easier to make out a long list of identities between the two plants than to make out a long list of differences. But we might really lengthen either list to infinity by subdividing in detail characters which have been mentioned in the abstract. I have little doubt however that in microscopic structure of petals, pollen-grains, &c there would be some striking identities, hardly compensated by differences Still we can see at once that no ratio between number of identities and number of differences can be constructed which will tell us anything—the number on each side is *almost* purely arbitrary The *value* or *importance* is what we have to consider

In what does the value of characters consist as a basis of natural classification or as a proof of common descent? Largely no doubt in their connection with the number and general arrangement of parts Evolution only accounts for essential changes and their consequences, and though it may modify the number of parts and their arrangement, at first superficially and in course of generations more profoundly, yet an older general arrangement survives long beneath the modification and can as a rule be traced there.[1] The arrangement of those four or five whorls of leaves on a shortened stalk, which we call the flower, is thus one dominant feature in the analogical estimate of a plant's affinities Its inferential value is the same whether we call it one point of

[1] See Darwin's beautiful verification of the modifications of the orchid-blossom by following the spiral vessels which indicate the position of the original petals in the modified corolla, Fertilisation of Orchids, p 289 ff. Homology is an analogical conception in the logical sense

identity, or five, or twenty It will be observed that in considering a plant in the light of evolution we have a combination of the higher and lower forms of teleology, related negatively to each other Just as in the example employed above of the 'French microscope-stand', we have here within the basis of analogy *both* a definite purpose suggesting definite means, *and* the mere tendency of individual or racial characteristics to perpetuate themselves The local manufacturers' custom is *gradually* modified towards the better mechanical adaptation, as the organism is *gradually* modified towards the better mechanical adaptation At any moment a manufacture or an organism is a compound of recent useful change, and of survival, some of which survival is obsolete, and some, the major part probably, has never ceased to be useful

Thus, in the establishment of common descent, there is a special value in what recent evolutionary modification is likely to have spared This would include both underlying arrangements which evolution would take very long to touch, and trifling details which it would have no reason for touching.

The account which I gave of the differences between the plants in question is the account of a common observer; the account of their identities is the account of a botanist This, it may be said, is enough to vitiate the argument against a ratio, for of course knowledge and judgment are to be used in making the enumerations But the idea of enumeration gives us no right to employ botanical knowledge It is only the idea of a presumption resting negatively or positively on teleology that enables knowledge to operate in assigning value to index-characters Thus in judging of the plants in question we have to distinguish the element of heredity, or self-maintenance in the lower sense including the deeper and also the more trivial survivals, from self-preservation in the higher sense (though even here not involving conscious intention) in the set of recent differentiations introduced by evolution. Even thus we omit much that is most interesting and important Evolution can for example assimilate plants of different origin as well as differentiate plants of the same descent, and we ought properly to show that any identities on which we

rely cannot have been initiated by such assimilation [1] We have so far anticipated this demand by requiring the identities to refer to matters with which evolution is not likely to have recently interfered

The plants of which I have been speaking are *Cornus sanguinea*, common dogwood, and *Cornus suecica*, the Swedish or dwarf cornel. They are in fact species of the same genus. But the four yellowish bracts round the flower-heads of the dwarf cornel have the appearance of petals and form a striking superficial difference between the two plants, not to mention the enormous disparity in size And now, possessing the names of the two plants, and having thus opened to us what is known of their local distribution, we can confirm our analogical estimate based on passive self-preservation or heredity, by a presumption drawn from the coherence of the modifications which that estimate ascribes to active self-preservation, i e to evolution since the divergence of the species. The smaller plant is sub-Alpine and Arctic, the larger belongs to southern England and to temperate climates This fact suggests that the smaller plant, whether driven northwards by a change of climate, or simply maintaining a portion of its old habitat, has been dwarfed or has not grown larger, and has compensated for its smallness by the brilliant simulated flower Its distinctive leaf-growth and flower-growth may be summed up as a dwarfed or at least as a miniature growth, stalks and branchings having disappeared or not appeared How the contrast between the inconspicuous small dark purple flower of the small plant and the larger white flower of the large plant is to be explained I am unable to suggest. But it is hard to suppose that the petal-like bracts of the small plant are not in some way a compensation for the inconspicuousness of its flower.

Concurrent Analogies Negative Confirmation.

IV Before reducing this example to regular form it is worth while to remark that, by assuming the two plants to be given us to compare, we presuppose the work of enumerative induction to be done to our hand. And in fact, where a subject-matter falls under an existing science, we are already in general

[1] Such identities are called in biological language 'homoplastic', as opposed to 'homogenetic' The daisy, for example, is a head of flowers that mimics the appearance of a single flower

beyond the stage of Enumerative Induction, though it may of course operate in particular unfamiliar instances But speaking generally, the abstract ideas which guide Comparison are active in every special science as precepts filled with a content capable of guiding elementary observations. We approach an element, or a plant, or a part of speech, just as the state approaches a taxpayer, with a schedule in which the heads of our requirements are already jotted down, forming an abstract analysis of the predicates with which, in the class of cases in question, we are concerned But if, supposing ourselves unfurnished with such a schedule, we construct a conjunctive Induction for the case before us, it would run in some such fashion as this .—

These two plants [1] have similar berries ,

These two plants have similar leaf-nerves ,

. The conjunction of similar leaf-nerves and similar berries may not be an accident [2]

Then the Analogical argument would fall into some such shape as—

Having similar berries is conjoined in these plants with a pervading identity of underlying (and so long inherited) structure ;

Having similar leaf-nerves is conjoined in these plants, &c , &c ;

·. Having these similar berries is connected by an underlying (and so long inherited) structure with having these leaf-nerves

And, as we saw, two further analogies would confirm this —

The resemblance in the berries is conjoined in these plants with trivial identities of structure (e g closely appressed hairs on the leaves) not likely to be modified by evolution ;

The resemblance in the leaf-nerves is conjoined in these plants with trivial identities, &c , &c ,

[1] 'Plant' here = species This equivalence itself rests on analogical argument, which however is presupposed in any highly developed language, though not for all classes of objects See above on Individual Judgment, and Lotze, Logik, sect 14, on 'first universals'.

[2] In strict form, 'Similar leaf-nerves perhaps are (involve) similar berries.'

∴ These attributes are connected with each other by attributes probably hereditary

And contra-positively, giving affirmative content to the negations, but leaving them their negative value in inference —

What is not identical in the fruit-growth (e g the clustering and the isolation of the fruit) is not a property likely to be remotely hereditary (because obviously modified by alteration of length of the stalks, i.e. by dwarfing) ;

What is not identical in the flower-growth (e g the presence and absence of the four white bracts) is not a property likely to be remotely hereditary (because obviously related to the inconspicuous flower, i e to dwarfing) ,

∴ What is not identical in the flower-growth of these plants is united with what is not identical in the fruit-structure as parts in a set of properties not likely to be remotely hereditary.

The true relation of these arguments to each other would be that they should form a single analogical inference, in which each positive premise and the positive conclusion should be *materially* defined and limited by the corresponding negative judgment When this reciprocal adjustment was completed, we should have analysed each of the plants into two related systems, in respect of one of which systems the two plants would coincide, and in respect of the other of which they would differ The one system would point to the construction of a common ancestor , the other would point to the evolutional history of the species since their divergence As their boundaries would precisely fit each other we should, in arguing on the basis of either, be supported by the defining influence of the other , that is to say, every judgment A is B would be supported by its converses Not-A is not-B, and Not-B is not-A But though such negative relation of positive contents is valuable in analogy as elsewhere, yet to be fully effective it presupposes great accuracy and exhaustiveness of analysis, which is not usually to be obtained where analogy flourishes, and which, if obtained, takes us beyond analogy By its negative aspect such inference leans over to Scientific Induction, while by its appeal to the coherent nature of a system it tends to pass into definite or philosophical subsumption

v Analogy, like Enumerative Induction, is a critical point from which two tracks of knowledge diverge In assigning the coherence of attributes within a system we cannot but be confronted with negative relations, which are the conditions of all precise determination and of all causal or necessary inference This feature of deepening analogical consideration points forward to scientific induction—the analysis of the teleological whole, or, *a fortiori*, of the unformed datum of perception, into its definite and necessary constituent relations The goal of this path is the abstract Hypothetical judgment which forms, as we have seen, the point of transition between inference by combination of abstract relations and inference through the nature of concrete subjects. Divergent tendencies in analogy.

On the other hand, if we continue to regard the concrete subject from the point of view of its totality, which has begun to dawn upon us in analogical inference, our principle of inference tends to assume the shape of a concrete whole, understood as a synthesis of abstract relations Such a subject combines within itself, in perfect equilibrium, the two aspects of the universal which have occupied us throughout—the aspect of concrete reality and that of abstract interconnection Inference based upon contents of this nature may take the shape of the complete subsumptive syllogism in fig 1, or, when more adequately expressed, of inference under a disjunction, or finally of the *explicitly* teleological inference respecting beauty or goodness

It is plain that to employ in inference such a subject notion as I have just indicated presupposes a detailed mastery of the abstract relations which enter into it, and therefore presupposes the advance, which was above described as a divergence taking the direction of the hypothetical judgment. The subject can be known as an embodied purpose only by inference based on its necessary constituent relations Why then should we regard the abstract hypothetical judgment as belonging to a track that diverges from the direct high-road of concrete knowledge? The reason is simply that in the formation and combination of Hypothetical judgments we sacrifice reality for the sake of necessity, and lose an element which was present in Analogy In other words, the Hypothetical judgment with

the combining inference that belongs to it is itself an apex or climax of one whole tendency of knowledge—of the mechanical view of the world which considers necessity apart from reality, and to which disease is as orderly a sequence as health. This is the view of the eye of *purely* physical science, which in a catastrophe that should extinguish life on the surface of the globe might see 'no more disorder than in the sabbatical peace of a summer sea'[1] This aspect of knowledge has been sufficiently analysed and discussed in our treatment of mathematical inference which is its purest form My excuse for constantly recurring to it must be that a thorough understanding of its range and consequences is the primary condition of any clear thinking on the subject of the reign of law, which, as thus isolated, is absolutely indifferent to the purposes and interests that give reality its relation to mankind The apparently self-dependent completeness of this analytic view of the universe gives it a right to an independent development, although this right of independence which it claims may also be regarded as a limitation to which it submits. Science, professing to be purely physical, has, as we have partly seen and shall more fully see, in our own day at least occupied itself with ideas which fall outside the categories of abstract necessity. And this was hardly avoidable; for we have seen over and over again that necessity must rest upon reality, and that therefore the self-completeness of the mechanical view of things is in this ultimate instance merely apparent.

[1] Professor Huxley in Contemp Review, Feb. 1887. For a further consideration of this point of view, see chap. vii, below.

CHAPTER IV

Scientific Induction by Analysis

The moment we begin to demand precise definition of relations and to attempt analysis, we are, as the development of analogy proved to us, face to face with inference from negations.

Negative Inference. Its general nature and conditions

1 I will now speak shortly of the nature of this inference, and will then attempt to explain its function in inductive analysis.

1. All inference depends on the relation of differences within a universal, and negative inference, in its fundamental nature, does not deviate from this principle It may be laid down at once that the ideal of negative inference is to be looked for in Inference under Disjunction, as the ideal of negative Judgment is to be looked for in negation under Disjunction. Our discussion on the connection between bare negation and significant negation will have prepared us for this conception

But negative inference in this sense is not a peculiar or separable form of reasoning Negation and Affirmation in disjunctive or precise thinking are respectively double-edged, and though this character which thought acquires from being imbued with negation is pre-eminently distinctive of thought that has reached the stage in question, yet it does not admit of being ascribed to negation as contrasted in the abstract with affirmation. It is for this reason that in treating of calculation and of geometrical reasoning it has been unnecessary to devote special attention to negative inference. Negation appears no doubt in mathematical principles and theorems, e g. in the definition of parallel straight lines, or in the theorem that if two circles cut one another they shall not have the same centre. But as a general rule—I will not venture to say universally—it is easy to substitute for a negative expression of this kind a definite though not thoroughly particularised positive expression, which possibility goes to show that the negative expression was adopted rather for some rhetorical convenience —e g. for brevity—than because a positive expression was unattainable. Often, as in the second of the above examples,

the negative form of a conclusion arises from the employment of an indirect proof And an indirect proof can only operate under a disjunction True, in the present example the disjunction *seems* to be merely formal,—i e to consist simply in a positive judgment and bare negation; 'have the same centre,' 'have not the same centre.' But 'have not the same centre' in case of circles means 'have different centres', the relations of which are easily seen in general from the content of the indirect proof itself In mathematical or pure mechanical reasoning there is no room for anything approximating to bare negation—the excluding motive must be definitely demonstrable, and contains the idea, though not necessarily the particulars, of an assignable quantitative difference between the excluding and the excluded content [1] Incommensurable quantities are, *so far as incommensurable*, not quantities at all Throughout this region of inference therefore negation and affirmation have as a rule their ideal complementary position, and no ground is given for a special and separate treatment of negative inference For this same reason, however, *where* and *in as far as* negative inference formally occurs within this sphere, it *formally* falls outside the character required of combining inference, and must be technically referred, not indeed to subsumption, but to a special genus which also includes any negative reasoning that may arise within the limits proper to subsumption

But when we turn from calculation with precise scales of difference to the traditional form of the subsumptive syllogism, the inherent paradox of negative inference immediately stands in our path The syllogistic rules undoubtedly contemplate inference *from* bare negation, and also, so far as they are concerned, inference that has bare negation for its conclusion. But all inference, we have been insisting, rests on an identical nature or a pervading universal, which prescribes a relation, whether definite or indefinite, between its differences. How can a universal prescribe a relation between itself and a content which falls wholly outside it, and is absolutely disparate and alien to its nature?

[1] So that we need not say 'is' or 'is not', but 'differ by a yard', 'the same to a yard'

We were met by this difficulty in the discussion of the negatively infinite judgment, and of the ultimate indemonstrability of the negative *as such* Where there is absolutely no connection it is impossible for denial to be intelligible ; and what is not intelligible cannot convey a truth Denials which though frivolous seem true have some shade of assignable meaning beneath them [1] Therefore the only true meaning of an inference in Celarent is e g.—

To be a man excludes being a monkey (in virtue of a certain universal nature which *including both* assigns an intelligible difference between the two) ,

Socrates is a man ,

∴ Socrates is not a monkey.

But the ordinary graphical representations of the extensive syllogism slur over this difference, and leave us to suppose that from an *utter* absence of connection together with an assignable connection we can infer an absence of connection, which is to introduce the infinite judgment into inference It is worth pointing out, however, that even the true reasoning in Celarent, considered as starting with its conclusion as a suggestion to be proved,[2] *begins* with something nearer a bare denial, and *ends* with an intelligibly motived exclusion. 'Socrates is not a monkey ,' 'Socrates having the nature of a man, cannot be a monkey.'

Thus in syllogistic negative inference we find (1) the erroneous idea that negation is utter disconnection, and that negation in this sense, bare negation, can be intelligibly asserted and inferred And we have (2) the true idea that negation in order to be significant must fall within a controlling identity, although its ultimate shape is indemonstrable, and *qua* indemonstrable or irrational falls into a genus by itself, and outside both subsumption and construction. And as a testimony to the inevitable power of formalism in any symbolic scheme of inference, we may point out (3) that the accepted syllogistic

[1] For examples, see Book I, chap vii

[2] I am confident that this is as a rule the most instructive point of view from which to analyse inference, corresponding best to the vital process of thought The essential question is, what difference is there in the conclusion as a judgment, before and after, or in and out of the inference

rule that there can be no inference from two negative premises is really an offshoot of the idea of bare negation.

No conclusion from two negatives.

11. If we have two bare negations or mere disconnections —negatively infinite judgments—nothing follows, because nothing is said. And as two negations will always present the external appearance of two mere disconnections, and will be ambiguous in interpretation, concealing their positive aspect, it is well to maintain this rule in a symbolic scheme of inference Once at least in every inference, the rule says, you must show your hand, and develope your universal in terms of its positive content Then, with one positive relation of content before us, we shall not be far out, it is implied, in interpreting the denial which is subjoined to it

Nevertheless, it must be maintained that the negative syllogism acquires scientific value just in the degree in which this rule is disregarded and in which the syllogism is consequently informal If negative inference has any value it is the establishment of exact and self-consistent boundaries between the species of any genus, or the modifications of any principle.

The rule that two negatives give no conclusion has been impeached by good authorities [1] in respect of the third figure—the figure which we followed in our account of Enumerative Induction. This figure is obviously adapted for the expression of a positive instance, or of an exception—of an instance which comes under the condition of a rule but of which the consequent annexed by the rule to its condition does not hold good.[2] Why should it not also express a true *negative instance*, i e. a negation which does not conflict with but corroborates the rule by coming neither under the condition nor under the consequent? In insisting on such a principle as 'Whatever gravitates is matter' we may often find ourselves relying on such instances as 'Light is not matter, Light does not gravitate, ∴ Something which is not matter does not gravitate', or *vice versa*,

[1] Lotze, Logic, sect 89, Bradley, Principles of Logic, p 254, quoting Jevons.

[2] Or in case of a reciprocal judgment, also *vice versa*. Even in a rule which is not reciprocal, a great extension of the consequent beyond the condition is always suspicious

making in favour of the contra-positive converse either of 'Whatever gravitates is matter' or of its reciprocal 'Whatever is matter gravitates'

It has been objected against this case of a conclusion from two negations that either the argument has four terms or one of the premises is affirmative If the two premises are mere denials, then neither of them can furnish the negative predicate required to be subject of the conclusion. For this subject must be a positive content merely determined in one aspect by a negation A bare negation cannot be subject in any judgment If on the contrary in one premise such a positive content negatively determined is the predicate, and the fallacy of four terms is thus avoided, then that premise is affirmative in form and the conclusion is not drawn from two negatives.

This objection is not only sound in form, but has substantial justification It is well known that to attach the negative closely to the predicated content has a tendency to transform the idea so negated from an excluded content into a positive opposite A form like 'not-moral' cannot maintain itself in living thought It must advance to 'immoral' or fall back to 'what is not moral' And if we admit that in the third syllogistic figure the same judgment can be both affirmative and negative—for accepting the above case of inference amounts to accepting this—it is hard to say why the same double character should not be adopted, and conclusions from two negatives introduced, in the remaining figures as well. In the second figure we should be tempted actually to take an affirmative conclusion from two negative premises, but as the ambiguous term is here the middle term, and not the subject of the conclusion, we cannot do this without treating both premises as affirmative (to secure an identical middle) and thereby reducing our conclusion to a problematic judgment,[1] thus,

Good workmen do not complain of their tools,
My pupils do not complain of their tools,
. My pupils are probably good workmen

[1] Both premises must be taken in the same way, though both may be taken as either negative or affirmative It is only the contrast of exclusion with assertion that can give a certain result in the figure

Or again,

Not good workmen are not satisfied with their tools;
Not-my-pupils are not satisfied with their tools;
∴ Not-my-pupils are probably not good workmen.

Our treatment of analogy would also be illustrated by the case in which Not-A and Not-B join in a positive C. But as both premises would then be formally affirmative, the case does not come under the present head.

It is impossible to deny that arguments like the above may have material weight. Their value rests on the possibility of gathering up the phenomena just bordering on a system we are investigating into a system of their own, α, β, γ, limiting and limited by the former A, B, C at every point. We nearly achieved this—rudely of course—in our comparison of *Cornus sanguinea* and *Cornus suecica* We then obtained two systems, A, B, C and α, β, γ, such that A, B, C were respectively not-α, not-β, and not-γ, while α, β, and γ were respectively not-A, not-B, and not-C. B and β, the two dominant or middle terms, stood, it will be remembered, for 'inherited from before divergence of the species' and 'modified by recent evolution' respectively, and the object was to attach all the peculiarities of the two plants systematically to one or other of these conceptions. Obviously in such a case it depends merely on our point of view whether we take as premises 'Not-A and Not-C are Not-B', or 'α and γ are Not-B', or 'Not-A and Not-C are β', which are the various forms suggested above.

In the same way it might be argued in fig. 1 that

No mere animal has language;
A deaf mute is no mere animal;
∴ A deaf mute has language.

Horrible as these arguments must appear to any one conversant with syllogistic rules, I do not see how they are to be kept out if the argument from two negations in fig. 3 is admitted.

There is however an indispensable condition on which alone any value can be ascribed to these inferences. This is that the negation of a content should in all cases be merely an aspect of a positive content[1] which is really in question, and

[1] There is a puzzling inconsistency in this identification, because the treatment of 'man is not-mortal' as an affirmative judgment, which

this we know to be the case in all significant negation, although not reckoned upon in the technical rules of the syllogism. Moreover, we have seen, that we are not bound to omit in the conclusion of inference any relevant matter given in the premises. But if not, we can in any case secure the positive significance of the denial of a content by supplying in the conclusion the middle term of which it is denied. Thus in one of the above examples we may conclude 'Light is something which does not gravitate, and is not material'.

The Negative Instance

111. We have now obtained the logical formulation of the Negative Instance. Like the Exception, it begins in Enumerative Induction, and is capable of development through Analogy. I shall assume throughout my examination of its working, in order to avoid uninstructive complications, that the rule or law suggested by Enumerative Induction, of which the negative instance is confirmatory, has come to begin with from affirmative instances, and is expressed in an affirmative judgment. Then we may formulate the cases supplied by mere Enumerative Induction, with their sequels in Analogy, as follows —

AFFIRMATIVE INSTANCE suggesting	EXCEPTION against	NEGATIVE INSTANCE confirming	Symbolic scheme of Instance
the *prima facie* rule that B is probably C			
A is C; A is B, . B is or may be C	A is not C; A is B; B may not be C.	*a* is not C, *a* is not B, . Not-B may be Not-C.	Enumerative Induction.
C is $x\ y\ z\ R^1$, B is $x\ y\ z\ R^1$, · B is for good reasons likely to be C	C is not $x\ y\ z\ R^1$ i e is $x_1\ y_1\ z_1\ R_1{}^2$), B is $x\ y\ z\ R1$, · B is not exactly C.	Not-C is $x_1\ y_1\ z_1\ R_1{}^2$, Not-B is $x_1\ y_1\ z_1\ R_1{}^2$; Not-B is Not-C i e C is B).	Analogy.

I have regarded as making 'not-mortal' a positive or significant content, was historically, as reference to a *class* 'not-mortal', the origin of the 'infinite judgment' which is the very type of bare negation. The interpretation employed in the text regards 'not mortal' not as a fictitious class but as a positive attribute excluding mortality

[1] Analysis of A

[2] The analysis of *a*, which is not-A, i e. not $x\ y\ z\ R$

Example of Instances	AFFIRMATIVE INSTANCE suggesting	EXCEPTION [1] against	NEGATIVE INSTANCE confirming
	the flower-structure in these two plants probably is characteristic of a common descent shown also in the leaf-structure		
Enumerative Induction	In these two plants there is similar leaf-structure, In these two plants there is similar flower-structure, The flower-structure may be an element in an inherited group of qualities to which the leaf-structure also belongs (or, in brief, may be at bottom one with the leaf-structure)	In these two plants there is not wholly identical leaf-structure, In these two plants there is wholly identical flower-structure, Flower-structure seems not to be generically connected with leaf-structure	In some aspects of these two plants there *is* a difference, e g of leaf-stalk, In some aspects of these two plants there *is* a difference, e g of flower-stalk (umbels v cymes), The difference of leaf-stalk may be connected through those aspects of the two plants) with the difference of flower-stalk
Analogy.	The leaf-structure in these two plants can be connected with a whole set of identical generic properties, The flower-structure in these two plants includes a whole set of identical generic properties, The flower-structure in these two plants probably is characteristic of a common descent shown also in the leaf-structure	Leaves have not the same stalk-arrangement (i e are sessile in one case and stalked in the other), Flowers have the same stalk-arrangement in both plants (i e are stalked in both), Flower-structure does not follow variations of leaf-structure (in these two plants, i e species)	The difference of leaf-stalk belongs to a connected set of aspects [2] of these two plants *not* concerned with their remote hereditary properties, The difference of flower-stalk belongs to a connected set of aspects [2] of these two plants *not* concerned with their remote hereditary properties, The difference of leaf-stalk is connected with the difference of flower-stalk by a relation *not* concerned with their remote hereditary properties

[1] The Exception of course cannot be made successful if the Negative Instance is to be so I have therefore treated the Exception as a mistaken interpretation of the facts which the Negative Instance interprets rightly

[2] Viz, the recent dwarfing of one plant

2. The object of Scientific Induction is, *given a suggested coherence*, 'B may be (probably is) C,' which has become through analogy a hypothesis in germ represented by an 'importance' attached to the mediating content *x y z* R, to bring such a coherence into the form of one or many pure Hypothetical judgments The outward and visible side of this process is to modify the rule, i e the contents B and C with their connecting content *x y z* R, so that there shall be no exceptions 'B is not quite C', and that the two contrapositive converses 'Not-C is not-B' and 'Not-B is not-C'[1] shall be true when filled up with positive contents precisely excluding B and C respectively The inward and intellectual side of the process however simply consists in grasping a necessary relation based upon some fundamental reality. This essential activity of the scientific spirit can only be characterised beforehand in respect of its most general attributes, which are embodied in the external process to be described as Scientific Induction We can affirm from the known nature of the logical universal that it must be purified by exceptions and finally limited by negations But as all data presented to us are thoroughly concrete, it follows that there is an endless possibility of erroneous abstraction and construction in all adjustment of contents to one another, so that the outward and visible side of induction, though knowable in respect of certain essential phases, can never assume the character of a mechanical method or royal road to knowledge In the same way the ultimate necessity of the law or principle at which we arrive can be guaranteed by no general considerations It depends in general, we know, on the systematic necessity of the negations, which, representing its relation to the reality within which it falls, hedge it in on every side and exhibit it as no longer itself, but as transformed, whenever and in as far as their limits are passed But the specific necessity of individual truths cannot be assigned by any general theory of science.

Scientific Induction

1 Induction then in its most general sense consists in satisfying the principle of sufficient reason by an analysis of

Induction and other Inference.

[1] The contrapositive converse of C is B, which affirmative judgment, and therefore its contrapositive converse, must be true if B is C is to be a pure or reciprocal judgment

experience, directed to revealing the true coherence of differences within universals But as soon as this is stated, a difficulty arises in distinguishing Induction from Inference as such, which has precisely the same object And this difficulty has, historically speaking, prevented the range of Induction from being consistently defined As in Jevons' theory of Induction, the most recent and, so far as I know, the most thorough and appreciative account of the operation, so in Mill's famous analysis of the four methods of experimental enquiry, we are dealing with processes essentially deductive On the other hand, if we try to confine ourselves to what has been termed 'Inference from particulars to particulars' we cannot meet the requirements of Scientific Induction. The name Scientific Induction is indeed something of a contradiction in terms Induction *is meant to mean* the treatment of instances In this meaning the idea of enumeration and even of the calculus of chances is confused with the idea of an analysis of observations—a confusion all the harder to disentangle, because number of observations does as a rule assist analysis and contribute to eliminating error Scientific analysis as such, however, does not deal with instances, but only with contents When we speak of a scientific treatment of instances, we mean a precise determination and skilful resolution *of their content*

Therefore the distinction between Induction and other forms of Inference, erroneously described as the distinction between Induction and Deduction, is chiefly a distinction of *aspects*, largely based on a confused idea of Induction, but yet in some degree justified I have just spoken of the confused idea in virtue of which Induction is regarded as a treatment of instances pure and simple I need only add that a semi-numerical content may often have to be added to an inductive analysis of causes, where our knowledge of conditions falls short Here we *really* fall back on number, on ratio of instances to instances If a self-fertilised flower is fertilised 90 times in 100 cases, and an insect-fertilised flower only 20 times in 100 cases, then the number of cases strengthens the unlikelihood of any exceptional variety and relevancy of unknown conditions, and we take self-fertilisation to be the more effective

process, because there are fewer unknown conditions which stop it, or more which assist it. This helps the confusion which regards Induction in contrast to Deduction as an affair of number of instances.

Again, Induction does not exclude Deductive processes. All Induction whatever is guided by principles; and Induction as considered in Jevons' theory essentially consists in processes of mediate Inference, which he explicitly calls Deduction, and which operate by deriving data deductively from hypothetical premises And usage bears him out. The verification of hypothesis has been considered from Bacon downward as an integral part of scientific induction. And nothing can be more deductive than the connection of a hypothesis with the consequences by which it is verified.

But the distinction, as one of aspects, is justified. It is nearly akin to, but not identical with, the distinction between discovery and proof. This distinction indeed we refused to recognise, because what is not proved is not really discovered. Nor does Induction coincide with discovery. For discovery may include as in mathematical science construction and proof, which no one would call inductive [1]

But in a deeper form an analogous distinction to that meant to be drawn between discovery and proof does hold good between Induction and Deduction We may take Induction as Inference viewed from the side of the differences, Deduction as Inference viewed from the side of the universal. In Induction *par excellence* the Real presents itself in concrete and more or less isolated data, in virtue of which the universal nature, or the system of further differences charged with the universal nature, is referred to reality. In Deduction *par excellence* the Real presents itself as qualified by an intelligible system—e g. by mathematical attributes; and further differences are referred to reality as constructed by and out of this system. It may be doubted whether Newton's discovery of Gravitation was Inductive or Deductive. That in process it was largely

[1] There may be and indeed must be true induction in mathematical matter in so far as instances suggest underlying laws The case of gravitation, to be discussed below, illustrates the degree in which this is possible

deductive there is of course no doubt The popular story however, about the falling apple,[1] would indicate, if true, an inductive aspect—that of a problem set by concrete data, and resolved by analysis and hypothesis But we must not suppose Newton's mind to have been as empty of mathematical generalisations as our own He probably brought a systematised qualification of Reality, drawn from elements in the researches of previous mathematicians, to meet the facts that demanded explanation In this example the aspects of Induction and of Deduction are about equally balanced, and we see the whole principle involved in the distinction together with its merely transitory importance The relation of the universal to its differences is not affected by the order in which they have presented themselves to us as qualifications of Reality. But it is this order alone which furnishes the differentia of Induction

Regarded as relations within a system, i e in the light of the principle of Sufficient Reason, all inductive explanations point beyond themselves They demand in the first instance the explicit statement of the system from which their necessity is derived, and thus they appeal as we have seen from the pure Hypothetical judgment to the Notional or Disjunctive judgment But the underlying real systems themselves are in various degrees limited and incomplete, and in virtue of their finite nature, as we have seen to be the case with space and time, demand explanations which go further and further afield in accounting for the boundaries which persistently present themselves The task of explanation imposed upon the mind by the principle of sufficient reason is therefore an endless task The principle of sufficient reason, as Schopenhauer says, is not like a cab which you can send away when it has brought you to your destination Nothing is isolated, but as the connections which debar isolation reach to infinity, nothing is complete, nor has what it requires in order to justify its existence This is the standpoint of relativity, which applies in a degree to all known matters. How far we can escape from this standpoint, which has been called the stand-

[1] For comments on this and for an excellent criticism on popular notions of Induction, see De Morgan's Budget of Paradoxes, pp 49, 81.

point of the Understanding, and which as thus stated is merely an abstraction of our own minds, will appear when we return to more concrete forms of thought.

Two observations may be made about the account here to be given of Scientific Induction I shall not speak in it especially of Causation I have attempted to show in Book I[1] that cause is a merely popular idea, indicating one or another ill-defined grade in the process of inductive explanation. The only distinctive peculiarity of Cause contrasted with Reason is that it refers to operation in time I believe that all which has value in this idea will be elucidated by our account of inductive explanation, taken together with the analysis of the idea of Cause to which I have referred

And I do not propose to give any account of inductive disproof. Disproof is for the theory of Inference only a form of correction or modification . If at any point we are unable to perform the processes necessary to correction, then we have *pro tanto* a disproof—if e g. we fail in accommodating a suggested rule to actual exceptions, or actual exceptions to a suggested rule But for theory such a failure is not a positive phenomenon We must suppose that there is a true rule, which, if we could but hit upon it, would cover the facts and appear as a correction of our disproved rule The failure to light upon a hypothetical rule fulfilling these conditions is a mere delay in making the required correction, of which theory need take no account Bacon's complaint that the 'axioma distinctione aliquâ frivolâ salvatur' is, but for 'frivolâ' which is its sting, an account of the sole and inevitable process of knowledge.

In order to exhibit distinctly the variations which impede a clear definition of Induction, I propose to speak separately of Induction as perceptive analysis and of Induction as inferential explanation These two varieties, together with Analogical Inference and Enumerative Induction, are all confused together in the popular idea of Induction as opposed to Deduction.[2]

[1] Chap vi

[2] Compare Mill, II 25 ff He tries to separate Hypothesis from Induction, but really includes, though he denies doing so, much Hypothesis

Induction as perceptive analysis.

11. Induction in the narrowest sense is perceptive analysis.

Symbolic expression of the problem

a. We suppose ourselves to have obtained from any source whatever, all such sources being ultimately reducible to analogy, the problematic judgment that the attribute or occurrence B in virtue of a nature *a b c* R, has probably a necessary coherence with the attribute or event C. This is so far only a presumption arising from the value for cognition which we have been led to attach to the nature *a b c* R—a value depending, in all the higher and truer applications of analogy, on the ultimate identity of human purposes and necessities,[1] and in the lower walks of inference on the identification of self-maintenance or self-preservation with some such idea as that of purpose I follow Lotze in employing an expression of the type *a b c* R in which *a*, *b*, *c* may be taken to stand respectively for definite attributes or relations and R for the residual nature of the concrete whole before us, considered as only contributing its normal support to the operations of *a b c* and not as actively interfering to modify them. It is worth mentioning that Mill's account of the Experimental methods, otherwise at least suggestive, is rendered terribly perplexing by his use of corresponding letters A and *a* to indicate from the first the several antecedents and consequents underlying concrete phenomena The result is that his first statement of every problem presupposes in symbolic form its explicit solution. When the phenomenon can be resolved into antecedents A B C and consequents *a b c* the work is already done. Mill, no doubt, does not *mean* to have determined by his expression the fact that a particular element A of the given content is from the first known to correspond to another particular element *a* He intends A and *a* to be empty forms, indicating the problem which our analysis has to solve But the correspondences of the symbolic letters are undoubtedly misleading

in Induction His test seems to be that where you have a *vera causa* you have Induction, not Hypothesis But he admits that in Induction the *vera causa* may not be known to be present in the case under investigation

[1] The process of learning a foreign language, and ultimately of understanding language at all, is an excellent example of this We are guided throughout by the assumption that identical aims and feelings underlie the different systems of expression

I mention this question partly because it illustrates our present task The *problem* is just to break up B into $\alpha\ \beta\ \gamma$ R, so that we can say of each element in turn, 'If α R, then α R, and if α R, then α R' In each case we must understand all the elements which we are not observing to fall back into the mass of R. This neglect of the other elements is capable of two interpretations Either the other elements may be taken to retain their normal relation to the α under investigation, and are not especially and abnormally operative upon it under the conditions of the observation, or they are actually inoperative and might be removed This latter interpretation can never be justified without special proof, which must address itself to a definite analysed R For every conjunction of conditions whatever is *an* R, i e an inexhaustible concrete, even in the most precise experiment, and all that can ever be done in the way of isolation is to exclude some portion x of the whole concrete R, by substituting for it an element y which has the effect of turning R into R_1 We have then excluded x, but not R as such, i e we can only exclude R in as far as we can analyse it.

And to end the subject of symbolic expression, I may point out that for simplicity's sake I shall not consider the whole analogical suggestion 'B is probably connected with C, both being conjoined with *a b c* R', but shall confine myself to one member at a time, as we should have to do in a practical investigation, e g to 'B is conjoined with, and probably coheres with, *a b c* R' This is not an inadequate treatment. It would be easy to add C as a character to *a b c* R, indicating that their conjunction must be taken subject to unknown conditions, and in any case the investigation of B in relation to *a b c* R is certain if pursued to the end to lay open the track of coherence between *a b c* R and C The defect of symbolic modes of expression in these higher forms of reasoning is that not only are all elements of the content most variously interconnected, and far from being on the same level in value, but also every element of the content is undergoing transformation from the beginning to the end of the whole process Therefore, as Mill no doubt really intended, corresponding symbols like *a* and α represent a pair of series or a pair of continuous

developments within the inference rather than a pair of fixed contents

Establishment of Ordinary Hypothetical Judgment

β. I will begin by analysing at some length an example of perceptive analysis conducted *chiefly* through observation as opposed to experiment—though experiment was at times applied—and in respect of its content just on the borderland between analogy and scientific induction

It might be suggested without doing violence to facts that the Linnaean classification in botany corresponds on the whole to the stage of enumerative or conjunctive Induction, the mere natural classification to Analogical Inference, and the analysis of plant-structure and evolution in the light of the Darwinian hypothesis to scientific induction—to perceptive induction where we deal with the visible adaptations of particular species, and to generalising or reflective induction when we lay down universal conditions as controlling the evolution of the organic world

Let us suppose that Analogy, the habit of ascribing what I have ventured to call *de facto* purposes to adaptations in the organic world, has made it probable to us on inspecting the flower of the Bee Ophrys that it (B) is adapted for self-fertilisation (*a b c* R)

Here the expression 'adapted for', in consonance with the notion of *de facto* purpose, refers not merely to the mechanical adjustment of a contrivance, but to the fact of that contrivance actually achieving in normal use the purpose which it suggests A case in which we cannot make out this additional element of meaning will be mentioned below, and in it the purpose will not rank as established by Induction

I should observe, too, that the element C which we usually spoke of in analogy and which we mean to omit here for the sake of brevity may in the present example be identified with any peculiarity the conjunction of which with the general appearance B might have first attracted our attention to the flower B.

Of course my analysis is only rough and typical. I select two or three prominent characters out of a whole apparatus of converging contrivances.

The object is now to analyse the flower B in the light of *a b c* R. We may attempt this roughly as follows —

(*a*) Caudicles (stalks of pollen-masses) are of the right length to (*a*) reach the stigma.

(β) Anther-cells open of themselves, and (*b*) let the pollen-masses fall to the level of the stigma

(γ) Hanging pollen-masses oscillate in the wind till (*c*) they strike the stigma

R in this example has the significance that the remaining parts of the flower and plant are necessary to give the process its value, and to make it possible for the contrivance to operate, by nourishing and mechanically supporting the flower. But all this is involved in the nature of a plant, and therefore assuming a, β, γ to be in a living plant, and that, of course, the right plant, R need not be further considered in the analysis *at present*, i e until it in some way interferes with the possibility or reality of the action we are investigating

The very important relation of γ to c in the above analysis assumes the operation of an external cause, and requires a confirmation without which the whole analysis is futile, for as the pollen masses when liberated do not fall on the stigma, but only hang like a pendulum on the level of the stigma, it is incumbent on us to show how they can be and are brought in contact with it There is a further interest at this point in affirming or denying the action of insects, which are usually necessary to cross-fertilisation, but the need for whose intervention would impair the certainty which is the purpose of self-fertilisation Here we have recourse to the negative instance which, here as usual, contains an element of experiment For the essence of the negative instance is to obtain a positive content equivalent *ad hoc* to an exclusion, and this can only be done by a disjunctive limitation of possibilities, and an exact ascertainment of the reality within the possibilities so limited The limitation of possibilities consists, not in removing all R, which is impossible, but in securing an R analysed and believed to be passive, and artificial combinations give the best chance of obtaining this condition. And the exact ascertainment of reality consists in observing a positive or negative condition, or both, whose nature we can exhaustively analyse Here again artificial production gives the best chance To test the connection of γ (movement by

wind) with c (contact with stigma) Mr Darwin put a spike of Bee Ophrys in water in a room Thus he secured an R, residuary conditions, which he could ensure to be passive (absence of touching by animals or by any unknown cause of motion), and having thus limited the possibilities he was able to observe with certainty and with a high degree of exclusiveness *the absence of wind*, not-γ, which resulted in *absence of contact*, not-c, the pollen-masses continuing to hang freely in front of the stigma Thus he obtained the confirmatory or true negative instance 'not-γ is not c', which is the contrapositive converse of 'c is γ', i e 'contact arises from wind' We have here left the ground of formal logic,[1] in which 'not-γ is not c' could only rest on the knowledge that 'c is γ' In the process now considered 'c is γ' actually rests on the knowledge that 'not-γ is not c'. The corroborative power of the negative instance in induction depends on the fact that it has a positive content within the same *ultimate* system as c and γ, and, within that system, related by way of definite negation to them. Thus the negative instance is capable of *independent agreement* with the positive case. 'Not-γ is not c' = 'Free caudicles without wind give no contact'

But it will be said that we have gone too fast. We read the 'free caudicles in a room give no contact' as 'not-γ is not c' But it was probably also 'Not-G (no insects) is not c', 'where no *insects*, there no contact,' i e in searching R we have found a not-G, an absence of a condition, which, it is suggested, may not be, as R was meant to be, indifferent. We may treat this as a positive suggestion from analogy, 'G is probably c,' for in the absence of such a positive suggestion we should have no more cause to note the absence of insects G from the experimental R than to note the absence of direct sunlight L or extreme changes of temperature T, But there is plenty of analogy for insects fertilising plants, so 'G is probably c' demands attention

Mr. Darwin provided against this suggestion by exposing some of the flowers under a net, which excluded insects but admitted wind In the cases so treated contact was effected.

[1] See i 305-7 What is true of the double negation is true of the contrapositive converse which implies double negation.

We may read this off as an *exception* in the form 'not-G is *c*' against the suggested rule 'G is (probably) *c*', or 'insects (probably) produce the contact', and as at the same time a positive instance in favour of the suggested rule 'γ is *c*', 'wind makes contact'. This double-edged character, proper to a negated content at this stage, is justified by the experiment above-mentioned which might be read off as 'R γ not-G is *c*', R standing for the mass of conditions presumed to be indifferent, not-G for the exclusion of insects, γ for the presence of wind, and *c* for contact

Establishment of Reciprocal Hypothetical Judgment

γ. And this connection γ is *c* (wind acting on the pendent pollen-masses produces contact with the stigma) has been defined and confirmed—i e *re-inferred in a precise form* through the two conjunctions claiming to be connections, R-not-γ is not *c* (pendent pollinia without wind [1] do not touch the stigma) and R γ-not-G is *c*, i e pendent pollinia with wind and without insects [2] do touch the stigma From not-γ is not *c* (R being disregarded as the common basis) we infer *c* is γ, i e. 'contact comes from wind', the reciprocal of 'wind produces contact'. And by 'γ-not-G is *c*' we confirm this reciprocal 'contact comes from wind' by overthrowing the suggestion that G may be the operative agent in *c*, and consequently that *either* wind *or* insects may be concerned in the contact.

It is true, however, that we have not obtained, against 'G may be *c*', the more fatal exception 'G is not *c*' (in presence of insects no contact is effected); the exception which we obtained is strictly an exception against the reciprocal of this, viz. against *c* is G or contact comes from insects, i.e. against the suggestion that insects are the *only* agency in producing contact. Thus we have not strictly *proved*, as against insect agency the only suggested alternative, that wind is the *exclusive* agency in the self-fertilisation of this flower, for when wind was excluded, insects were probably (in the room) excluded with it. In fact the γ with which we began included G, and

[1] Experiment of the flower in a room

[2] Experiment of flowers under a net in the open air This experiment goes far to give the pure judgment '*only* γ is *c*', which no ordinary Judgment-form will express for Logic, except the clumsy equivalent All *c* is γ'. I have written it 'γ-not-G is *c*'.

was really 'wind-or-insects', and it is of *this* γ that the reciprocal 'c is γ' was proved by the experiment not-γ is not-c But we subsequently make it probable that this γ *ought* to mean wind only, by making it certain that it may mean wind only This shows the transformation which a content undergoes in course of an inductive inference

And for the kind of matter with which we are dealing this conclusion is perhaps sufficient We are studying the use of an adaptation, which use any normal agency suitable to it will suffice to establish. We could not hope to prove that no insect, or that no human hand, has ever fertilised a Bee Ophrys by pushing the pendent pollen-masses When we know that the wind can do it, and does it without other aid, and that wind is a common occurrence, and that in the absence of wind (though in the absence of other things at the same time) the adaptation fails, then we are justified in saying that here we have the only agency which is normal enough to account for the growth of a contrivance adapted to it Logically, these considerations are represented by the claim of every judgment to become reciprocal, which formal claim has different values and interpretations in different kinds of matter Here, for instance, we might make our judgment truly reciprocal—and purely truistic—by transforming the content of γ into simply 'a sufficient cause of motion' This would suffice for a mechanical construction of our problem, but not for an organic explanation of it An organic adaptation demands for its explanation a definite regular agency to which it is adapted, it *need* not exclude agencies of diverse origin, but it is pretty certain to shape itself on some one well-defined type of operation Thus in speaking of agencies to which evolution has adapted structures, the claim of any actual and normal agency to be *the* exclusive agency is *prima facie* very strong To make it absolute we should proceed by analysing c as we have analysed B itself, and showing that γ, wind agency, as $\xi\ v\ \zeta$, is the *only* agency corresponding to c as $\lambda\ y\ z$ But this I at all events am unable to do, further than by pointing out that 'normal' and 'general' in γ correspond to 'gradual growth' and 'need of a reliable agent' in the contrivances concerned in c

Thus far we are left with B as $\alpha\ \beta\ \gamma$ R is $a\ b\ c$ R or S The flower of the Bee Ophrys as having flexible caudicles of the right length and self-opening anther cells, and considered as acted upon by wind, is adapted for self-fertilisation by the pollinia falling to the level of the stigma and oscillating till they touch it

Conversion or Generalisation

δ In formal logic the affirmation of one attribute can have no influence on the affirmation of another about the same subject unless an explicit contrariety between the two affirmations is within our knowledge In short, difference does not justify negation To say that a flower is self-fertilised does not formally warrant us in denying that it is cross-fertilised But in science every content claims to be treated as a system, and every attribute must either quarrel with any other attribute suggested of the same subject, or must make peace with it on definite terms Therefore the inductive conclusion B is S, 'The Bee Ophrys is self-fertilising,' which we have thus obtained, contains in its claim for reciprocity, i e for predominance or essentiality on the part of the attribute, a further suggestion to which in material or actual knowledge we are bound to pay attention We cannot indeed expect to show that every self-fertilised plant is a Bee Ophrys, i.e we cannot reduce self-fertilisation to mean solely the adaptations of the flower in question, nor can we extend our idea of the flower in question to include all adaptations that in any plant might ensure self-fertilisation. The attribute 'self-fertilisation' is not sufficiently concrete and specific to be identified in this way with the nature of a particular species of plant But though we cannot reduce self-fertilisation as such to mean simply and solely the fertilising contrivances of *Ophrys apifera*, we are confronted by the reciprocal tendency of judgment with another problem which Darwin, with his usual exhaustiveness of apprehension, has frankly stated and discussed We saw that B is S, or, to prepare for our present enquiry, B is *b* S, i e. The flower in question is characterised by its own peculiar contrivances for self-fertilisation Can we convert this judgment materially? Can we say 'Self-fertilisation S, not cross-fertilisation F, is *the* characteristic of this species', or in hypothetical form, 'If S pure and simple, then B?' This

suggestion might be embodied with more formal correctness in double negation or in contra-position, as 'B is not not-S', or, 'If any not-S, then not-B,' and the question would thus arise whether F (cross-fertilization) was not-S in the sense of being incompatible with S in B But Simple Conversion without limitation (formally impossible) expresses the guiding idea more effectually, in demanding that an *essential* attribute of a subject shall be the *sole* attribute in the relation to which it belongs Here however this suggested reciprocal is not true The contrivances which have their meaning in subserving cross-fertilisation, the viscid discs, sinking caudicles, and elastic threads tying up the pollen-masses, are present in the Bee Ophrys without the least trace of becoming aborted, and therefore a strong *analogical* inference holds, to show that B being *d e f* is F, and so is not-S in as far as not-S is identified with F, in other words, that S, and not-S in the sense of F, are not contrary or incompatible in B, and so if we like that F is in this case not to count as not-S, or else that B is both S and not-S, to which, if not-S only means different from S, there is no objection

A certain methodical gain is drawn from affirming this conjunction of S and not-S, although unintelligible to formal logic. Having failed in Conversion, we are driven to Generalisation For though S and F form no logical contradiction, but are *prima facie* quite compatible with one another, yet ultimately and from the point of view of a harmonious theory there is a contradiction *until we reconcile them.* Difference without a reason,—i e difference in the same relation, or difference of means[1] to the same end *qua* the same,—*is* a contradiction We express this problem justly by saying, 'The flower B is self-fertilised S, and apparently may also be cross-fertilised Not-S' We are here in need of a further suggestion by which to generalise S and not-S into one conception This suggestion cannot be mechanically obtained, but must be drawn by analogy from our general knowledge of the organic world Combining what Darwin says in the place under discussion

[1] Apart from the insufficient amount of the one means, which therefore may need supplementing by another If the one means is as easy to provide as the other, this reason falls away

with his views in other works, we might give this suggestion the form, 'Σ (healthy preservation of the species) demands some F (cross-fertilisation) at least,' under which we may infer by analogy from 'B has besides S some F (not-S)' that 'B the special adaptations of the Bee Ophrys have for their all-embracing and determining nature the tendency to Σ the healthy self-preservation of the species, including both S and not-S'

Beyond analogy, in this final inference, we cannot go, for cross-fertilisation is not, according to the passage upon which I rely, affirmed of Reality as a datum in the content of *Ophrys apifera*, but is itself only inferred from analogy, and therefore the general conclusion, though a suggestive concurrence of analogies, cannot be considered as a truth resting upon scientific induction. The operation of the contrivances by which the self-fertilisation of this flower is secured may on the other hand be regarded as made good by precise perceptive analysis at every point

In establishing this positive attribute of self-fertilisation considered as significant of a *de facto* purpose, we have about reached the limits of perceptive analysis In establishing the probability of cross-fertilisation we have in one sense gone beyond the limits of perceptive analysis into the region of hypothesis, if in another sense we have retrograded into mere analogy Such a hypothesis as we have just recommended by analogy, if drawn out into a variety of precise details and supported by their precise verification as real data, would be the essence of reflective, mediate, or generalising Induction We must however bear in mind that hypothesis was present in a germinal form throughout perceptive analysis, throughout analogy, and even throughout enumerative induction, where it was represented by the content of a common name; so that there is no *saltus* between these phases of inference. The fascination which attaches to the researches of the great masters lies just in their power of absorbing, by exhaustive analysis, the mass of perceived data into intelligible conceptions

111 In order to estimate the logical character—the position in the evolution of thought—of such a process as this which Logical character

of Perceptive Induction

I have attempted to describe, three special points must be briefly treated These are, α What is the essence of the inferential process concerned ? β What is the purpose of the symbolic representation of it by letters ? γ What part in it is played by number of instances ?

Its essence as Inference

α Ordinary mediate inference, either subsumptive or constructive, may be detected in every step of the process which we have examined, as in any complex judgment of perception As we analyse, for example, the flower into its parts, and its parts into mechanical adaptations, we obtain the material for a three-term inference by which the adaptations in their mechanical aspect would be formally brought home to the flower Or again, the contra-positive conversions and the ideal reciprocity of the judgment, to which we have so freely appealed, may be held to require explicit proof through syllogistic or disjunctive argument based on abstract principles The mediate inference thus involved is of two kinds.

With regard to the mediate inference involved in every *complex* judgment of perception, and therefore in every *precise* one—this is *really* present in the Induction of which we are speaking, and may sometimes need to be explicitly drawn out in order to correct the results of an overhasty perceptive analysis Especially this is the case when we are employing experimental apparatus which embodies whole chains of reasoning and concentrates on a single datum a multitude of precisely determined conditions The observer e g with a microscope must always bear in mind what it is that his instrument does in virtue of the principles of its own construction, and in many classes of observations is liable to be thrown back on constructive optical inference, in order to determine the interpretation of the appearance presented to him,—whether it means a true line or an interference-line, whether an elevation or a depression, whether absence of structure or complete transparency of structure (in which latter case the use of polarised light will sometimes detect the illusion) Such mediate inference as this is really and genuinely present in the processes we have been considering, being concerned with material principles relevant to the special subject of the inferences But yet such mediate inference does not belong to the

differentia of Induction, but is shared by all Inference whatever, being inherent in the nature of Judgment.

On the other hand, it appears to me that mediate inference from abstract principles of knowledge, such as principles of disjunction, of causation, or of sufficient reason, is not a genuine element of scientific Induction at all, although it may be the duty of the logician to point out a relation between inductive inference and such principles as these. The active form of thought, to which these principles belong, loses, as we have seen,[1] its active nature if it is made a mere content within an inference The relation, for example, of the judgment that embodies a 'negative' instance to the affirmative judgment which it corroborates is a case of the active form of negation engaged in acquiring a definite content within a certain complex system We should gain nothing in such a case by erecting an argument to the effect that What is not A is not-A The problem is, given the forms A and not-A, the positive and its limit, to bring these two forms into material agreement in respect of the matter to be organised

To reject abstract argument from principles of knowledge is however a different thing from the omission to exhibit the material of inference as permeated and articulated by the active forms of thought Such an omission I hold to be unjustifiable I have argued elsewhere that Mr Bradley goes too far in holding that an inference *qua* inference is not bound to exhibit its principle or *rationale*

The essence of induction in this, the perceptive stage—and beyond this stage it more and more transcends mere induction —is in the peculiar parallelism between the positive connection which suggests, the negative connection which defines in corroborating and corroborates in defining, and the 'exceptional' connection which modifies either itself or the affirmative connection I have explained[2] why I do not take account of the sustained exception which overthrows The logical peculiarity of the process is in the positive and consequently independent value of the negations, which are established without being derived from the affirmations, but operate on the latter through the formal interdependence of negation and

[1] Cp chapter I of the present Book [2] p 116, above

affirmation The process is of course not mechanical. Mechanical Induction is an idle dream The reciprocal adjustment of the negations and affirmations consists in the revelation of intelligible systematic ideas which are thus inferred to be true of reality

Theoretical purpose of representation by symbols

β. The symbolic representation of these processes by letters may seem to have an external affinity with the processes of equational logic But the two systems are to be regarded in precisely opposite aspects In the above discussion not-a and not-a have been employed to designate contents which are positive, but have, towards a and a respectively, a boundary or negative side The only object of such designations was to emphasise, for theoretical purposes, the negative relations subsisting between certain inter-connected positive contents But for practical use the events or attributes in question must be taken in their concrete form, upon which everything turns By manipulating them in the shape of abstract symbols no progress can be made in the task of Induction, which is a problem of material suggestion and adjustment '*Just* where' and '*just in so far as* x fails to be a it fails to be a', this is the meaning of the inductive 'not-a is not a' And no handling of symbols[1] can express or can warrant this 'just' and 'in so far as' which are the whole essence of the process What warrants these expressions of definite relation is and can be nothing less than a ground or real system containing parts which negatively determine each other It is the business of Induction in the form of perceptive analysis to initiate the disentanglement and reciprocal determination of elements within such systems, in the light of ideas—germinal hypotheses —suggested by analogy Analogy, in fact, does not cease to operate in Induction Induction is Analogy fortified by negative and precise determination

Part played by number of instances

γ. Induction, we saw, is popularly identified with proof by instances, and owes its recognition as a distinct method of inference to this identification.

[1] Of course this remark does not extend to true calculation, which has been independently treated and does not fall within induction The two processes have some common ground in statistics, as will appear from γ below

In perceptive anaylsis proper.

(1) But scientific induction does not depend on or in any way deal with instances as such, i e particular occurrences or observations with reference to their particularity—their number or recurrence Here we have an antinomy, to which at the present stage we need only draw attention, as it has really been solved by the distinction between Enumerative Induction and the subsequent diverging phases of the Inductive process All that scientific Induction demands is a content referred to reality, in how many observations or cases or occurrences the content is presented is a matter of entire indifference to science. If, to put an extreme supposition, the entire content, positive and negative, employed in the above analysis of the Bee Ophrys, could be observable in a single flower, that single flower would, subject to one reservation to be mentioned presently, form a sufficient ground for all the conclusions that were then drawn What characters can be and what cannot be united in a single or continuous observation is a question of the nature of the object concerned, and not of logical theory The same flower cannot be both fertilised, and ultimately not fertilised at all It can be both fertilised (later) and not-fertilised (up to a certain point of time). Or it can be both self-fertilised, and not-self-fertilised in the specific sense of being cross-fertilised. The first of these three comparisons requires two 'instances', the two latter need only require one apiece, or indeed one between them. And then is an 'instance' a plant or a flower? If a plant, one instance would probably do all we should demand

Known effects of unknown conditions.

(2) There is, however, one case to be distinguished to which the above remarks do not apply The self-fertilisation of the Bee Ophrys, Darwin says, is markedly 'successful' The Bee Ophrys, which is self-fertilised, in many dozens of plants had a capsule (seed-vessel) for every flower[1] The Fly Ophrys, cross-fertilised by insects, had only seven capsules in forty-nine flowers This is again '*a* is *a*',' not-*a* tends to be not *a*.' And here number of instances is essential to the result, *because we are dealing with the operation of conditions not fully known.* This throws us back at once into enumeration of instances, statistical methods, or even the statement of chances. We

[1] *Exceptis excipiendis*—deformed flowers

proceed by the comparison of hypotheses explanatory of observed ratios, as we saw in discussing the statement of chances If, to use an extreme illustration, we could say 'Fly Ophrys is exclusively insect-fertilised', *and* 'the observed plants of Fly Ophrys are in a place inaccessible to insects', then we should not need a single instance to fortify the conclusion that all these plants must remain unfertilised But as we do not know with precision what conditions are operative, and to what degree, in securing or hindering the approach of the right insects to the flower at the right moment, we are reduced to enumerating observed instances in order to obtain an actual ratio between successes and failures, upon which we may base an estimate of the nature of the cause or causes, whether in the flower or outside it, which would probably have produced the observed ratio of successes to failures We have as data, say, forty-eight cases and forty-eight successes in self-fertilisation, compared with forty-eight cases and only six successes in cross-fertilisation by insects. We have to conjecture or construct the kind or type of causes which are most likely to have produced these two observed series

Supposing indeed that we take into account *all* unknown conditions whatever, no question can be raised, for it is a mere transcription of the series to say that the one flower is less adapted to the conditions which *have* acted on it, than the other to the conditions which *have* acted on *it* And supposing that Darwin excluded interfering causes in counting both his sets of instances, as he did in one, there is no more to be said The self-fertilised plant meets the unknown conditions wholly and the other does not. In order to compare probabilities we must have a suggestion as to some special kinds of causes that normally operate with an assigned frequency,[1] and for the

[1] We must in short take by way of hypothesis some 'natural cycle', or what comes to the same thing, some cycle external to that observed, otherwise there are no two ratios to compare in respect of each observed series. See Book I, chap VIII We might indeed compare the probabilities that each series proceeded from a supposed cause, and from chance (unknown independent conditions) respectively, but this would be, where we know some of the conditions operative, to forfeit the use of knowledge which we possess If we wished to reckon the probability of either series occurring by chance, I suppose we should have to take (*faute de mieux*) the chances of failure and success as even for each flower.

sake of illustration I will assume that obvious accidents have not been excluded As every adaptation has its limits beyond which conditions become abnormal to it, i e are accidents, I am able by this means to suggest the idea that the two kinds of flower may be equally well adapted to normal conditions, but that *in the observed series of cases* the Fly Ophrys may have been the victim of a set of disasters which destroyed the flowers inspected, by causes lying outside the limits of adaptation of either flower Abortion of the flower or destruction by insects before maturity may easily prevent seeding in one flower out of four Now of course in the first place it is an impossible assumption that Darwin would not have noticed any extraordinary prevalence of abnormal interfering causes confined to the flowers of the Fly Ophrys In order to obtain a useful 'not-*a* is not *a*' the two negative contents must diverge from the positive *a* and *a* only in *a*-ness and *a*-ness They must be, as we have insisted, within the same real system ; i e the R of general conditions must be the same in both, or in the same relation to both [1]

But in the second place, making this false assumption for the sake of illustrating our point, we will suggest that one flower in four of the Fly Ophrys is *on the average* destroyed by accidents which no adaptation could avert And then it becomes not impossible that in a given series of forty-eight inspected flowers, these accidents have been heaped together by unknown causes , and that the flowers, though adapted to *all* normal conditions,—i e as successfully adapted as those of the Bee Ophrys,—were nevertheless in forty-two cases out of a given forty-eight hindered from being fertilised by a series of extraordinary accidents. We have then to compare, as regards the Fly Ophrys, the hypotheses of maladaptation to normal conditions, such as to cause failure in forty-two cases out of forty-eight, and of perfect adaptation to normal conditions, hindered of its effect by abnormal conditions in forty-two cases out of forty-eight The possibility of making this

[1] It is not indeed fair to say that in order to a just comparison the Fly Ophrys must have its insects as the other must have its wind, because the question is whether the Fly Ophrys was wise to rely on so capricious an agency as that of insects But there must be no extraordinary influence known to be keeping the insects from it

comparison by calculation depends on our being able to assign an average ratio of operation to the abnormal causes Taking them to produce on an average established by general observation one failure to be fertilised in every four flowers, and excluding probabilities derived from the non-appearance of extraordinary hindrances in the instances of the Bee Ophrys, we have to determine the probability that in forty-eight independent flowers, with three favourable chances and only one unfavourable for each flower, we should obtain a set (in any order) of forty-two failures and only six successes I presume that this problem is the same as to estimate the chances of drawing a black ball exactly forty-two times in forty-eight out of a box containing only four balls, being three white balls and one black ball These chances would be expressed I suppose by $(\frac{3}{4})^6 \times (\frac{1}{4})^{42}$—the chances of six successes in drawing a white ball on *assigned* occasions—multiplied by the combinations of forty-eight things taken six together, in order to add together the number of independent ways in which six successes and forty-two failures can be realised

This probability, which must be very low, owing to the enormous number of failures required, with the low chance $\frac{1}{4}$ for each, has to be compared with the high probability with which the assumption that, *normally*, insects only visit one flower in eight, and that therefore the flowers are maladapted to seven sets of conditions in eight, would give as a consequence the ratio of forty-two failures in forty-eight flowers This would I suppose be the same as the chance that out of a box containing seven black balls and one white, forty-two black balls should be drawn in forty-eight trials The combinations remain the same as in the former case, and the factor supplied by the fractions expressing the chances would be $(\frac{1}{8})^6 \times (\frac{7}{8})^{42}$, involving a high power, the forty-second, of the very favourable chance $\frac{7}{8}$ The comparison of these probabilities would be our warrant for deciding, on the assumptions which we have made, that Fly Ophrys is much worse adapted to normal conditions than the Bee Ophrys In fact, our conclusion is much more certain than on these assumptions, for it is certain that any violent interfering cause which destroyed one flower in four would have been noticed and excluded by any such

observer as Darwin But in as far as we rely on the exclusion we are ceasing to rely on number and are going back to analysis of content The exclusion, however, in such varied and uncertain conditions is probably imperfect, and therefore, in our actual inference, I take it that we eke out our reliance on Darwin's accuracy of comparison by a reliance on the probability of a normal feature in the conditions (viz a degree of unsuitability to the flower) as against the very low probability of a variety of accidental conditions which Darwin did not exclude We shall illustrate this particular application of the inference from number of instances directly

The above case may serve as a type of all Inductive processes in which number of instances, as number, plays an essential part Their essence consists in selecting as most probable that cause or class of causes which would produce, as an alternative bearing the largest proportion to the sum of possible alternatives, the ratio actually observed among the phenomena Apart from the assumption of any particular cause, every additional instance enormously increases the improbability of *every* single definite succession by making every such succession one among an immensely increased number of possible alternative successions (or conjunctions). If therefore any cause can be alleged or supposed, which would give that particular definite succession which exists in reality as sole alternative or as one of a comparatively small number of alternatives, the principle of impartial ignorance urges us to decide for that cause as giving to the actual observed succession the nearest approach to its actual position as real—i e the largest share of estimated reality. Or if two or more causes are suggested, from which each component event of the actual observed succession can be derived as one out of different numbers of alternatives respectively (e g as one out of three alternatives in one case and as one out of twenty alternatives in another), then we compare the probability of these two causes just on the same principle as that on which we compare the probability of a single imputed cause and that of the total absence of any single cause at all, forming a case in which the actual succession must be attributed to a succession of what we call accidents

A particular class of accidents however, such as that assumed above as destroying one flower in four before maturity, is of course a class of causes, and may be defined and treated for purposes of calculation as 'a cause'

Number of instances thus operates by increasing the improbability *per se* of every particular conjunction or succession of phenomena, and therefore increasing the probability of any cause which can be proved capable of producing the given conjunction or succession as one out of fewer alternatives than the number derivable from any other suggested cause or from the whole series of instances treated as accidental Probability is estimated by counting, on the basis of impartial ignorance, hence the opposition between enumeration of instances and analysis of content

The case of so-called elimination of irregularities by accumulation of instances may readily be exhibited as an application of the above principle A class of causes, or common element in a variety of active conditions, is for our present purpose a cause or ground It, on the accumulation of instances, there appears in the observed succession or conjunction any feature, e g of recurrence in certain cycles, or *a fortiori* of persistence in a single character, which can be hypothetically referred to any common element in the wholly unknown conditions, then the accumulation of instances progressively increases the relative probability of causation by this common element, by progressively decreasing the probability of every conceivable sequence, including the one observed, if considered as the result of accident, i e of independent causes [1] In other words, it becomes more and more probable that, the unknown irregularities notwithstanding, the unknown conditions include a common element, however composed, relative to the persistent feature of the observed conjunction or succession, and uninterfered with by the unknown irregularities of the unknown conditions Material considerations of content, suggesting an approximation to exhaustiveness in the enumeration of kinds of instances and consequently of kinds of conditions, are almost invariably

[1] See below, p 170, on Kirchoff's proof of the presence of iron in the sun

present to reinforce in some degree the argument from sheer probability

Observation and Experiment

IV Experiment is observation under artificial conditions What is artificial bears to a certain extent the impress of human intelligence, and is, to this extent, abstract and idealised Human action, in virtue of the human thought which directs it, is definite and selective And 'artificial' means produced or arranged by human action. The difference between observation and experiment therefore is in the degree of definiteness and ideal selection which is present in the material conditions of the latter process.

Natural Experiment

a. It is obvious that natural conditions would serve the purposes of enquiry as well as artificial conditions on the assumption that they were exhaustively known And if exhaustive knowledge of natural conditions were in no case possible, observation could not exist as a scientific process But it must be noticed that in giving effect to the knowledge which guides it, observation itself tends to take on the character of experiment The transition between the two processes is therefore gradual Experiment would usually be considered to begin where we pass from intentional selection of our standpoint and from the use of contrivances auxiliary to perception, to actual analytic interference with the object under observation Before the line is reached, however, observation passes into something which may properly be called 'natural experiment' I quote an excellent passage from Jevons [1] in illustration of this point

'It may readily be seen that we pass upwards by insensible gradations from pure observation to determinate experiment. When the earliest astronomers simply noticed the ordinary motions of the sun, moon, and planets, upon the face of the starry heavens, they were pure observers. But astronomers now select precise times and places for important observations of stellar parallax, or the transits of planets They make the earth's orbit the basis of a well-arranged *natural experiment*, as it were, and take well-considered advantage of motions which they cannot control Meteorology might seem to be a science of pure observation, because we

[1] Principles of Science, pp 400–1

cannot possibly govern the changes of weather which we record Nevertheless we may ascend mountains or rise in balloons, like Gay-Lussac and Glaisher, and may thus so vary the points of observation as to render our procedure experimental. We are wholly unable either to produce or prevent earth-currents of electricity, but when we construct long lines of telegraph, we gather such strong currents during periods of disturbance as to render them capable of easy observation.'

Observation with accurate instruments

β There is a further point connected with this transition which calls for remark We habitually speak of telescopic, microscopic, or even of spectroscopic observation Considering what an enormous artificial interference the instruments thus employed exert upon the image of the object to be observed, it may seem strange that we call the result an observation and not an experiment The instinct which guides our use of language is however just, at least so far as concerns the ordinary applications of telescope and microscope as magnifying instruments An apparatus which merely brings the object nearer our perception is *par excellence* an observing instrument In the compound microscope the image is variously transformed, and often goes through some degree of chromatic dispersion, *in transitu*, but as it is reconstituted before reaching the eye, these transformations do not amount to experiment This question turns on the employment of interference not merely to make an object accessible to us, but to analyse its content Thus the moment we modify the object under observation itself, e g by applying heat, electricity, or chemical reagents on the stage of the microscope, we say that we are experimenting When we use spectroscopic devices to observe the real prominences of the sun, without waiting for a total eclipse, we are really analysing the solar image, though not the sun, but strong analogy from the general use of optical instruments makes us still say that we are observing In actually compounding coloured lights with a colour-box for the purpose of equation, however, there is no doubt that we are experimenting The fact is then that experiment is not merely observation under artificial or determinate conditions, but *observation under determinate conditions which constitute an integral part of the image or*

product to be observed Thus common dissection is not experiment, though it introduces conditions in the way of separation and demarcation as definite as anything can be, but vivisection is experiment, because the determinate conditions it produces enter as factors into the action of the organism observed

Experiment expressed in logical symbols

γ. Returning for the sake of brevity to the symbols which we used before, we may say that the function of experiment is to exhibit both a and not-a as determinate cases of β, γ, δ, &c which form the ultimate analysis of R so far as R is relevant to a The cases of $\beta\ \gamma\ \delta$, &c obviously may include zero values of any one or more of these factors, and apart from a special hypothesis to be tested—or rather if the hypothesis to be tested is merely that $\beta\ \gamma\ \delta$ are concerned in a—all possible combinations of values of the three or more series must be tried a moreover is certain to be continuous, and to admit of variation within itself, for no phenomenon is utterly atomic But in order to secure a distinct correspondence between phases of condition and of effect, it is well to treat every appreciable phase a, within the general a, in turn, as a bounded by not-a, so as to identify its condition a, viz a phase of $\beta\ \gamma\ \delta$, with absolute precision, as being on both sides bounded by not-a, viz other positive phases of $\beta\ \gamma\ \delta$ excluding that which is a The goal to be attained, if the experimental conditions admit of it, is a reciprocal Hypothetical judgment, consisting of an affirmative hypothetical judgment in the form, 'If a (a determinate phase, or series of phases, of $\beta\ \gamma\ \delta$), then a,' supported by its *simply* corroborative equivalent, 'If not-a,[1] then not-a' (viz determinate phases of $\beta\ \gamma\ \delta$ excluding the phases a), and by its reciprocally corroborative equivalent, 'If not-a, then not-a'

Can this reciprocal, equivalent to 'If a then a', be justified by experiment, which can at best take the shape, 'in order to remove a you must remove a,' and not 'by removing a you remove a'? It can be thus justified on the assumption

[1] Not-a under experimental conditions is of course itself positive and exclusive of a If there are different not-a's, as is quite possible at first sight, they determine different a's E g aerial impulses too slow to be heard as musical sound, and discord, are two not-a's as against a's in musical sound

that α considered as a phase of $\beta\ \gamma\ \delta$ is an ultimate analysis of R For in this case all possible combinations of the ultimate components of R have been exhausted, and we can lay down throughout them all the demarcation between α and not-α. But if we take R in its primary meaning, of which we cannot stop short without special justification—viz as the entire system of the universe—this assumption can never be true except in virtue of a consistent abstraction by which it is taken as true

Such an abstraction is the source of mathematical necessity I will not say that in mathematical construction we are secure from the irruption of any conditions beyond those which we have put there, because we may be guilty of omission or oversight on mathematical ground, and the fact that discoveries can be made in mathematics seems to show that such omission constitutes the gradually receding limit of the science But it may safely be said that in mathematical construction we are secure against any conditions which do not fall within the definite general type of those which we have put there

Apart from such an abstraction, the assumption that we have a perfect analysis of R is always erroneous, and the postulate that we must express α and not-α in terms of the ultimate analysis of R is theoretically incapable of being fulfilled. The approximate or presumptive fulfilment of the postulate depends chiefly on our general systematic knowledge of the course of things, which enables us, as we think, to draw a line between R_1 the whole irrelevant residuum, R_2 the real basis of both α and the positive not-α which make up the phenomenon *prima facie* in question, and therefore *as such* irrelevant to the distinction between α and not-α, and R_3 the limited number of precise positive conditions on the combination of which, including their reciprocal interference, the precise distinction between α and not-α depends R_3 is finally reduced to the general α. It is obvious that R_3 has its roots in R_2, and R_2 in R_1, so that the distinction between these remainders cannot be pressed far except on the ground of specific knowledge. Gravity for example belongs to R_1 in relation to an acoustical experiment which I purpose to describe presently

No doubt gravity is essential to this experiment in the same sense in which it is essential to all that takes place on the surface of the globe. But the variations of gravity within their actual limits do not affect the experiment appreciably or at all. Thus general systematic knowledge operates through confining the immediate problem to R_1 or at most to R_2 and R_3, by setting down R_1 as for this purpose not-R, viz not a residuum within the problem, but one outside it R of the problem (viz R_2 and R_3) is related to not-R of the problem (viz R_1) just as α is related to not-α in the immediate experiment.

And secondarily, in as far as R_1 is *not* materially known, but is an unknown residuum, a presumption of its irrelevancy may be supported by the *number* of instances in which R (as R_2 and R_3) is a sufficient R for the experiment, i.e presents α and not-α as required This confirmation of the line drawn between the R of the problem and the not-R of the problem is precisely the same in kind as the confirmation by number of instances of a material difference in the adaptation of two plants to their environment, worked out above, p. 135 Every successful trial to generate α and not-α on the basis of the R of the problem alone, increases the difference between the probability of the result on the hypothesis that it is due to the known factors included in the R of the problem, whose certainty of existence is assumed,[1] and the probability that the successive occurrences of α and not-α are due to independent causes, some of which must therefore fall outside the persistent conditions which make up R_2 and R_3.

If, on the other hand, we confine ourselves to the R of the problem in its most limited sense, viz to R_3 analysed as β, γ, δ, then the assumption that we have in β, γ, δ an ultimate analysis of R can only be questioned on the ground of a further analysis suggested or presumed. The possibility of a *positive suggestion* needs no explanation, it would arise from conjunctive induction and pass through analogy in the ordinary way, having the form 'δ is probably complex, consisting of λ, μ, ν', and would be tested by further experiment in the ordinary way, some part of such experiment probably

[1] Because if we fail to produce them all, we do not expect α, nor count the case as a trial

coinciding with passing $\beta\ \gamma\ \delta$ through their possible variations This experiment would however then be extended by taking account of the variations and zero values of $\lambda\ \mu\ \nu$ and including or excluding these, as the result might require, in the analysis of a and not ι A *presumption* of further analysis very commonly arises when no positive suggestion is forthcoming We have a strong presumption e g from the history of chemistry and from the nature of ordinary substances that we shall not constantly be lighting upon new elements, and therefore we do not assume an unfamiliar substance to be an element—i e we presume that further analysis is possible—even though we should fail to analyse it at the first attempt

After using the above example, however, I must guard myself against the idea that 'analysis' is for this logical purpose to be understood in a sense borrowed from chemistry Logical analysis is the understanding of any whole in reference to its constituent parts or factors, and chemical analysis is only the understanding of a chemical whole as chemical It is a trite observation, but perhaps necessary to be repeated here, that the analysis *of* an organism, if it is not its analysis *as* an organism, may destroy rather than display its inmost nature If δ is an organic element, and λ, μ, ν are its chemical constituents, then our first business in 'analysing' is to ascertain whether we want these constituents λ, μ, ν, which are, strictly speaking constituents not of δ but of δ_1 (δ as a *merely* chemical substance) or whether we want other constituents o, π, τ, which if we can we may then further construe into forms of combined chemical action $\lambda\ \mu\ \nu\ (o)$, $\lambda\ \mu\ \nu\ (\pi)$, $\lambda\ \mu\ \nu\ (\tau)$ In speaking of *organic* characteristics as capable of analysis, I have in mind such questions as the precise degree, direction and mode of transmission in which irritability in plants sets up reflex action or how far certain movements are reflex and how far purely mechanical—e g in the case of circumnutation[1] The description of analysis needed in

[1] I have not the smallest desire to deny that the joint action of $\lambda\ \mu\ \nu$ as chemical agents may make up, and, for all I care, initiate the action of δ as organic The question of abiogenesis is an open one for Logic I am only pointing out that, combined as δ, the constituents λ, μ, ν, acquire organic attributes o, π, τ, which are capable of having their nature precisely determined by experiment

each particular case must of course be determined by the nature of *a*, and the consequent nature of parts or factors with reference to which it is to be understood

Experiment with the Siren analysed.

δ. A very simple and beautiful example of the progressive reciprocal definition by experiment of *a*—*a* and not-*a*—not-*a* is to be found in the well-known verification of the connection between rapidity of periodic vibrations and musical pitch, as given by help of Helmholtz's Siren The reader would do well to study the account and figure of this instrument in Helmholtz's Popular Lectures [1] In the light of the account which has just been given of the logical purpose of experiment we may roughly analyse this arrangement as follows —

R_1, or the not-R of the problem, may be typified by the action of gravity, which pervades all matter, but the variations of which within their actual limits are indifferent to the phenomena now in question

The R of the problem, consisting of R_2 and R_1, within which *a*—*a* and not-*a*—not-*a* are to be sought for, is in general terms the musical sound produced by the machine, and its conditions, including the machine Of these we may take as R_2 (extending, as we knew it must, continuously into R_1) the air in the room, the hearing ear, and the machine itself, as operative in the production of a physical effect which together with a hearing ear results in musical sound R_2 is relevant as the proximate basis of the phenomenon itself, and as such, regarded by contrast to R_1 or not R, is itself an *a* defined by a not-*a* and connected with an *a* But *prima facie* and outside the experiment itself, R_2 does not demand much analysis The air in the room or some conductor of sound must act as a medium between the machine and the ear—sound cannot pass through a vacuum,—the ear must not be outrageously abnormal—this is included in a reasonable interpretation of 'hearing ear,'—and of course there must be no other source of sound undistinguished from the machine itself. And as R_3, the unanalysed whole which is the phenomenon, we must take the action of the machine as already somewhat idealised by analysis, i e in respect of its quantitatively specified effect upon the air in generating aerial impulses with

[1] Engl Transl, published by Longmans & Co, Series I, p. 57 ff

a measurable rapidity of succession restricted to certain limits, and in one series, or in two simultaneous series, at pleasure *a*, the musical sound,[1] is most conveniently considered as included in R_3, but as also distinguished by anticipation and confronted with the gradually narrowing R's and their analyses as the problem to which they are all directed. Ultimately, however, *a* itself will of course be reacted upon by analysis, and will be found to include distinct elements both in kind and in degree

If we omit, for the sake of brevity, to speak of the characteristics of quality and loudness in musical sound (and these are in fact not especially illustrated by the Siren), we find that R_3 is immediately reducible to a very simple relation, the relation of comparative rapidity of succession between series of puffs of air, which series differ in no other assignable respect R_3 is, in other words, assumed *ad hoc* to be exhaustively analysed. Confining our attention, to begin with, to the case of a single series (and not two sounding at once) we become aware *first* of a constant relation between any given rapidity and the pitch of the note which is heard while that rapidity is maintained, and *secondly* of a relation of quasi-proportion, sufficiently explained above,[2] according to which rapidity is to rapidity in a definite numerical ratio, while pitch is to pitch in a definite recognisable relation, measurable by intervals but not made by summation of intervals First, then, every particular rapidity is to us as an *a*, deviations from which on either side are *to it* as not-*a*, corresponding to an *a* deviations from which on either side are *to it* as not-*a*. But as each and every deviation from *a* brings a 'proportional' deviation from *a* with it, every such not-*a* is to some corresponding not-*a* as an *a* to an *a*, and we have therefore the most perfect case of negative relation between positive contents. But, secondly, when the quasi-proportional character of the two series attracts attention (which it did from the earliest times, as a fact demanding explanation, owing to the relation

[1] 'W*hat* musical sound?' the reader may ask I reply, *in fact* some musical sound in particular, but as a problem musical sound in general It is impossible to particularise the sound you have heard, except as a result of advancing analysis.

[2] See p 74, above

of the length of strings to the notes which they sound) it becomes an essential element in the relation which constitutes R_3, and presents itself as a further determination of the mere principle 'rapidity has a constant connection with pitch', which forms the first simple a—a We thus obtain the suggestion of a *law*, and the verification of this suggested law becomes the object of the experimental process A law is treated just like any content a The object is to show that a the realisation of the condition is attended by a the realisation of the consequent, and that not-a, any deviation from the consequent (ultimately perhaps a variation of the consequent), is preceded or attended by a deviation from or variation of the condition Thus as always our goal is in the Hypothetical judgment, 'If a, then a,' with its reciprocal, 'If a, then a'

It is further worth while to mention how the experiment in question obtains minute measurable variations of a and a at pleasure. If rapidity corresponds to pitch in a certain proportion, then two rapidities in the right proportion correspond to a definite harmony Deviations from a harmony are recognisable by a trained ear with extreme minuteness. Helmholtz's Siren will sound two sets of impulses, of controllable rapidity, together; and in the first place by adjusting the two precisely to the proportion required by the law, it verifies the law a—a in a compound case. But then by a contrivance for very delicately, and measurably, altering the rapidity of one of the series, a slight or considerable discord can be produced at pleasure If we were interpreting a—a as '1·2 gives note and octave', this result reads as a confirmatory content, 'not a—not-a,' i e 'deviation from 1 2 gives deviation from note octave' But of course, as before, this not-a is a case of a, rapidity corresponding to pitch, though not to two notes, one an octave above the other *because* the sets of impulses are not as 2·1 in rapidity.

But here, with the Siren alone, we are pretty much at an end of our analysis. That R_3 is not merely a law of rapidity in succession, but a complex theorem concerning shapes of vibrations and their decomposition into pure pendulum oscillations, related to the *quality* of a and to discordant beats in a (if a includes two notes sounding together), does

not appear from the above experiment It would perhaps not have been suspected but for the obvious fact that an air wave must have some shape, on the one hand, and that pitch is not the only element in sound, on the other.

The point of the above illustration consists especially in displaying the various senses of not-*a*, and the various stages of its adjustment to *a* R_1 is not-*a* in one sense, R_2 in another, and in a relative and shifting manner portions of R_3 are not-*a* also The same applies throughout to not-*a* Induction consists in separating the R's and in establishing those variations of *a* which appear, against any fixed starting-point, as relative not-*a*'s (and the same with *a*)

I may conclude this chapter by calling attention again to what I have insisted on in another work,[1] as the claim of an experimental apparatus to be considered in the light of a reasoning machine It must be granted that in any logical engine whatever we have to make the conclusion, i e to read it off as a conclusion, and if we are to do this we may read off the connection of imperfect ratio and false harmony from the Siren as distinctly as we can read a combination of letters from Jevons' logical machine In the operations of nature, causes have their consequences, but the causes are not precisely known, and the consequences are therefore not consequents In experimental instruments we find the attempt made to generate actual consequences which shall also be consequents as arising from conditions precisely known in respect of the mode and degree of their combination Any instrument which does this may be called a reasoning machine, whether it deals with combinations and eliminations of letters as logical symbols, or with the same relations of actual number, or of any definite motions with their effects. The value of the connections thus demonstrated is of various degrees but a complex experimental apparatus has the advantage in the synthetic variety of the contents which it exhibits as relevant to each other, if the logical machine has the advantage in the abstract generality of its formal conclusions

[1] Knowledge and Reality, p 327 ff Cp Jevons, Principles of Science, p, 282 ff

CHAPTER V

Scientific Induction (*continued*).

Hypothesis and Postulate

1. Hypothesis is a name that may be applied to any conception by which the mind establishes relations between data of testimony, of perception, or of sense, so long as that conception is one among alternative possibilities, and is not referred to reality as a fact

Hypothesis falls outside Postulate.

1 From Aristotle onward, indeed, logicians have been anxious to consider a hypothesis as the suggestion of a real agent—a thing or occurrence in a thing—related to the data as 'cause' to 'effect', and to distinguish such a suggested 'agent' from a mere suggested 'reading' of the phenomena —a principle, law, or definition Of course there is a primary difference between a material agent and an ideal law or principle, but the distinction is not ultimate in theory and appears to be, for this reason, incapable of being sustained in scientific practice A 'working hypothesis'—and most of the great unifying conceptions of modern science are working hypotheses—is the suggestion of a real agent taken as equivalent to the suggestion of a mere law or principle It is worth while for the sake of clearness to look at the distinction between law and real agent in a form recently given to it by Lotze, viz as the distinction between Postulate and Hypothesis

In the account to which I refer [1] the name of *Postulate* is given to the conditions which are absolutely and essentially involved in a given set of appearances, and apart from which 'the content of the observation with which we are dealing would contradict the laws of our thought'. These conditions, it must be observed, need not be abstract, except in the sense of being definite and precise They might therefore, I infer, exhaust or define the nature of a real agent, in so far as a real agent is capable of being determinately known But it is plain that as a rule they will not suffice to do so The concrete

[1] Lotze, Logik, sect 273

nature of a material thing will contain much that is indifferent to the conditions precisely involved in any determinate effect.

By *hypothesis*, therefore, in this same account, is meant a conjecture which specifies the natural agents taken to be at work in a phenomenon and to be the means of fulfilling the postulate involved in it, in the case under investigation. In other cases, it is implied, the same postulate might be satisfied by means of other agents And, it should be added, by a *fiction* is meant the reference of an effect to a cause or principle which we know to be incapable of producing it, but from the real effects of which it only differs by an error which is capable of being determinately assigned Omitting the case of a confessed fiction, and including a fiction, not confessed to be such, under the title of a hypothesis, we may throw the remainder of our discussion into the form of an enquiry into the distinction between Hypothesis and Postulate

The Postulate sets an abstract problem which Hypothesis has to solve in the concrete The distinction *prima facie* coincides with that upon which Mill lays stress in his treatment of hypothesis, between a quantitative law of action, and the thing which acts according to that law But it would certainly seem that every hypothesis in order to be established must be passed over into the content of the postulate, in the sense that, without the matter suggested in the *hypothesis*, no less than without that suggested in the Postulate, 'the content of the observations with which we are dealing would contradict the laws of our thought.' For this is ultimately the ground on which we affirm of Reality everything that we do so affirm. When the postulate is shown to contain the hypothesis, by a concrete proof that the suggested thing or fact is necessary to prevent self-contradiction in our thought, then we have a hypothesis with a *vera causa* (see II below). When the hypothesis is moulded into the postulate, not or not exclusively by proof of the concrete supposition, but in a great degree by attenuating its content into a 'law of action', then we have a 'working hypothesis', i e materially an abstract postulate, but formally a supposition of a real agent Such a hypo-

thesis is a fiction which may or may not be a confessed fiction. In Mill's notes on Whewell [1] we see the process of attenuation at work, reducing hypotheses to fictions which are confessed by Mill and not confessed by Whewell Modern science seems to the outsider more and more tending to substitute explanation by laws of action for causation by unknown real agents. But, in theory, a determinate agent may be involved in the postulate just as much as an abstract law, supposing that the agent is operative in the content in modes sufficiently many-sided to assign it a determinate nature For logic, law and agent are alike conceptions by which thought constitutes the content into an organised whole, both may be 'within' the content, if we include in the content what is needed to constitute it rightly, neither can be within the content if we separate it, by an unreal and indeed impossible distinction, from the work of thought in determining it Every object of perception is such a conception, by which data of sense are determined in a way necessary to make them intelligible.

The real distinction which Lotze should have drawn is not between the law of action and the concrete real agent, but between the concrete real agent as known to be necessary for the explanation of the observations, and such an agent as not so known, but arbitrarily imagined, or identified with something known from other sources. If we assume a thing thus arbitrarily, or on the ground of extraneous knowledge, then (considering the thing in the latter case apart from the extraneous knowledge on the ground of which it is assumed) we have the relation 'If a, then a', but not the reciprocal 'If a, then a', i e in other cases other agents than a might satisfy the same postulate, or minimum of conditions, involved in a But this unnecessary element in a hypothesis cannot of course be acquiesced in The supposed real agent must either be elevated into the content of a postulate, or depressed into that of a fiction. Obviously, however, before deciding

[1] Mill's Logic, i, p 335, and ii, p 24, e g 'Can an agency undulate? Can there be alternate motion backwards and forwards of the particles of an agency?' Mill is maintaining in effect that Whewell's view of the imponderable agents reduces them to laws of action He is distinguishing an *agency* from an *agent* (the ether) And compare Clifford on Causation, Lectures, &c, vol i, p 153

that the latter course is the only one open to us, we must concentrate *all* available knowledge upon the supposed real agent in order to test its right to become a postulate One science e g may need one aspect of it, and another another

Hypothesis with *Vera Causa*

11. Thus to meet the difficulty that many characteristics of a *thing* assumed hypothetically to account for certain data are likely to fall outside what those data demand and justify, it is usual to require of a hypothesis that a the supposed agent shall be a *vera causa* This can ultimately have but one meaning It must come to this, that a, though containing elements which are superfluous for the explanation of *the data from which we happen to have started*, yet contains no elements which are not necessary to the explanation of *some data or other*. It is commonly said that a *vera causa* is one independently known to exist, or accessible to direct perception. Of course we do not restrict our conviction of reality to matters accessible to direct perception—the centre of the earth, the inside of a block of marble, the other side of the moon, are cases in point And if we did attempt this restriction, what is direct perception? All perception is inferential, and proceeds by furnishing conceptions which bring data of sense into intelligible relation And if we require that the cause shall be independently known to exist, this is a mere question of the range of observations which it is to explain A *vera causa* then is a thing, or occurrence in a thing, whose reality we are thoroughly convinced of from the necessity of reconciling observed data,[1] and there is no reason in the nature of things why a single science or a single range of reality should not suffice to produce such conviction '*Direct* perception' is a mere popular phrase without logical meaning The question is simply whether our data are determinate enough to guide us to the nature of a real thing as explaining them What is *really* demanded in the *vera causa* is probably

[1] The most thorough and simple way of classifying matters known from testimony or history is to include them under the head of conceptions which are necessary to determine observed data, the observed data being the books, speech, &c which bring the facts to our individual notice As to ranking agents under the head of conceptions, I may say that this is not reducing agents to mere conceptions As known and established to us, they *are* conceptions, though they may be more

independent evidence of the thing's reality, with an eye to the doctrine of chances A single coherent set of errors may vitiate a whole coherent system of appearances, but the chances against errors in *independent* sets of observations are the same as the rapidly increasing chances against coincidences of independent events [1] This is a parody (as the doctrine of chances is always a formal parody of some material truth) of the operation of multiform data in moulding a concrete hypothesis, which will be illustrated directly

Thus in a 'working hypothesis' we have postulate and hypothesis tending to identification by attenuation [2] of the hypothesis, in a hypothesis with *vera causa* we obtain the same result by extension of the postulate to cover the alleged cause and turn it into a *vera causa*

Phases of Hypothesis.

2 In an ultimate sense, there is no knowledge without Hypothesis

Rudimentary Hypothesis

1 'All science may be rightly described as progressive "colligation of facts" through superinduction of conceptions' [3] if it is understood that, though such conceptions are present in the real facts and are not mere additions out of our heads, yet in the progress of our knowledge such colligation does not operate upon the real facts themselves, but only on the facts as imperfectly understood by us Thus the whole course of the present work has been an attempt to trace the progressive determination of feelings, or of facts imperfectly understood, by conceptions which may be regarded as hypotheses in course of development and proof. The continued identity of an individual, for example, which is the soul of the individual judgment, may be regarded as a conception or hypothesis which is superinduced (though without conscious reflection) upon the successive appearances which we observe, and 'colligates' these facts And as we have seen in speaking

[1] If the chance of error in one set of data is $\frac{1}{2}$, the chance of independent error in two sets (of the same but independent liability to error) is $\frac{1}{4}$, in three $\frac{1}{8}$, and so on

[2] A working hypothesis often, and perhaps usually, partakes of the character of a fiction, being in fact suggested as a *vera causa*, and subsequently attenuated till it is clearly not a *vera causa*, though retaining its original claim to be so

[3] Green, Philosophical Works, II, p 288

of Induction, hypothesis in a genuine sense, as a conscious activity, begins to operate where the individual judgment begins to be employed in explaining the conjunction of attributes, in conjunctive or enumerative Induction From this point, at which Hypothesis is represented by the content of a generic or specific name, we have watched its development through analogy and through scientific analysis, till in the experiment of the Siren we found ourselves testing by determinate perceptive comparison a relation which can only be completely explained by a complex mathematical theorem [1]

Mediate Hypothesis

ii Procedure by Hypothesis proper is mediate

Hypothetical Nature of Induction

a It is clear from what has been said that we must assent in substance to the view of Jevons and Sigwart which is in the main that of Whewell [2] and De Morgan,[3] so far as it asserts the essential identity of Induction with procedure by Hypothesis And indeed Mill himself might almost be reckoned on this side He shows [4] triumphantly that the Method of Difference will test the premises of a Deduction, and the fact that the 'instances' on which it operates are in that case obtained by Deduction, he sets down as of no consequence, i.e. as not interfering with its Inductive character But it is not so clear that this method, which unquestionably will test the consequences of a precise deduction and therefore the truth of its premises, will perform any other function that could be called Inductive Mill's objection [5] against Whewell's hypothetical method, on the ground of insufficient provision for excluding unproved or unproveable elements of hypothesis, is an objection which arises from the impossible demand for *merely* negative and exhaustive determination It is very probable that Whewell makes too little of the necessity for showing or for its being possible to show that *nothing but* *a* could produce *a*, but what Whewell seems to have rightly felt is that this is after all in its essence a material and positive question, depending on the degree and mode of connection between *a* and *a*, and being for logic the same as the question

[1] The theory of wave-propagation, which explains among other things the discordant beats produced by sonorous impulses which have not certain definite ratios of rapidity

[2] See Mill's Logic, II 24

[3] Budget of Paradoxes, p 49

[4] Logic, II 12

[5] See Mill's Logic, II, p 24

whether *o as such* produces *a* The possibility of proof or disproof, which is claimed as essential to the 'legitimacy' of a hypothesis, must be a material or real possibility, and reduces itself to specific presumptions that proof may be had, which are in themselves grades of proof But while accepting the general view to which I have alluded of the importance of hypothesis in Inductive Inference, I am unable to agree with some important results which have been held to follow upon such a view

Example of fusion between hypothesis and data.

β I shall begin by endeavouring to lay the true doctrine very briefly before the reader, in the sense in which I understand it and in which it seems to me to follow from our previous discussions It will then be necessary to speak of the relation between Induction in the scientific sense and the work of generalisation which is popularly ascribed to it, and I shall conclude the present chapter with some observations on the above-mentioned misapprehensions, and on the true relation of Induction to Inference as such

The purpose of the example which I propose to analyse is to exhibit the mediate identification of a hypothetical cause, at first sight somewhat remote, with a given effect. I intentionally select an instance in which the identification is not quite perfect, in order to display the full nature of the difficulty to be overcome

As a datum to be explained, we will take the curious fact, long known to scholars, that the Greek god Apollo, especially the Apollo of the Troad, is associated with the mouse, both in his appellation Smintheus and in recorded usages—there were sacred mice and figures of mice in his temple, and so forth [1]

The conjunction of aspects which excites surprise in this fact is the association of an insignificant animal with the worship and the temples of a comparatively pure religion. A large choice of analogies lies open to us, any one of which might furnish some sort of mediation between these two extremes, and of these that which is at first sight the most

[1] My example and my arguments are all drawn from Mr Lang's Custom and Myth, p 103 ff My purpose however only permits the most meagre reproduction of some points out of this interesting study

remote may perhaps on a consideration of all the phenomena be considered the most hopeful

It seems that in the Peruvian religion we find this same conjunction of aspects, the association of insignificant animals with the worship and the temples of a comparatively pure creed And in that instance, it further appears, we have a definite and complete mediation or explanation of the two terms or extremes' Before the establishment of Sun-worship by the Incas as the creed of the state, the Indians of the various tribes worshipped tribal animal gods, including all sorts of insignificant animals the Indians of each tribe believing themselves to be descended from some one of these animals 'After the establishment of the purer religion, the Incas had the good policy to collect all the tribal animal gods into their temples in and round Cuzco, in which the two leading gods were the Master of Life and the Sun' This toleration of an older and cruder in subordination to a purer faith is a very common phenomenon, as Mr Lang truly observes in religious evolution And he cites an example of a festival described by Theocritus which still continues in a Catholic country

Here then we have a content the whole of which is given (I assume) in perception or in the *proximate* interpretation of perception, viz in history Analogy or Induction would not commonly be held to apply within the limits of this content, but nevertheless in as far as within the single 'instance' or range of reality—which is really the life of a whole nation—a *principle* is detected by our thought, there is operative what constitutes the essence of inductive as of all other inference But no details have to be referred to reality solely on the strength of the principle, because it happens that they are all warranted by testimony [1]

Now if the content which perplexed us in Greek religious history fell *bona fide* within the lines of the content thus warranted and interpreted in Peruvian religion, no inference

[1] It is probable, and appears I think from Mr Lang's account, that the interpretation even of the known succession in Peru into an intelligible evolution would involve, as almost every interpretation does, some remodelling and supplementation of details So far we have inference in the popular sense

would be necessary, or rather, the purely formal inference which recognised the identification would suffice to include the Greek problem under the same solution as that which supplies itself for the Peruvian problem But the very slight and superficial abstraction which is all that we have thus far formulated of the Greek problem can warrant no such material identification—so far as we have yet stated the point, almost any hypothesis might explain it, the misunderstanding of a name, or the caprice of a priest or king What we must now do is to look in the Greek problem for the facts and relations of which we have seen the significance in the Peruvian problem. But as historical data such facts and relations are wanting; and here we have the essential difference between Induction by analysis of Perception, and Induction by mediate Hypothesis Our hypothesis is *prima facie* a conjectural matter of fact falling wholly outside the content which has to be explained The view which I wish to illustrate is that our proof of the hypothesis must ultimately depend upon the characteristic positive connection between the hypothesis and its consequences This connection is as we have seen elucidated and purged of irrelevancy by the establishment of limiting negations, but is not otherwise dependent on the disproof of an indefinite number of alternative hypotheses, and is no more restricted to mere probability than is the determination of any perceived data by any conception which makes them intelligible [1]

We have before us, as a datum of fact, a surprising conjunction between Apollo and the mouse, especially in Apollo's temple. We have as a suggested fact which might explain this conjunction, a previous state of Greek or neighbouring tribes in which they worshipped animals such as the mouse, together with a religious evolution in which the earlier cult survived by the side of the later and purer worship According to the ordinary process of Induction as inverse Deduction,

[1] Contrast with this the mechanical views of Jevons, Principles of Science, p 152, and Sigwart, Logik, II, p 357 Jevons thinks that no proof by Imperfect Induction (Induction falling short of complete enumeration) can be more than probable Sigwart thinks that a hypothesis is refuted by refuting its consequences, but not proved by establishing them, though it grows in probability as its consequences agree with the facts

we proceed to 'deduce the consequences which might be inferred from the hypothesis'

In drawing consequences from a hypothetical state of facts we have to apply that state of facts to the reality on the basis of which it is supposed, and to examine in detail the results of the combination. This analysis of the content of the hypothesis is not a contrivance of demonstration, but an inevitable necessity of knowledge In working out, for example, the hypothesis now before us, we must take into account the customs relating to marriage and to names which belong to that phase of savage life which we are conjecturally imputing to the Greek race in the past Among savages named after tribal animals which they worship and bear as name or emblem, and from which they trace their descent, the members of one family do not intermarry with people bearing the same name or emblem, and the children of every marriage take the mother's name or emblem (totem) These names consequently tend to become scattered throughout a large region, and are associated with the well-known phenomenon, for which in very early society there are obvious grounds, of counting kinship through the mother and not through the father On the other hand, when this state of society passes away, as in European nations it has passed away, it is plain that a powerful family will crush out the names of the other families in a district, and form a local tribe called by an animal name. From this hypothesis thus analysed, if applied to 'mouse families', there follow primarily four results, which briefly stated amount to this —

(1) There would be places named from mice, and mice will be held sacred in those places This was so in the Troad.

(2) The mouse-name would be given locally to the god who superseded the mouse. This was so in the places called after the mouse

(3) The figure of the mouse would be associated with the god in his temple, and used as a badge or local mark in places where the mouse had been venerated The former usage was found, and the latter was not uncommon, in Greece

(4) Stories would be told in the district in question to explain the worship of the mouse This was so in the Troad.

I do not say that these four points, thus baldly stated, carry us very far But in so far as they support the hypothesis at all, they do so not merely as an arrangement of coincidences due more probably, in a calculable degree, to a single cause than to independent unknown causes, but, like an arrangement of results *which some person has the power and a strong motive to produce*, they support the conjectural cause by the material connection of the data with it, or a material extension of the data towards including it One of the above points for example is the appearance of the figure of a mouse as a badge or city emblem in Greece This, when referred to an actual race of men exceedingly conservative in its customs, is a point, though a trifling one, actually in common between hypothesis and data The badge or crest of a city is not the same thing as the totem of a family, but the connection of parts of cities with local tribes is too well made out in Greece and elsewhere to give us pause And the veneration of an animal by the people of a city in ways strikingly analogous to totem worship is made out in the case of Egyptian cities Of course this point *may* be otherwise explained than by the suggested hypothesis, and so may all the others, but they all, as referred to the life of a race, demand some explanation, and the only difficulty is to model that explanation rightly It is this idea, that of moulding a hypothesis, that should be substituted for the idea of gauging its probability as something attaching to its definite and irrevocable form To meet paradox by paradox, rather than admit that a hypothesis can only be established by the refutation of infinite others [1] and the non-refutation of itself, I would maintain that *of every set of data some positive hypothesis* (viz that 'something or other' conditions these data) *is within our knowledge demonstrably true*, and that the problem of induction by the inverse method or by hypothesis is merely to further determine this 'something or other' In this work of definition, as we have abundantly seen, negation is all-important, but it must be motived and relevant nega-

[1] This is the root of the idea that no results of hypothetical Induction can be certain The idea is ridiculous when it is once seen that hypothetical Induction is identical in principle with common perception and with all Inference whatever

tion, 'not *this*, because *that*, which has a determinate relation to this'

I should weary the reader by further discussion of the mouse hypothesis, which moreover space forbids me to treat in its interesting details But I must point out that by considering the peculiar marriage customs (e g maternal kinship), sacrificial and festival rites, and animistic beliefs, which are traceable throughout Greek life, and which are characteristic of the primitive phase that forms the content of our hypothesis, we can remodel hypothesis and data once more, and this time into a really intimate approximation to each other Hypothesis and data approach amalgamation in the conception of a finely gifted race still bearing in its prime the traces of a natural though characteristic evolution out of a savage past We might almost claim that a savage phase of life is a *vera causa*, apart from the proof of our special hypothesis,[1] not only in the Peruvian but in the Greek race Is not the conception of a past and natural evolution, in the case of *any* race of men which we may be considering, a conception 'apart from which the content of the observation with which we are dealing would contradict the laws of our thought'? But if so, then, according to the distinction accepted above, the conception, *although* that of a real agent or event, is a postulate and not a mere hypothesis, and therefore is the conception of a *vera causa*, and the work of induction is, as said above, to assign to the postulate in detail its actual content or law of action

Other hypotheses, independent of that which the author advocates, are carefully dealt with in the chapter from which I have been quoting And I think that any one who considers the matter in the light of this or any equally genuine piece of research must feel that just in so far as the adverse hypotheses are independent, their refutation, although a *sine qua non* of the establishment of the hypothesis advocated, can never genuinely contribute to that establishment In other words, the refutation of other hypotheses is a genuine assistance to one hypothesis only when it elicits positive content which goes to model this latter hypothesis

Before leaving this subject I must refer back to the dis-

[1] Every hypothesis *when proved* is a hypothesis with *vera causa*

cussion of Book I[1] on cause and ground, and must explain that the content of a hypothesis may correspond, according to the degree of its purity or relevance, to any member of the series there described, to cause, to effect, to antecedent, consequent, or ground The reason for treating an example in which the popular sense of cause is dominant was simply that the equally popular sense of hypothesis, from which the theoretical difficulty of its use arises, corresponds to the popular sense of cause (or effect) The more scientific type of hypothesis approaches more nearly *prima facie* to the nature of a postulate or ground—of a principle included in the facts, or of a systematic reality which they constitute. And hypotheses which are, to begin with, of this type do not present the great apparent difficulty of passing by sheer inference from isolated data to actual things and facts not included in them As we have seen, a hypothesis which to begin with is not of this type, necessarily tends, in course of demonstration, to approximate to it, just as, in the case which we tried to analyse, certain isolated data and isolated suggestions about the Greek race tended to coalesce into a systematic conception of that race as developing in a normal fashion under the natural influences and conditions which appear to be common to mankind

Generalisation

3 It is unquestionably the case, that a process or result which may be termed Generalisation is somehow connected with Induction The only question is how to state the connection

'From many to all' exploded

1 I trust that the popular idea according to which Induction is a process from what happens often to what happens always, from particulars to the totality of particulars, has been set in its true light by the whole scheme of our account of Induction The conception, to which unhappily Professor Jevons adhered, of Perfect and Imperfect Induction as corresponding to complete and incomplete enumeration is hopelessly fallacious It cannot, I think, be necessary to pursue this error at the present stage

By mere determination

ii There is a sense in which all abstraction, i e all becoming conscious of the determination of sensuous data by explicit

[1] Chap vi.

conceptions, operates as Generalisation Such generalisation is embodied in the hypothetical judgment, which of course is not hypothetical *qua* judgment and is in no sense a hypothesis, but is an affirmation, based upon a reality illustrated by a hypothesis The explicit conditions forming a determinate case which, as the antecedent in the hypothetical judgment, illustrate or qualify Reality, take the consequent with them wherever they go, and in this sense the judgment is general, absolute, or universal Supposing the judgment to be absolutely true—and formally we can suppose nothing else of a judgment which we make, though of natural phenomena no judgments are absolutely true—it is absolutely universal Varieties of detail may fall within it, but they do not affect it Against such unessential variations, and against mere number of examples in time and space, the explicit antecedent appears as general, and in selecting and defining it a work of generalisation has practically been accomplished There is no advance from known to unknown in the strict sense of the case we are now considering There is no advance from known to unknown in saying that a pair of parallel straight lines which you may intend to draw to-morrow will never meet If the 'intent and purpose' of the antecedent 'hath full relation' to the example to be adduced, there is *prima facie* no inference, no extension, no advance, in affirming the consequent true of that example

A still stronger and a far more important case of such *generalisation by mere determination* is afforded by mediate or inverse induction through hypothesis A hypothesis is a hypothesis because it is not to begin with present in the data, and has to be brought there by mediation But to supply matter for modelling a conception which is not furnished by mere direct interpretation of sense, i e by perception or by testimony, a set of data must have wide range and be capable of a high degree of systematic inter-relation Thus e g in an anthropological hypothesis about the past of the Hellenic race a considerable portion of the history of Europe is ultimately involved, and the data bearing on it are inter-related and elucidated

In this sense a hypothesis or mediately obtained determination of observations by a conception superinduced upon

them, is likely to involve as a result an elucidation and articulation of a wide range of reality Sparta and Athens, Crete and the Troad, Sicily and Magna Graecia, all fall within the region of reality which through determination of our knowledge about it by the anthropological hypothesis in question would acquire for us a certain set of highly important common properties and relations Here the generalisation results from the range of the explicit system, and not from the mere abstract precision of a hypothetical antecedent What we are systematising is a reality, and the judgment which expresses our conclusion may indeed, like all precise judgments, be thrown into hypothetical form, but its content makes it really categorical In it therefore we have two kinds of generalisation, one depending on the range of the system which we have constituted, the other on the hypothetical abstractness which makes even this concrete system a case, within the lines of which systems differing from it in other relations may conceivably fall

Material or Analogical Generalisation.

iii The generalisation that falls within the limits of strictly scientific Induction is confined to what has just been described as Generalisation by mere determination It is not generalisation in the sense of an advance from the known to the unknown Obviously there can in no case be such an advance except in the sense that the unknown becomes known And this advance is made in the determination itself, the case, when determined, is known, and is generalised in so far only as it is known Water is composed of oxygen and hydrogen in certain proportions There is no further generalisation in applying this to water such as we have analysed, and if there could be a doubt whether a certain liquid was in that sense 'water', the judgment gained by previous analysis would not *prima facie* determine it The hypothetical character of this judgment is an automatic apparatus for excluding material generalisation In the extreme instance of such exclusion the antecedent 'water' is little more than a name, to which 'composed of oxygen' &c supplies the content Then if 'composed of oxygen' &c is not true in a particular case, that case *ipso facto* falls outside the hypothetical judgment Or in a less extreme instance, 'water' may indicate certain

visible or other properties which are synthetically related in the judgment to the chemical composition But here again the least variation in those index-properties may formally throw the case in which it occurs outside the hypothetical judgment.

This is all very well so long as we interpret the hypothetical judgment to be strictly hypothetical abstract or necessary But the account given in Book I of the Universal Judgment in its sub-forms of Corporate and Generic Judgment reminds us that we have a Categorical element to deal with in the characteristic nature which binds things together into ideal or actual totalities, and we have to face the possibility that the nature of these totalities may conflict with the content of hypothetical judgments in which they are set down as antecedents Every universal is an identity in difference, and the identification of the conditions by which inductive enquiry has determined a content is not, as we assumed just now, a purely formal activity, but is, like every judgment, a synthetic and material operation Reliance on the accuracy of our analysis will lead us back to a Lockeian formalism, if we neglect the identification of the data analysed 'Man is rational, because if a creature turns out not to be rational, it is not what we called a man,' is an argument which, unless specifically justified, has no more content than A is A because we called it A If no distinct elements are fixed within the synthesis, the synthesis itself is destroyed But on the other hand, if we affirm our analysis directly of a generic content which is categorically taken and includes a system of differences, it is clear that we are pledging ourselves to a material generalisation Does 'water' for example, in the judgment above cited, include steam and ice? Apart from specific chemical knowledge, I do not see how we could predict that it would do so And if we say 'water' for chemical science is matter of a certain composition, whether in a liquid, solid or vaporous state, then we have got back to the nominalist judgment criticised above, '*a* is *xy* because what is not *xy* is not called *a*'

I do not mean to say that the above considerations are of serious importance in precise scientific induction, in which the

definition of the data as elements in a synthetic relation is always a primary problem But *either* in popular applications of science—the most fertile of all sources of fallacy—*or* in provinces of knowledge which are largely dealt with by analogy, it is essential to bear in mind that the identification of the datum, which has been analysed or determined by a conception, with either the popular import or the analogical generic content of that datum, is always a matter involving a material synthesis and not a mere formal recognition Where indeed the precise determinate conditions of a phenomenon are explicitly recognisable, there the identification of the phenomenon is a formal act, and the hypothetical judgment applies *ipso facto* But where, as is constantly the case in practice, and, *owing to the nature of a universal, always in theory*, the conditions are not unambiguously recognisable, but only either a *part* of them, or a *case* of them, or an *index-mark* of them can be discovered, then we must form a synthetic judgment of identification or distinction, based on the general principles which we have seen to govern argument by analogy, viz on the reality of a system of recognisable types and purposes

Apart from such a material assumption the truth of principles derived from Scientific Induction itself would be practically though not theoretically destroyed [1] A system of unapparent deviations in the properties of natural objects, such as to defy classification, might be such as to destroy the applicability, while not interfering with the formal truth, of hypothetical judgments In our earlier discussions on the hypothetical and categorical aspect of judgments, it was conceded that the *pure* hypothetical judgment does not allege the existence of the elements which it explicitly puts in relation But it is also clear that no ordinary result of Induction ought to be taken *bonâ fide* as a judgment of this type A judgment which cannot be denied but which has no range of real application has only abstract and not concrete truth If all our knowledge were of this character, as *qua* mere determination it conceivably might be, it would have no hold upon reality

[1] i e would be destroyed for concrete and real theory, but not for abstract and formal theory

So long indeed as the variations of natural objects observed a continuous and mutually coherent progression, we should only have a state of things not unlike the system of animated nature, which would be in some respects favourable to knowledge by the clearness with which it would mark the course of evolution, and might not be incompatible with human life But it is easy of course to imagine varieties in fundamental properties of substances not indicated by external appearance, which though following strictly from natural antecedents, and not in any way miraculous, would yet be incompatible with such a degree of knowledge as is necessary to maintain human life on the surface of the globe No analysis of water would help us, however true under the conditions under which it was made, if something which we could not distinguish from water except by renewed chemical analysis were liable to arise out of water by a concealed process of causation, and were endowed with the properties of sulphuric acid No formal principle will meet this conceivable difficulty Many distinctions all-important for human life *are* only learnt by degrees or are only drawn with imperfect success—e g between pure water or milk and the same liquids when contaminated with sewage-poison We can only say that *if* we are to live on the surface of the globe the results of scientific induction must not only have formal or hypothetical truth, but must also have that degree and proportion of categorical application which is necessary to enable us to adapt ourselves to the environment. This degree of categorical application, of which mere determination, except in the case of an extended system of reality like the Hellenic race, or Europe, or the British Constitution, or the Solar System, can tell us nothing whatever, measures the work of recognition or of generalisation which is over and above the work of generalisation by mere determination. When I say 'Water is composed of Oxygen and Hydrogen', I must mean by water, not necessarily all, but some large proportion of what I commonly take to be water If not, my judgment, however true in the abstract, fails to grasp reality in the concrete

The ideal of knowledge, no doubt, is not in this analogical generalisation, but in the second and larger kind of generali-

sation by mere determination, viz in the progressive reduction of reality to a single system or to comprehensive single systems It must be remembered, too, that the synthetic nature of every universal or identity is double-edged If all sets of conditions have to be recognised and interpreted as universals, all sets of conditions should be fixed and determined in the inductive analysis as universals—i e with the full prevision that differences, variations, extreme cases, will arise within them Nevertheless, it seldom happens even in geometry that a principle when first established is established in its full content and application Inductive analysis can never make full provision for the application to fresh cases of a principle which it discovers, except in as far as it discloses the nature of a comprehensive *individual* system of Reality within which other individuals fall

4 Our results as regards Induction are then as follows

General view of Induction

1 Our view of Induction as an inverse process differs essentially from that of Jevons by its dependence on material and positive connections, which are only defined by negation Inverse procedure by hypothesis is for him essentially a matter of probability, and depends on the exclusion of alternative hypothesis simply *qua* alternatives, i e ultimately on the statement of chances[1] or the number of cases out of all conceivable cases which are in favour of the result in question upon the hypothesis proposed This view essentially depends upon the false conception of generalisation which has been frequently alluded to, and according to which the ideal of Induction is perfect Induction, i e the summation of an infinite series As we have seen,[2] the statement of chances admits of valuable and extended application where we are dealing with classes of unknown conditions, i e conditions known to us simply as furnishing such and such numbers of 'equal alternatives', and in astronomical and other exact science it is justified by the splendid success of its results I will venture however to point out, in respect of one example adduced by Jevons, that other considerations seem to have contributed

Difference from Jevons.

[1] Sigwart seems to waver between this view, and the true view of proof by content Logik, II, pp 428 ff E Tr II 308

[2] p 135, supra

to the inference beyond those which fairly arise out of the statement of chances Kirchoff's proof of the presence of iron in the sun [1] depends upon the exclusion of the alternative that the 60 dark lines of the solar spectrum coincide *by chance* with the 60 bright lines of the incandescent vapour of iron The probability of a chance coincidence being (from the distance between the lines on the spectrum) about $\frac{1}{2}$ for each line, the probability in favour of a chance coincidence for all 60 lines is about $(\frac{1}{2})^{60}$, i e less than one in a trillion 'But on the other hypothesis,' Jevons continues, 'that iron exists in the Sun, it is highly probable that such coincidences would be observed' Here the proof seems to appeal to some of the considerations which belong to the positive connection between given content and hypothesis, or to material generalisation, or to both. Why should '*the* other hypothesis' be 'that iron exists in the Sun'? The answer must be, I should suppose, either that the 60 bright lines have a connection with the nature of iron, as *a characteristic* or exclusive differentia, which would dispense with the proof by calculation—or that though there is no exclusive connection between the nature of iron and the production of 60 bright lines, yet *in fact* no known substance but iron produces such lines, *and* it is very improbable on general grounds that a substance unknown to us but sharing this property with iron is present in the Sun [2] The former of these considerations would belong to the nature of true Inductive determination, the latter group to the postulates of material generalisation The real function of number and ratio in Induction has been sufficiently illustrated above

Ultimate nature of Induction.

11 After the discussion on p 117 above of the connection between Induction and other inference, I need only sum up the view which I have taken in a very few words Induction is

[1] Jevons, Principles of Science, p 245

[2] The fraction $(\frac{1}{2})^{60}$ represents, I suppose, the chance of 60 coincident cases all produced by independent causes, but ought we not also to consider the probability not merely of one unknown cause producing all the cases, and that cause being iron, but that of all the possible alternatives in which 2, 3 and so on up to 58 inclusive, of the coincident cases, are produced by a single unknown cause, and the remainder in each alternative by accident?

not a species of inference, as calculation, geometrical reasoning, analogy and subsumption, are species of inference It has not, that is to say, for its differentia any peculiar nature in the universal which carries the conclusion It is consequently, like Comparison or Recognition, like Observation or Experiment, a transient and external characteristic of inference An Inductive proof, when completed, may be a geometrical construction or an arithmetical calculation, an articulate subsumption or a morphological analogy Its Inductive character belongs exclusively to the process of discovery, and depends on the relation between the elements of the content and the qualification of reality from which the process of cognition starts Inferential connection is one, and is necessary and invariable, but the points at which a single and coherent system may be in contact with the real world as known to an individual cognitive subject are infinitely various From these points, whatever they may be, the cognitive subject has to build up the single and coherent system, which he then refers to reality When these points are isolated perceptions, occurrences or qualities, then the task of building up the system which they necessitate is called Induction

Inductive proof rests, like all Inference, on systematic and necessary connection of content How many observations, what experiments, how many and how favourable conjunctions of phenomena, may be needed to disclose the connection to us, is, as Aristotle implied in the Posterior Analytics,[1] theoretically indifferent The observations do not give us the connection, but we judge the connection on the basis of the system demanded by the observations, and this systematic or reasoned judgment is the essence of the proof

Is a principle then proved by the number and variety of its verified consequences? It gains nothing from any repetition of identical consequences once established to be fact, but variety of consequences may be said to prove it by displaying its nature as actual and modelling it into concrete identity with themselves I incline to think that the truth upon this

[1] Anal post, p 90, a 24, cp p 87, b 39, and see the author's Knowledge and Reality, p 285.

point is best stated through the paradox proposed above[1] Every fact, every sense-perception, every datum of testimony, absolutely and irrevocably proves *something* and necessitates the assumption of *some* agent or principle Repetition of the same datum, *qua* the same (i e assuming that it was completely and correctly observed at first, which is *never true*), can add nothing to what it proves But every further datum which can be connected with the first goes to develop the content of that agent or principle which both the data prove If therefore we speak of the mere proof that *something or other beyond the datum* must be assumed, one datum is as good as a host to prove this, and the proof of it is absolute at first, and to allege variety and range of data as contributory to *this* proof is to fall once more into the fallacy of generalisation from number of instances What is proved once does not need to be proved again Every datum proves irrefragably the reality of the system to which it belongs, *whatever that may be*

But this representation of the matter, though it leads up to the truth, is in itself a paradox without real import A proof which proves the reality merely of something or other is a proof of nothing at all But if we speak of the proof of a determinate agent or principle or real system, then both range and variety of data are essential to the proof, and the proof of the whole is not absolute at once, and therefore the proof of any part, *as a part in that whole*, is not absolute at once For the proof depends upon the intelligibility with which the hypothesis—to use the terms explained above—is adjusted to or included in the postulate, that is to say, with which the alleged real system is identified with the real something demanded by all the data taken together And from the nature of knowledge as a system the necessity of this synthetic connection can only be evident in an extended range of applications, and hence it is—not from number of consequences, but from the varied determinations which are indispensable to define any universal in its inter-connected differences, that range and variety of data are contributory to the proof of a hypothesis[2] Thus we

[1] p 161

[2] I am omitting, to avoid confusion, the consideration of repeated observation as eliminating accidental errors, which depends on the

may say, if we like, that variety and range of data contribute nothing to the proof of a hypothesis, but only aid in its definition [a] But we must then bear in mind that the proof to which range does not contribute, and which each isolated datum effects absolutely and ultimately, is a proof of something in general, but of nothing in particular

Induction, then, is the reference to reality of a system on the ground of particular differences within it by which reality is taken as qualified, and may involve, in the constitution of the system for knowledge and in its identification with those differences, any process known to Logic It is essentially an advance from the Individual or concrete Generic judgment to the pure hypothetical, or to its higher form, the Disjunctive judgment When we are able to start from a reality qualified to us by pure hypothetical or by disjunctive judgments, then we can go at once from the differences as in the universal to the relations of other differences, and we can refer these differences to reality on the basis of the universal itself which is accepted as real We do not in this case employ species of inference unknown to Induction, but the process in which we employ them has not the peculiar relation to given Reality, e g the gradual emergence of negative determination, which constitutes Induction

principle of chances illustrated above Accidental errors are errors arising from a variety of unknown causes Repeated observation distinguishes series such as are likely to be due to a single cause, from series likely to be due to unknown causes

[a] It is a good way of stating how far a hypothesis is proved to ask ourselves what we should have to disbelieve if we disbelieved it The more it is efficient as an interpretation of the data, eliciting and depending on their individualities, the more difficult it is to disbelieve it without disbelieving the data, or at least without disbelieving a certain reading and colouring of them which *ex hypothesi* is included in their verification The proof that 'nothing else can explain them so well' is really dependent on this reading and colouring of them expanding into the detail of the hypothesis, not on the detached destruction of competing hypotheses And then of course the wider the range of the data, the more difficult it is in turn to disbelieve *them*, without disbelieving the whole of the rest of our experience There can be no doubt I think in principle that true conclusions—data as deduced—support their premises

ON DEFECTIVE FORMULATION OF THE INDUCTIVE PRINCIPLE

Inference and Repetition

111 A point which I have touched upon above, relating to the basis of Induction has recently been brought into prominence by the attitude of M Bergson with the imitation and repetition theorists whom he appears to follow [a] to the creative and constructive activity of the intellect I cite a typical passage (*Évolution Créatrice* p 218) L'intelligence a pour fonction essentiel de lier le meme au meme et il n'y a entierement adaptable aux cadres de l'intelligence que les faits qui se repetent'

'Same cause, same effect'

a Such a statement is in the sharpest possible conflict with the view of intellectual activity which to many of us seems natural and obvious But when we refer to the most accredited expositions of the logical theory of Induction which attempts to deal with the characteristic working of the scientific intelligence in the advancement of natural knowledge we find them dominated by ideas which appear to justify M Bergson's position What I wish to attempt is a brief reconsideration of the exact meaning and function of these ideas in Inductive Logic

The basis of Induction is usually stated in some such formula as 'Same cause same effect' It is unnecessary for our present purpose to raise the questions connected with the converse formula 'Same effect same cause' It is enough to understand the simplest truism of Identity, that a thing does what it is its nature to do under given conditions [b] and cannot do otherwise except by some change in the conditions, from which it follows, that if in an alleged causal nexus the alleged effect is sometimes absent while the alleged cause is present, *ceteris paribus* it is impossible that the alleged cause should be the real cause of the effect in question The principle is sound beyond any doubt as far as it goes It is, in fact nothing more than can be read off from the law of non-contradiction, as formulated for example by Plato The same thing cannot behave differently to the same thing in the same relation If

[a] Cp Tarde Les Lois de l Imitation, p 15 Il n y a de science que des quantites et des accroissements, ou en termes plus generaux des similitudes et des repetitions phenomenales

[b] Cf Mr Joseph s Introduction to Logic chap xix

it seems to do so (Plato's condition 'at the same time' is superfluous), you can infer that there is a difference in the supposed agent The same, so far as *it* is concerned (i e if no condition is altered) produces the same, what produces something different, out of itself, is not the same If this much is not to be assumed, we cannot treat anything as having an assignable nature Truth ceases to have a meaning. Anything might behave anyhow

Now it is from this law or truism that, according to current logical theory, the fundamental Inductive test of causal connection is derived. The Inductive process is thus regarded as one of elimination [a] You have before you, it is assumed, one or more suggested connections of cause and effect, and you labour to eliminate from among them all alleged causes that are present in the absence of effects with which they claim to be connected Such elimination leaves, it is presumed, a surviving statement which approaches more and more closely to a true, i e an invariable, causal connection. The principle is simply that which M Bergson refers to What is the same, does the same, if the same product is not there, the same agent is not there The same produces the same And yet, if this were all, we should have a difficulty in denying M Bergson's doctrine which I began by stating It *would* then seem to be the case that the essential work of the intellect lies in binding the same to the same, and that the true type of the logical universal—the essence of cognition—is, as M Bergson says, the relation of an abstract statement to examples which repeat its tenor wholly without variation That water boils at sea level at 212° Fahr would be such a generality, and according to the number of instances in which people boiling their kettles at or near sea level [b] found

[a] The rules of elimination which depend on the further principle, 'Same effect, same cause' (i e on the exclusion of plurality of causes), rest merely on a more precise consideration of the ideal of identity, which it is not necessary to take account of in order to understand the point at issue in this discussion See Joseph, chap xvii

[b] I am satisfied to take a case in which strictly accurate repetition is all but impossible, because it illustrates the real fact, which is that the interest of the generality lies in the differences which it binds together. A *strict* repetition could have no interest at all

the water to be about 212°, would be its rank and power as a piece of knowledge

This a tautology.

β But why *should* we deny M Bergson's doctrine? Perhaps it may be the truth As a *primâ facie* answer to this suggestion, we need only refer to such a criticism of tautology as we find, for example, in Mr Bradley's *Principles of Logic* [a]

M Bergson's doctrine is logically bound to deny not only the advance from one truth or connection of fact to another, but the possibility of apprehending or of uttering any significant truth at all It may appear that this criticism is exaggerated, because the doctrine explicitly treats (so far as I am aware) as outside the principle of the intelligence, only the difference between the corresponding terms of one nexus and those of another nexus, and not the difference between the terms themselves—alleged cause and alleged effect—which are constituents of a single nexus But there is no escape by this road. If tautology is the principle of the intelligence, the connection of any two distinct terms, say, as cause and effect, stands on the same ground as the connection between two different connections With tautological identity as the principle of intelligence, all systematic coherence, between term and term, equally as between judgment and judgment, inevitably vanishes

But in fact there is (i) some misinterpretation involved in setting up the principle 'Same produces same' as the dominant principle of scientific Induction and as governing the nature of the generalisation which is the aim of that process, although

(ii) I admit and maintain that the current logical statement of the theory of Induction lays itself open to this misinterpretation The view which I have taken above [b] indicates, I believe, the right theoretical direction, with its consequence as regards the place of elimination of erroneous hypotheses in drawing Inductive conclusions But I hope that the point will be made clearer by a few additional remarks

Its meaning

(1) When you postulate as the basis of Inductive Inference, the principle 'Same cause, same effect', you do not mean that

[a] e g p 29 [b] p 164

the effect is the same as the cause * They must be different, if the relation of cause and effect is to be worth establishing You do mean (α) that assuming the truth of an alleged causal nexus A—B, it only applies in cases which are absolute repetitions of it, i e where you have exactly the same A as before without any variation, and (β) that in examining the truth of an alleged causal nexus A—B, your rule must be that if you find a case in which, *ceteris paribus*, B is different (o or B_1) and A is unvaried, your alleged causal nexus A—B is disproved For if it were true, the same cause would be producing, *ceteris paribus*, two different effects, which is impossible (If A is plural, or rather, various, and B singular, this is not literally a case excluded by the formula 'Same cause, &c', which is strictly taken silent about it, i e about plurality of causes, and no negative inference follows, unless we are making what usually counts as an additional postulate 'Same effect, same cause')

What you mean by 'Same produces same', then, is an assertion that the cause, in a nexus guaranteed by this principle, is unvarying compared with itself, and the effect unvarying compared with itself You imply no comparison between cause and effect

And your principle makes no suggestion towards the estimation of any possible cause and effect allied to or developed out of those forming the nexus whose truth we assume to be accepted According to a proper interpretation of the word 'same' some such expansion would be permissible, passing from a—b to a—β and from a—β to A—B. But what makes it impossible is the demand for a methodic rule. Plainly there cannot be a general rule that will tell how much variation in your cause and effect, each from each, will be justified under the principle 'Same cause, same effect' And therefore, if you want a rule, you must take one which justifies no variation at all, and makes your 'generalisation' cover nothing but sheer repetitions, and degrades your procedure

* In a remote sense some such meaning might be assigned to the true Inductive principle which I desire to see established, and it may be that some hint of this possibility prevents the formula under discussion from seeming as naked as it really is Its strict meaning can only be that stated in the text

in connecting the same with the same into one which admits of no novelty or true inference

But the two types of connection thus disregarded, that of cause and effect, and that of any generalisation and its more advanced but kindred form, really contain the very life and mainspring of Inductive thought How the suggestion of the effect B issues from the fact of the cause A ; or how the more complex and advanced *a* (*def*)—*b* (*xyz*) came to be substituted for the cruder A—B , this is where the real work of the scientific intelligence lies This is the work of invention or discovery, of which the imitation and repetition theorists, whom M Bergson appears to me to follow, have never succeeded in giving any serious account [a] It is the process by which isolated observations are built up into a science, through an assignment of conditions which is always becoming more systematically complete on the one hand, and more relevantly precise on the other Examples of such an inventive pursuit of a universal relation would be the rise of the science of acoustics out of the old observation that the pitch of musical notes has a ratio comparable with that of the lengths of the stretched strings which produce them , or the development and limitation of the conception of equi-potentiality as applied to organic growth in recent embryology Here we have the plain fact, that it is the essential character of intelligence to bind different to different in binding same to same , and that it is for the former character that the latter is valuable, and, indeed, it is through the former only that the latter can exist. But the sameness here in question is not the sameness of M Bergson's doctrine or of the formal Inductive test We can see this from the nature of its aim The universality or generality, which is the goal of such a process, is not the relation of the terms of an abstract judgment, term for term, each to each, to the terms of repeated cases which fall under it It is the relation of the different terms of a judgment to each other, or of an organised system of conditions, representing a certain range of experience (e g our experience of musical sound or of embryonic growth), to the several connected factors or conditions, whether constant or varying, which it embodies and explains Its universality

[a] Cf especially Bergson, Évolution, p 177

is not measured by millions of repeated instances, but by depth and complexity of insight into a sub-system of the world

(11) The logical theory of Induction gives but scanty attention to this work of the universal in suggesting and pursuing new connections, because, for good logical reasons, that is, because of the individuality of truth, it cannot be reduced, like the eliminative test, to something like a formal rule Nevertheless, this work is the true spirit and mainspring of the inductive advance of knowledge , and to disregard it, while insisting on an eliminative test, is an error analogous to demanding a general criterion of truth But truth has no criterion except the fuller truth And the real interest of logical theory in the advance of knowledge is to note how, by the analysis and purification of its conditions, a perception passes into an organised system of understanding

True Spirit of Induction

The existing connections or universals with which the mind is stored, act as clues among the new experiences which confront us, selecting those that are kindred or complementary, and inventing new systematic ideas after the manner of what have been called proportional systems, and by means of relative suggestion [a] That is to say, that an existing connection of thought, when confronted with new matter, is able to reproduce itself in a new form which is at once appropriate to the new matter, and continuous with the connection as previously thought This is not a question of reproducing objects of thought which have previously been connected in the mind It is a question of continuing some elements of such a connection into new forms of nexus, because the connection between the new objects has a real kinship with the connection between the old, although differentiated by the nature of the new objects themselves, and made, as a connection, something new, and not a repetition of what it was before, like the

[a] Cp Stout, Anal Psych , II, p 80 I note that Professor Stout here observes that relative suggestion 'would not of itself enable (the discoverer) to fix in exact detail the special variations' In the case he is dealing with, calculation was necessary In our instance from embryology, observation is necessary But it seems to me that the question what calculation? what observation? is answered by the governing idea in both cases, and the relevant conclusions are selected by it, and it is it that they develop

continuation of a varying curve from the datum of a given fragment of it.[a] Such a continuation is plainly not a repetition, and I think that in view of the current theory of Inductive generalisation, the notion of repetition as a condition of knowledge is not meant to apply to such an inventive construction as that of which I am speaking.

Let us look at an example. In recent embryological discussions,[b] covering the old ground of preformation and epigenesis, we read of experiments which *primâ facie* suggest two precisely opposite causal connections.

Half an ovum, we are told, in certain cases will produce only half an embryo, but in other cases the half ovum may develop into a perfect embryo. The former fact suggests a complete preformation of the organism, each part of it in a fixed part of the ovum, the latter suggests that the ovum has a structure of which 'every part may become anything'. It is of great logical interest to look at the course which these two alleged types of connection have imposed upon Inductive research. Sheer prelocalised preformation is an idea, it would seem, that the experiments undertaken to confirm it immediately destroy. And if a universal nexus had no power of developing into novelty, this check would have been checkmate, and the idea would have been dead. But a universal can take on new shapes as demanded by new matter, and though, as it seems, the 'mosaic theory' (of the independent preformation of parts) must be abandoned in its rigid shape, yet the most various experiments on the tissues of organisms in later stages have shown that some of these are necessary to the development of some organs, and that therefore something essential to special development (perhaps 'organ-forming substances') is preformed, though not necessarily pre-localised. The logical interest is, that the idea of preformation, defeated in its primary and rigid shape, has been able to act as a clue to new experiments in a different region, such as to confirm it when restated in a more subtle and flexible form.

So with the idea that every part of the ovum has the capacity

[a] Cp. Bradley's Principles of Logic, p. 281 ff.

[b] My example is drawn from Driesch's Gifford Lectures and Jenkinson's Experimental Embryology.

of becoming a whole It is easily seen that this capacity is limited, and is sooner or later lost, but the idea of the kind of causation at work modifies itself according to the limitations which are discovered, and seems to suggest new lines of research which promise to account both for the capacity, and for its limitation and arrest And the logical interest is, that by means of this suggestion, that of 'organ-forming substances' and their distribution, it appears as if the two universals in question, 'preformation' and 'epigenesis', might coalesce in an idea different from either, but satisfying the requirements of both

Of course I am offering no opinion upon the value of these investigations I only adduce them as striking examples of the ordinary course of a universal in its Inductive development. What works throughout is a continuity through differences, and its value is in the differences it connects This is throughout the essence of creation and invention, which permeates the whole of life, and so everyday a process as the use of language is a striking example of it No one ever used the same word twice in precisely the same sense, in 'finding the right word' there is always a creative effort

Now the general rules of Inductive elimination, based on 'Same produces same', are simply the minimum negative criterion of truth, and can do by themselves no Inductive work at all. To rely on them alone is to reduce Induction to trial and error [a]

γ Thus, I do not think it is true to say that 'Same cause, same effect,' is the basal principle of Induction, and if this is so, there ceases to be any ground for maintaining that it is the essential function of the intelligence to connect the same with the same The true principle I should rather state in some The True Principle

[a] It is a subtlety that in fact the underlying positive nature of negation often asserts itself, and the 'just-not *a* gives just-not *b*' affords a positive extension of the nature of *a* and *b* respectively, which may be theoretically valuable, see above, p 134 Thus in Driesch's tubularia experiment, it is now alleged, the capacities of different cells are just not equal, as they just belong to different elements of the body And this suggests that differentiation is present in a certain degree—a positive correction and extension of Driesch's conclusion Driesch, Gifford Lectures, I p 128, and Jenkinson, Experimental Embryology, p 292 n

such form as that every universal nexus tends to continue itself inventively in new matter It is true that to guide this process we can have no general criterion, because, as we have said already, the only criterion of truth is the fuller truth—the science at a more developed stage And, therefore, there can be no rules for it, and it tends to drop out of logical theory But none the less, it is this process to which the whole positive construction or invention of our inductive knowledge is due, while the principle Same produces same' can only eliminate what, having been suggested is found on further trial not to produce the minimum characteristic of a real nexus We have seen, indeed,[a] how a good experiment may sometimes reveal a correlation of serial variations, which is in itself a positive expansion of the suggested nexus But this is only incidental to the strict process of Inductive Elimination

The neglect of the positive continuity between differences as the inventive factor in Induction appears to me to show itself in the doctrine that Inductive progress consists strictly in mere elimination of the unfit,[b] in reducing the number of nexuses that can claim the position of the true invariable law This doctrine seems to me to subordinate the more important process and element of proof, because it can have no abstract criterion, to the less important, which is nothing but an abstract criterion But if the aim of logic is not to give rules of practice, but to understand the nature of knowledge, this ground of subordination is invalid, and it remains true that the mainspring of inductive advance in natural knowledge, as of knowledge in general, is the power of ideas to make experience coherent, and that therefore the demand of continuity between term and term or between nexus and nexus—of a positive explanatory character attaching to the nexus—is a fundamental requirement of inductive science, which is in fact merely an elementary stage of knowledge, and shares all its positive characters[c]

Developing the Nexus

4 The modification outlined above in the idea of Inductive

[a] P 175 and p 181 footnote

[b] See Bradley's criticism of one form of Disjunctive reasoning Principles of Logic p 515 and above, p 156

[c] But see Mr Joseph's example from the discontinuity between

universality or generalisation follows from this conception. The value of an Inductive conclusion, as of any piece of knowledge, lies in the amount of reality which it enables us to grasp, and this is very slightly tested by the number of cases in which the nexus is repeated in fact And if the idea of identical repetition could be realised (which it cannot, for every so-called repetition is differenced by a new context) the frequency of recurrence would have no connection with universality at all

What is here advocated as the true view of Inductive advance has been suggested by Green's treatment of logical theory,[a] and has in some degree been embodied in the present work, at the point[b] where it dealt with true Inductive generalisation as consisting in the range of differing data and conditions welded into a system by any investigation, as contrasted with the number of recurrent cases which may fall under a single abstract statement and there is a definite logical necessity for making the former type of universal the goal to which the latter is a halfway house or less For, as Plato[c] pointed out and as Mr Bradley[d] has recently emphasised, statements of fact (implicit statements of nexus), but slightly hedged with conditions, must always be at the mercy of unexpressed factors for their truth or falsehood They tumble backwards and forwards between 'is' and 'is not', Plato's famous expression, which Mr Bradley's argument in the passage just referred to strictly and precisely justifies. The remedy, as Mr Bradley says, is to get the conditions into the subject, and this means either an explicit or an implicit reference to a complete system[e]

physical cause and psychical effect, p 453 I believe, however, that more could be done than is usually done to remove this discontinuity, though of course there can be no resolution of the ultimate difference of kind But I suggest that a consideration of the way in which elements of brain excitement reinforce and modify each other, would have made impossible the defects, say, of Kant's ethical doctrine An idea unsupported by outlets of activity could not be a powerful idea, and therefore not one ethically fitted to be paramount

[a] Works, II, pp 288 90 [b] Above pp 164 and 169

[c] Rep, p 479, c [d] Mind No 72 p 499

[e] See above I p 245 for judgments which imply though they do not expressly include a relevant scientific system

The normal and natural working of intelligence, then, is creative and constructive, tending towards the concrete and to continuity within differences. The universality which is its mainspring is in itself a nisus to the concrete. This operative continuity is not represented by the linkage of the same to the same. Its law—the law of intelligence—is not the law of Identity, unless the law of Identity is construed in a way that takes it deep into the postulates of organic systematisation.[a] And phenomena which should merely repeat themselves would present an absolute barrier to the central nexus of the intellect. Mere repetition, in fact, if it were possible, would be incompatible with understanding.

I am, therefore, still confident that the restriction of Inductive proof to the disqualification of competing hypotheses is a fundamental error of principle.[b] What really works in the proof is the same as what works in the discovery, the power, that is, of an idea to harmonise experience. No doubt the hypothesis which best satisfies this condition would also be the least likely to fall a victim to the rule of elimination. But yet, theoretically speaking, if accepted for this latter reason, it is accepted, so it seems to me, for the wrong reason, and its value as knowledge is not genuinely apprehended. But this point is only incidental to my discussion, and I will not pursue it here.

[a] See below, II, pp. 210–11.

[b] I am afraid that here I find myself in opposition to Mr Joseph, whose Logic I greatly admire. And I must add that I cannot at all follow Mr Lindsay's comment on the above remarks (The Philosophy of Bergson, p. 230), 'When we say, this is like A, and its effect will be like B, we are applying a result of that insight [viz. the insight that A causes B] and we are concerned with A' only in so far as it resembles A. This implies that A' repeats A for its difference with A must for our purpose be ignored.' Mr Lindsay has no doubt considered the relation of this statement to the conception of inference as a development of universals into their differences. But I really cannot understand it. Let A be a given depth of corolla-tube, seen to condition B, the length of an insect's proboscis. In applying this insight to A', in which the depth is different, how can we ignore the difference? Or would Mr Lindsay contend that no universals can be developed into definite new applications except in quantitative laws? But this would surely be an extraordinary thesis. *All* non-mathematical inference, except perhaps pure subsumption, would be abandoned. For nature of universals see especially Bradley's Logic, 281 ff. For further instances Stout, Anal. Psych. I, c. 10, or above, 180, cp. also p. 192.

CHAPTER VI

Concrete Systematic Inference

A pure hypothetical judgment, the outcome of scientific Induction or the embodiment of abstract relations in combination, expresses a synthetic connection based upon an underlying real system Analogical inference, from which scientific Induction was a divergence, depended rather upon an estimate, usually inadequate, of such real systems in their concrete import Now if, as a result of a highly exhaustive Scientific Induction taken together with an Analogical reasoning, we are able to recombine the abstract relations which the former has disclosed one by one, into a single totality which has an obvious significance, then this totality or system is the real determinate ground of each separate relational judgment that enters into our conception of it, and belongs, at the same time, to the concrete or categorical type of knowledge For the ground which warrants a hypothetical judgment is in the last resort always a real system, and moreover the content of every judgment is understood [1] to have such Reality as it is capable of

1 Therefore, in dealing with totalities which are thus thoroughly concrete and thoroughly rational, we are able to advance from the figure of analogy 'A and C are B, therefore A is probably C', to the first figure of the Aristotelian syllogism, 'A is B, B is C, therefore A is C' Philosophical Subsumption

In spite of all that has been written about and against the syllogism, I can find no more simple and natural expression than this for the reasoned judgment which embodies a *real necessity* Thus applied, the syllogism is subsumptive in so far as it appeals to unity of relations within a concrete subject, but has abandoned the differentia of subsumption proper, in so far as the definite form taken by the result of the appeal depends on intelligible coherence and not on mysterious con-

[1] See Bk I, chap II, on Categorical and Hypothetical Judgment

junction It is essential to such arguments that the teleological or quasi-teleological unity of the subject, which in analogy was conjectural and obscure, should be absolute and explicit It is only this absoluteness that can warrant the position of the middle term as subject in one premise, i e as a qualification which prescribes the precise content affirmed of it in the predication It is only this explicitness that can justify by a specific[1] necessity the determinate relations which the unity of the subject imposes on the two extremes The conditions thus demanded can only be fulfilled in subjects the nature of which is known as a definitely organised system We saw indeed, in the earlier discussions of Book I,[2] that such a system cannot avoid presenting quantitative relations between its parts, in as far as its pervading unity contains within itself differences of a common quality But in a true concrete individuality such quantitative relations are secondary, resulting from the nature of the system but not exhausting it, and therefore the system, although definitely intelligible, cannot be 'constructed' by geometrical or numerical combination Such combinations may however enter into it in various degrees Judgments which deal with these concrete individualities are at once individual and universal, and have been analysed in Book I as a combination of these characters [3]

Fluid though the distinctions between types of inference necessarily are, it will be convenient to distinguish the important class of inferences now before us by an unmistakeable differentia, at the risk of unduly limiting their province This differentia is the ascription of *real teleology* to the content analysed And by real teleology I mean the embodiment or operation of a conscious purpose entertained by a human intelligence All other teleological inferences, such as those depending upon the *de facto* teleology (quasi-teleology) of the organic world are most conveniently relegated to the category of analogy

The lowest case of real teleology is closely akin to that

[1] See Bk II, chap i, on specific necessity of Judgments

[2] See Bk I, chap iii

[3] See on the Corporate Judgment and the Individual Generic Judgment, above, Bk I, chap v

which was the highest case of analogy A tool, instrument, or machine, of which we know the use intended by the maker, furnishes this lowest case of real teleology, while any object of the same class the use of which we could only conjecture, furnished the highest case of quasi-teleology or analogy It was in part from the example of an instrument contrived by human intelligence that Plato introduced the conception of function or final cause into philosophy,[1] and the ultimate meaning of 'organism' is a system of tools or instruments. The term 'mechanical' in its modern philosophical acceptation abstracts from one-half of the import of 'machine', for though we are accustomed to think of *mechanical* determination as a resultant of any *de facto* combination of forces, yet we are not accustomed to think of a *machine* except as a combination of forces for a purpose consciously entertained

At first sight, then, we have in the tool, instrument or machine with known purpose, an adequate example of the type of knowledge before us 'A screw that is meant to turn one way only must have its head cut so as to give the screw-driver no purchase when turning the other way, a coffin-screw is a screw meant to turn one way only, therefore a coffin-screw is one which has its head cut,' &c, &c Or again 'A locomotive engine meant to drag a weight a at a velocity b must have boiler-space λ and cylinder-stroke between the limits z and z_1, a locomotive which is to work in the Newcastle coal traffic must drag a weight a, &c, &c, therefore a locomotive which is to work this traffic must be constructed as above determined'

It will strike the reader however on looking at such examples as these that the premises are very closely allied to hypothetical judgments, and are much more 'constructive' than 'subsumptive' It is true that in the analysis of a machine the inference does rest on the *system* of the mechanical combination,

[1] Republic, end of Bk I *δρεπάνῳ τῷ ἐπὶ τοῦτο ἐργασθέντι* The examples alleged by Plato in this important passage are of very different values, and are well worth careful attention The well-known description of the function of a thing as *ὃ ἂν ἢ μόνῳ ἐκείνῳ ποιῇ τις ἢ ἄριστα* leans to *de facto* teleology, and would not of course protect an object from a function alien to its nature but relative to human purpose *Such* a function could justify no analogy

and that this system with all the details dependent on it can, *in a machine that works well*, be deduced from the intellectual purpose which the constructor proposed to himself to realise in that mechanical combination. So far, as reading the significance of the parts in the coherent whole without which they would lose it, the inference is subsumptive. On the other hand, all tools or machines are liable to initial or acquired mal-adaptation. Their *de facto* function or actual result may diverge from their intended function. And when this comes to pass, their existence as mechanical combinations is not thereby terminated. A clock that has a hopelessly variable rate may not, philosophically speaking, be rightly called a clock (being absolutely useless to indicate time), but it remains a real mechanical combination in which co-operating parts produce a necessary result. In other words, though a machine embodies a purpose yet it only embodies it in a mechanical form, dependent, that is, on the right adjustment of a mechanical combination, and therefore on the continuance of that right adjustment. Therefore in every such inference there might be substituted for the statement of purpose a statement of the mechanical system in which the purpose is supposed to be realised, and as the purpose is only present in the actual system of adjustment, and not as an intellectual idea, such an analysis would be in one sense adequate to the nature of the object analysed. *Such* inference might fairly be treated as employing merely hypothetical judgment and constructive combination, taking no account of any significant unity in the content of inference, or of any special relation between it and the real world. The system would, by such a transformation, have forfeited its individuality and have become a mere necessary sequence of relations upon relations in the abstract world of force and mass, instead of an actual whole in the unique structure which we call reality. It must be noted however that the limitation or abstraction which is needed to make such an account intelligible, has by the change supposed become merely arbitrary. *Qua* mechanical result, every cinder that dropped from the fire-box and every cloud of vapour blown from the funnel would have as good a right to be described and deduced

from the mechanical combination which makes up the locomotive, as would the capacity of the engine in the way of traction

'But the purpose may be hypothetically inserted into the inference, as indeed was done above' We may state a purpose or any other content hypothetically, if we indicate that by intentional abstraction we are doing so I only say that, apart from any mark of forcible abstraction from reality, a judgment or inference that deals with a system having unity in a purpose presupposes the reality of that system because its content is adequate to reality, while a judgment which merely draws the necessary consequences of a determinate combination of forces, without reference to any purpose to which that combination is directed, is essentially hypothetical, for the particular combination has no pre-eminent individuality or *raison d'être*; and essentially imperfect, because in the absence of a *raison d'être* there is nothing to guide the selection of aspects or of consequences In *this sense* the hypothetical, the arbitrary, and the merely mechanical coincide

In the distinction between a machine which serves a purpose, and a machine which does not, we have in a nut-shell the question of categories Both are actual mechanical combinations producing results, and neither has in it one whit more life or intellect than the other But it is perfectly clear that our understanding of the useful one is incomplete if by preserving our ignorance of its purpose we remain on the same level of apprehension with reference to it which is the highest we can possibly attain with reference to the other And it is absurd to say that the category so implied is an accidental aspect and does not represent a fact It is true however that this category of purpose does not exist within a mechanical system in its proper or intellectual form, and that therefore the system can be regarded by a natural abstraction as on a level with a purposeless combination, and may by internal or external changes at any moment become such This hypothetical aspect of a combination of forces, in virtue of which it produces its resultant according to fixed necessities and in complete indifference to any purpose, is the purely 'mechanical' relation of a machine, and if exclusively pressed

home destroys as we saw all possibility of regarding it as an individual thing having its unity in a function

Beginning with the mechanical contrivances of which we have been speaking, there extend upwards in a series which forms the content of philosophy the phases and embodiments of man's intelligence and conscious will All of these the individual will with its complement in the moral order of society, the product of fine art, and the religious or philosophical system, are totalities which combine an explicit intellectual unity with determinate interdependence of parts The statement of the general character of these embodiments of mind may serve as an example of the argument we are considering 'The mind is a unity of reciprocally determinate but not reciprocally exclusive parts A feeling is an element in the mind, therefore a feeling enters into a unity of reciprocally determinate but not reciprocally exclusive parts' Or again 'The British Constitution is in its main features determined by the thoroughgoing application of ordinary law, the position of the prime minister is a function of the British Constitution, therefore the position of the prime minister is in its main features determined by the thoroughgoing application of the ordinary law' Or, finally 'The general will is expressed in the moral order of society, the individual will finds its freedom in the general will, therefore the individual will finds its freedom in the moral order of society'

When we consider the logical nature of such arguments as these, we notice two obvious characteristics of the content, and one, resulting from them, of the form

Logical Content Real System

1 In respect of their logical content they are at once categorical and hypothetical *a* The systems which form the content of such reasoned judgments as these are naturally taken as real systems in virtue of their individuality It is of course not impossible to construct a political or religious system on paper the consequences of which are laid down in hypothetical judgments and inferences from them, which in form might be identical with such judgments as are here employed The content of such judgments has an indeterminate place in reality so far as it has a meaning or objective reference, and depends on determinate reality so far as it

proceeds to affirm actual consequences But the content of a judgment which deals with an individual system is taken as real in our world unless the contrary is indicated, and even in hypothetical judgments that depend on the nature of the human mind, the real ground which would have to be made explicit in order ultimately to justify the consequences drawn is the intelligible and concrete system of that mind itself *Primâ facie*, therefore, we are dealing in these arguments with categorical judgments about reality, which explicitly postulate the real grounds that in the hypothetical judgment were latent

Apodeictic Sequence

β The nexus of the inferences in question is not, as in Analogical Reasoning and in Enumerative Induction, devoid of strict apodeictic sequence The systems of which we have been speaking, although they need not be capable of mechanical, numerical or geometrical construction,—which however, as in the case of a machine, may play their part in the analysis of the concrete whole,—are nevertheless invested with hypothetical or apodeictic necessity in two forms, in the relation of their parts one to another within the systems themselves as wholes, and in their own *ultimate* relation as parts to the unique system of reality as a whole

Of these the former is for our logical purpose the more important Within such a whole as the normal order of civilised society, regarded as the expression of the general will, it is obvious that there are parts united by necessary relations dependent on the nature of that whole and capable of being expressed in hypothetical judgments if we abstract from the explicit assumption of the whole itself We may say, for example, 'If a right, then a duty' The justification of this statement would be given by the affirmation, as a real ground, of the moral purpose involved in the moral order, which purpose exhibits itself as right or duty according to the attitude which the individual will may assume towards it Such an inference as this does not cease to be necessary when its ground, in this case the moral purpose and moral order, is affirmed to be real The basis of the synthetic transition is here as everywhere the nature of an identity or universal, and that the universal is affirmed to be fact makes no dif-

ference to its apodeictic force What in particular that apodeictic force may be, how it should come to pass that one thing can necessitate another, depends, as I have said before, on the ultimate fact of the nature of knowledge What we have more particularly to observe at this point is the coarseness of the illusion that systematic necessity can only exist in spatial and numerical perception Given the relation of man's intelligent will to an actual moral order, the relation of right and duty is as plain a consequence as, given the nature of space, the equality of vertical angles And apart from a given reality, there is in either case nothing, from which nothing can follow.

Hence we arrive at the second aspect in which individual systems, though real, are nevertheless hypothetical They are each and all of them, for us, hypothetical upon the *whole* given reality within which they exist When we speak of a thing as real, we imply that it is complete and self-existent, for if it is not, its reality includes a condition beyond the content which we have included in the thing, and *it* therefore, *as we have formulated it*, not including the conditions essential to its own reality, is falsely asserted to be real

Here we have the aspect of relativity which prevails throughout our knowledge, which is increasingly overcome by the work of intelligence in as far as it connects the actual and intellectual world into an organised whole, but is never thoroughly done away

Logical Form. Syllogism in fig 1

11 As regards the form of these inferences, it follows from what has been said that the only value of the syllogistic arrangement is to exhibit the structure of the reasoned judgment, which itself contains or displays the articulated universal. No question arises as to which premise we know first, and so which supports the other The prior or previous phase of the inference is not the proof of detached premises, but the entire thought in a less precisely articulated form If we are urged to say whether we know the major, the minor or the conclusion first, the only true answer is that, in their full import, we know them all simultaneously As detached fragments of experience we may know any one of them first And as each element of the universal when rightly under-

stood involves the others in their full determinateness, there can be no real difference in kind of import between major and minor premises, and no reason for preferring one order of the terms to another The order will in fact be subjective, depending upon the qualification of reality which we take as starting-point, whether in time—if our inference has the accidental aspect of a progress in time—or because of its individual nature The real purpose is the dominant essence of the universal, but the real purpose may be taken as conveyed by the general idea of the system in question as a whole, represented by its *name*, or as involved in the analytic scheme of its parts, or as concentrated into some special application by which some one part does the work of the whole And thus any one of these elements of the universal may stand as the middle term in reasoning, i.e. as the ground or universal *par excellence*. Hence there is no use in considering the syllogistic rules at the point we have now reached. They belong to calculative and in some degree to analogical argument, but the postulate on which they rest, of the absence of reciprocal determination between the elements of inference, does not hold good of a coherent system when thoroughly known We have thus arrived at a goal analogous to that attained by the theories of Quantification and of Equation in judgment, at a perfect reciprocal identity between the elements of the reasoned judgment, so that any one of the terms may occupy any place in the argument. But we have attained it, as we hope, without sacrificing difference to identity, and thereby destroying the identity itself. The equational form, though it symbolises correctly certain results of the reasoned judgment (the conjunctions which this judgment in fact involves), yet crushes into shapelessness its true living texture, and, as a simple sign of the deformation, forbids all growth and reconstruction within the inference itself, which reconstruction nevertheless, as we have seen and shall further see, cannot be avoided

As a particular case of the inapplicability of the syllogistic rules to the inferences now before us, it may be mentioned that we have here nothing to do with inference from negative premises. We must take the negative form to have

done its work and obtained positive significance, in the process which we have watched of constituting such a system as that which we are now considering. We are now considering these systems as real grounds, and so with reference to what falls within them, and not with reference to what falls outside them. For to what falls outside the system itself, unless with reference to a further system including that 'outside', the system can *ex hypothesi* only be related in the way of bare negation that has no import and is no judgment In analysing the completion of a type of knowledge it is vain to raise a question which would take us back to the beginning of the course we have traversed. But as a determining agency *within* a real system, and as invested by that function with positive import, negation reappears in disjunctive reasoning

Disjunction and Disjunctive Reasoning.

2. The nature of disjunction and its imperfect forms have been discussed under the head of the disjunctive judgment, and it only remains here to recapitulate the inferential nature of this the most complete and explicit form of the universal.

Inference under a disjunction is usually represented thus: 'A is either B or C, A is not B .· A is C,' or, 'A is B .·. A is not C.' Yet such an inference has no meaning except in the case of a disjunction of ignorance or a disjunction referred to a point of time. The categorical minor premise adds nothing whatever in the way of content to the disjunctive major premise. It only has meaning as resolving a doubt or as affirming one member of an alternative to be true in a given point of time This defect could not be removed by specifying in the minor the ground on which that one member of the alternative is affirmed, for this ground cannot really fall outside the content of the disjunction and its specification can only throw a doubt on the categorical nature (in the narrower sense) of the minor premise itself 'The signal is either danger or safety, it is red and so danger . not safety.' Obviously here, if we can conclude from 'red' to 'danger' in the minor premise, this relation must fall within the knowledge which constitutes the major; and moreover, by introducing a specific ground of assertion it exposes the minor to a charge of being hypothetical.

We saw in treating of the disjunctive judgment that the

disjunction of ignorance and the disjunction referred to time are not cases fundamentally distinct from the true disjunction of knowledge, being justified, in as far as they are justified, by the same type of knowledge which forms the basis and content of disjunction proper Only, being limited by an accidental condition (the speaker's knowledge, or an arbitrary point of time), they lend themselves to an appearance of progressive inference through supplementation by a perceptive or narrative judgment, which applies them under a limiting condition without expressing that condition In the true disjunction, which expresses the organisation of a system as such, the reference to an arbitrary condition falls away, and although the judgment is capable of inferential application under specified conditions, whether of time or of other kinds, yet this application is not essential to its import, and is not demanded by its form

We are thus driven to the paradoxical conclusion that the essence of disjunctive argument is included within the disjunctive 'major premise', in other words, that this judgment is in fact not a mere premise but at once a categorical judgment and a complete systematic inference, in which the content of a real system, thoroughly understood, is developed in its reciprocal positive and negative bearings The universal, or pervading identity, is developed in it as a system of *a*'s and '*just* not *a*'s',[1] such that in virtue of every 'not *a*' the system is positively determined to a certain definite *a*, and in virtue of every *a* the system is negatively determined to a certain definite not-*a* (which is *b*). Our ideal of inferential knowledge does not go beyond an individual system of this kind, of which every part is mediated in its turn by all the other parts and assigned by them its appropriate place in the whole, whose pervading nature is present in every part and prescribes the arrangement and content of all. Such a system contains its own applications, for the material conditions under which it developes its nature are given within it The mere realisation of one alternative member as fact or as a point in time, e g. in present perception, has in relation to such disjunctive knowledge the aspect of a case brought under it by an

[1] See account of Scientific Induction, chap iv, above

unknown condition, and so implies a defect in the disjunctive knowledge itself For if there is no such defect, then perception or testimony can add nothing to the *necessary reality* embodied in the disjunction A watch is either going or not going; and I do not need observation or testimony to tell me that at any time when the mainspring is broken it is not going. 'But your disjunctive knowledge will not tell you beforehand or apart from observation whether the mainspring *is* broken' I reply, 'Oh yes it will, *up to the limits to which it extends*' It will tell me the signs of breaking, the risks of breaking, the limit of breaking-strain, and therefore, supposing my knowledge of the world were disjunctively complete, it would tell me exactly when and how often the mainspring has been or will be broken That it does not practically tell me this is not owing to the defectiveness of disjunctive knowledge but to my not possessing it Therefore, as in all the affairs of life, I have to supplement scientific knowledge from testimony and unorganised observation, i e observation of what occurs under conditions not precisely known. But this observation, *qua* unorganised, adds nothing to knowledge, though in fact every content that is distinctly observed has necessarily *some* organisation, and leaves the disjunctive judgment a little richer than before But as mere abstract position or affirmation of a case fully known before, it adds in theory no element whatever to our disjunctive knowledge of a real system.

Therefore the disjunctive judgment must be taken to correspond not to the major premise of the syllogism, but to the whole syllogism. The syllogism must tell us, for instance, that the human will, being an activity of the human intelligence, sets its purposes before it in the form of definite ideas. The disjunction would in this case perhaps tell us that the human animal asserts himself practically either through the intelligence as will or through the sensuous instincts as appetite, or again, that he asserts himself through the intelligence either practically as will or theoretically as knowledge Here we obviously have the whole content of the syllogistic 'reasoned judgment' but in a more elaborate and more thoroughly articulate form It is clear that the whole conclusion, in so far as it is a conclusion that grasps scientific

truth as the definition of a real system, falls within this disjunctive judgment. The application of it in a special historical case can be of no importance, unless the new example suggests new matter for the definition of the term involved; in which case the content of the example must be taken up into the disjunctive judgment.

I need hardly remark that it need not be an objection against a disjunction of this class that the determinations of the system do not exclude one another in time. The essential point is to know how the system in question, e.g. the mind, is organised into parts which as such exclude one another. I am not prepared or concerned to deny that will and appetite may coexist in a mixed state of mind, or even that appetite may be included in will, but in as far as the mind merely has appetite, it does not will, and in as far as the mind distinctly wills, it has not mere appetite. The disjunction would only be false if appetite and will were essentially identical parts of the mental system, and not, *so far as the mind enters wholly into either*, reciprocally exclusive.[1]

'Then mere differents are disjunctively opposed?' Yes, *if the conditions are precisely assigned under which the real subject becomes capable of the one and incapable of the other.* A 'conjunction' or conjunctive judgment about a single subject differs from a disjunction merely by the non-assignment of the precise relations under which the various determinations attach. Thus it is, as Plato showed, that knowledge can solve the apparent contradictions of the perceptive judgment. 'A is both great and small.' Knowledge distinguishes cases and explains, 'A is compared either with x and then is great or with y and then is small.'[2]

The inferential principle of Disjunction is nothing more than the principle of all inference in its most explicit form. Every matter capable of being known consists in a common nature including within it and constituted by parts or differences, which are related to one another at first sight negatively

[1] Appetite, when it enters into will, must surrender its character as mere appetite, not merely by the addition of something else, but by taking on a new character in itself.

[2] See Plato, Republic, p. 524.

qua excluding one another, but further, through this very negation, are related positively because by their negative relation they positively determine one another Every such matter when explicitly stated in articulate form, is known as a disjunctive judgment And this is the nature of the ultimate judgment by which the individual consciousness sustains its real world The simplest cases of these reasoned judgments are to be found in the spatial perception, in which the determining differences take the shape of parts external to each other and so negatively related, but nevertheless by their position determining one another, and so through their negative relation positively related. But the most perfect cases are those intellectual creations that are the objects of philosophical science, in which the whole system not merely appears by its common nature in parts which remain external to each other, but tends to throw itself in its entirety into each of these differences, passing by an organic necessity from one difference to another Here, in short, the differences are not merely parts which remain outside one another, not merely phases which succeed one another, but moments which succeed one another so that the earlier are retained in the later through a progressive development, and yet the distinctive character of each moment is not weakened Such, for example, is the relation of the conceptions which by their development constitute the history of philosophy

It is usual to treat of classification as one special form, among others, of logical thought I am unable to regard it in this light. It appears to me to be merely an external consequence, reappearing in every kind of universal, of the relation between universal and differences The nearest approach to pure classification is therefore to be found in superficial arrangements destined merely to facilitate reference, in the dictionary, the index, the Linnæan system After this come the natural or morphological systems of botany and zoology, in which the universal appears though not explicitly, yet effectively, through analogy While in mathematical conceptions as in the true systematic disjunction we have classification relegated to its proper place, as a corollary of the comprehensive application of explanatory theory.

3. By introducing into logic the real or conscious teleology of the human intelligence, we have rendered unavoidable some consideration of the judgment of value, which rests upon the correspondence of a real system and the purpose for which it exists. This judgment obviously presupposes two conditions; i. our knowledge of the purpose for which a system exists, and ii. our knowledge of the degree in which the system fulfils that purpose. The judgment of Value.

1. The former condition demands a real teleology;[a] that is, a conscious purpose for which the system is intentionally recognised or maintained by the human will. We cannot here enter upon the questions, belonging to ethical science, which arise with reference to the objective justification of man's recognition of a purpose in the non-intellectual world. Indeed we cannot avoid extending such questions in some degree to the world of man's own volition by admitting that e.g. the systems of law and government which appear *prima facie* to be made and maintained by man with a view to a purpose which he consciously sets before him, have nevertheless an element of growth or development which goes beyond the knowledge or intention of any single individuals at any time concerned in framing them. The works of mind, in short, are something more, as the works of nature are something less, than the intentional achievements of any individual will, and therefore our estimate of their value is in many respects analogous to that recognition of a purpose which we apply so fallibly to natural objects. We may however—for we must—assume on the whole that the persistent purposes of mankind are represented within our own intelligence, and that therefore in our estimate of law and morality, of art and religion, of political and social institutions, there is at any rate some firm foundation of real teleology. Real Teleology.

ii. The second condition demands *mediation*. We have to ascertain whether a whole fulfils its purpose by comparing the Mediation.

[a] See above, p 99 note, on the defect of the conception of teleology, with the removal of which, however, it ceases to be teleology The standard of value should rather be expressed as Individuality, and the condition, which many would insist on, that value involves a feeling mind, is satisfied by the consideration that a feeling mind is necessary to individuality

operation of its mechanism with the idea which is intended to be its essence This mediation was involved in the philosophical syllogism, in so far as the purpose of the whole was taken to be its essence or unity. But after analysing, in the disjunction, the matter of the universal into a system of reciprocally determined parts and moments, we have forced upon us the question whether the totality of these parts or moments corresponds in detail to the purpose with which we credit it. Such correspondence is what we understand by goodness or value We may say for instance of a given social system that under it the people are either aristocrats who are not the best and do not rule, or a proletariate who pay no honour to those above them and who cannot be ruled And this contradiction between the effect of the system as realised in its parts, and its recognised purpose, entitles us to say that it is a bad system, in the form '*a* being either *ξ* not *x* or *ζ* not *z* is not A.'

Extra-logical as this judgment of value may seem to be, it is really implied in the constitution of knowledge from the point at which quasi-teleology begins, and with it the conception of 'a thing' takes its rise.[a] I shall have to return to this subject in the last chapter when I come to speak of the ultimate nature of dialectical or logical necessity to which the term æsthetic has sometimes been applied.

The main features of Inference.

4 Inference was first defined on p. 1 as the 'mediate reference of an ideal content to Reality', and further explained on p 3 as 'the indirect reference to reality of differences within a universal, by means of the exhibition of this universal in differences directly referred to reality'. And we have gone throughout on the principle that the species of inference are determined by the species of universal which occur in the realm of knowledge Having attempted to analyse these species, and to point out their affinities and their distinctions, we have not much more to say about the nature of inference.

But it may be useful by way of recapitulation to read off from the somewhat tedious treatment to which we have subjected the phases of inference a few answers to the vexed questions which concern it.

[a] See note on previous page If we replace the conception of teleology by that of individuality, the judgment of value will appear, as it ought, inherently logical, in the sense of Plato's *ἀλήθεια* and *οὐσία*

1. Is the syllogism a complete antecedent scheme, prescribing the shape and outcome of every possible inference? II. Is there any fundamental set of conditions to which all Inference must conform, and further, in what relation does the syllogism bear to such a set of conditions?

No antecedent scheme of Inference.

1. There is no such thing as an antecedent scheme prescribing, so to speak, a set of schedules in one or other of which every argument can be written out merely by filling up the blanks. The form of knowledge is an active and constructive principle, to the workings of which no abstract type antecedently prescribed can be adequate Not merely is Logic incapable of passing judgment on actual truth, but it is incapable of prescribing beforehand the type of relations which an inferential totality may impose upon its parts Granting that where we are dealing with imperfect subsumption, with the relations of attributes conjoined in individual subjects according to unknown grounds, the syllogism is able to anticipate the very indefinite form of combination that can result, yet we should not dream of claiming for it this capacity of prediction in the region of calculation, of mechanical or geometrical construction, or of philosophical subsumption. It is true that as regards the last-named process we found a type of reasoning which appeared to represent it adequately in the syllogism in Barbara But the reader must have observed in the examples which were given, if judged by the standard of formal logic, that irritating inaccuracy of form which is known to teachers in the first attempts of pupils to construct a precise syllogism In our examples and in their efforts this inaccuracy is due to the same cause, to the difficulty of moulding the vital and constructive action of thought into shapes prescribed by an artificial scheme, which does not precisely correspond to any single type of intellectual action. The violent transformations by which formal logic attains this end are not perhaps an undesirable scholastic exercise, for they unquestionably drag into light, though only as a meagre and skeleton framework, a certain ultimate community of type in all inferential operations. In so far as the difficulties of pupils arise from inability to transform or translate their intellectual operations *at all*, any exercise which demands such

transformation is perhaps better than none. But in the analysis of operations that constitute highly determinate individual totalities the difficulty of conforming absolutely to the scheme of the traditional syllogism rises into something like impossibility, because the parts within such a totality do not lie side by side like units in a 'class', but have peculiar and distinct relations, imposed, each upon each, by their individual place within the whole.

Thus we cannot preserve, or can only by a *tour de force* succeed in preserving, the identical correlations of terms demanded by the rules of formal logic. In our example 'the mind is a unity of determinate and not exclusive parts', we could not go on to affirm 'a feeling *is* the mind', according to the good old type 'Socrates *is* a man', and so we could not conclude that 'a feeling is a unity', &c But we were obliged to say either that a feeling is a reaction of the mind, or that a feeling is an element in and is a part of the mind, and could only conclude that a feeling is a factor or element in such a unity. And the other examples given in the same context, which preserve more appearance of correct formulation, are in reality no less charged with individual and diverse relations. The mere fact that the syllogism naturally leads on to the complete disjunctive judgment is the most striking proof of this. I may say at this point that to treat[1] the disjunctive judgment in systematic Logic as a form of thought needing completion by Induction, Analogy and Subsumptive Syllogism, seems to me to be a hopelessly erratic selection of phases out of the progress of the individual mind. This progress includes no doubt even in very early stages those imperfect shapes of disjunction which I have called the disjunctions of ignorance; and *these* disjunctions are expanded into systematic knowledge by the progress of determining thought in its various forms. But to make the complete disjunction prior to the imperfect forms of syllogism involves a retrogression from complete systematic knowledge of a real ground to the knowledge of the operation of this ground in individual cases and in a latent form

[1] As Lotze does, Logik, sect 97 ff

11. If we ask the more reasonable question, *not* whether a form can be laid down beforehand for every possible inference such that the inference can be drawn by merely putting terms into the blank spaces of one or another of certain prescribed schedules, *but* whether in the common nature of thought a system of conditions can be discovered which in one way or another is conformed to by every act of inference, on this head I think that an affirmative answer may be gathered from our previous discussions

Condition of Inference

(*a*) *Inference must have three terms and no more*

The explanations given in chap 1 of the present Book appear to me to justify this assertion They consist in a sharp distinction between *terms* and *data*—the number of data being accidental, while the number of terms or moments depends on the essential nature of the universal, *and* in the restriction of Inference proper to mediate Inference. We *admit* however that the function of thought from elementary reproduction upwards is essentially one, and we more especially *contend* that every judgment, in so far as it is explicitly synthetic, that is to say in so far as it affirms one definite content to be a consequence of another definite content, is an activity only separated from Inference by the degree of distinctness with which its parts are analysed. Every such judgment, and therefore ultimately every judgment, can by further reflection be expressed as a three-term inference, and this is especially the case with what we called the true Immediate Inferences, Comparison, Abstraction, and the rest.

(*b*) *An explicit Inference is a conclusion from two premises and no more, which assert relations between differences qua belonging to a single universal* Assuming therefore that the *propositions* which express the premises are not to be disguised purposely or through negligence, but are *bona fide* to express the judgments employed in reasoning, the two premises must have an identical term in common. And

(*c*) that *this identical term must be universal* follows necessarily from the theory of inference which has been developed, and follows also from the fact that this one term is able to stand in both premises For a universal is that which without prejudice to its identity persists through or contains in itself

different relations The simplest example may be found in what we termed the 'Inductive' Syllogism, in which, if we take the middle term as the meaning of a proper name, we argue that Socrates is both good and a Greek, therefore a Greek may be good Here Socrates, although *ex hypothesi* an individual, is universal *at least* in virtue of the double relation to good and Greek—i e of the synthesis, in the 'middle term', of these differences The universal or identity, however, need not, as in this case, be a 'subject', although it will be found ultimately to imply a subject The identical point in space, in which two lines meet in a spatial construction, is the synthesis of two relations in space, but *is* not, only *implies*, space itself as a whole containing these relations

And (*d*) I do not see how we can escape from saying that *not only one premise, but both premises must be universal* The only apparent exception would be the case in which one premise is negative, about which it might be urged that the common term does not stand in two relations, but in one and none, i e in one only. If this were so, however, the negative premise would be a bare denial, would be no judgment, and could give rise to no conclusion I do not see how a conclusion can arise without a synthesis of two positive relations

If then (*e*) negation means bare denial, it results that there can be *no negative premise* But as bare denial is not a case of genuine judgment we must interpret negation to mean significant denial only, and in this sense we must lay it down that *both premises may be negative* [1]

Relation of Syllogism to these conditions.

iii If now, in order to define our attitude towards the controverted questions which centre in the doctrine of syllogism, we enquire, In what relation does syllogism stand to the type of inference determined by the conditions just enumerated? we shall obtain the following results

We must distinguish the traditional syllogism with its apparatus of rules and its distinctions of quality and quantity from the syllogism as treated in the present work.

The traditional Syllogism.

a. The traditional syllogism is a hybrid between what we have called analogical inference and what we have called inference or induction by complete enumeration. It would

[1] See chap iv, above

therefore (*a*) *exclude many forms of inference which perfectly conform to the above conditions, and also some which have been included in our account of the syllogism* Not only would it exclude 'calculation' and what has been called 'construction' in the mechanical or geometrical sense, but it would find no place for Induction or Analogy or even for philosophical subsumption as above described Induction would be excluded by the conjunctive premise consisting in a number of individual judgments, Analogy by the material weight and stress thrown upon the definition of the predicate, which the ordinary half-numerical syllogism has no power of indicating; and philosophical subsumption by the genesis within it of new relations, not prescribed by any major premise. The traditional syllogism, in short, fails to recognise the synthetic activity of thought

(*b*) *The form of universality relied on by the traditional syllogism is vicious,* except for purposes of calculation, which it does not attempt It is true that its express form of totality 'All A are B' does not really cover what we understand to be its import; but its rules and transformations are derived from this express form, and exclude such vital and genuine processes as for example modal conversion The fact that we interpret the numerical totality into true synthetic connection only shows that the inadequacy of this form of universal is actually recognised.

(*c*) There is *no justification for the traditional pre-eminence assigned to one premise as the 'major'*, a pre-eminence which depends on the vicious quantitative form of the universal, and carries with it the *petitio principii* which has been irresistibly demonstrated to be present in the traditional syllogism.

(*d*) *There is no justification for the distinction between universal and particular premises,* except in as far as by a reservation depending on unknown conditions one premise may become merely *probable*

(*e*) *The true 'reduction' or transformation of arguments in figs.* 2 *and* 3 *into fig.* 1 *can only be effected by a material transformation of their content into the content demanded by fig.* 1 *through the processes of analogy and scientific induction.* Reduction without transformation of content is a grammatical *tour*

de force which illustrates no principle except that a simple inference can be awkwardly expressed.

The syllogism as reasoned judgment.

β. When we come however to make the comparison between our general conditions of inference and the syllogism as described in the present work, our results are somewhat different Syllogism as we have described it is a subsumptive reasoned judgment depending upon the unity of differences within an individual subject, and making the intelligible ground of this unity explicit in various degrees, according to which the unity displays itself as a conjunction or as a coherence

The syllogism as thus understood is (*a*) *co-extensive with subsumption, and exclusive only of calculation and construction.* The differences between the syllogistic figures in the sense in which we have retained them depend on the degree in which the Reality that stands as subject to the reasoned judgment is already qualified by antecedent judgment as a concrete unity or individual system. In the Inductive Syllogism the Subject is as nearly as possible a particular, a mere name or designative reference, in the Analogical Syllogism it is a particular as known under a universal characteristic, an individual; in the Syllogism of Philosophical Subsumption it is an individual thoroughly known as a universal in its particular differences, and so a concrete system

(*b*) The *difference between the syllogism thus understood*, and *the abstract combinations of arithmetical or geometrical reasoning*, lies merely in the correlative imperfections of the two processes. The syllogism begins with the perception of unanalysed individual unity, which it is unable to bring to bear as a determinate relation upon the attributes conjoined within it, and thus rests in the mere fact of their conjunction Calculation and construction begin with the perception of a specific determinate unity by which relations affect and generate each other, without making clear at the outset within what concrete system, and subject to what ultimate conditions, these determinate combinations (e g in space) are effected The two sets of Inferences ultimately involve the same elements.

And therefore (*c*) *if the determinate ground is made clear within a subsumption*, or *the individual subject is made clear*

which includes a combination of relations, the two types of inference fall theoretically together, and either may be classed as the perfect form of the other. But, as we have seen, this identification would remain formal and not wholly *bona fide*, because of the comparatively indifferent and unconstraining character of the abstract totalities within which geometrical or arithmetical reasoning is carried on It is true, on the other hand, that the syllogism as we have treated it has no repugnance to the genesis of constructive relations within the unity that is expressed in the inference.

Here we see the true interdependence of the classificatory ideal of knowledge with the ideal which takes the shape of explanatory theory. The former is teleological, categorical, and concrete; the latter is mechanical, hypothetical, and abstract. It is only by a combination of the two—which are not ultimately separable—that a real and coherent world can sustain itself in the judgment which is knowledge.

CHAPTER VII

The Relation of Knowledge to its Postulates

The formal postulates of knowledge

1. It is usual to devote some discussion in a logical treatise to the principles or axioms on which the possibility of knowledge is supposed to rest. Adhering as I do to the conviction that 'The truth is the whole', I cannot be expected to attempt a justification of any abstract principles as points of attachment antecedently furnished upon which the truth of knowledge could be supposed to depend. But as postulates, as general characteristics of known Reality, which it is convenient to state in an abstract form in any systematic treatment of knowledge, because they are inwoven in the whole texture of the real world, some of these axioms call for comment both on their actual import and on their alleged necessity.

It is convenient to distinguish the abstract principles or postulates which are thus found to be involved in the nature of knowledge, as 1. Formal, 2 Material Postulates. This distinction must not be understood to mean that some are drawn from the nature of intelligence exclusively, while others are merely drawn from the content furnished by perceptive experience It would be more correct to say that those which we call formal are drawn from the character of experience merely as experience, existing no doubt solely for intelligence, but for that very reason not separable in its source or nature from any other source or nature which could be described as intelligence pure and simple Whereas those which we call material are drawn from the actual significance which we ascribe to the content of experience as developed in a concrete system, and being ultimately coincident with the conclusions of philosophy and of science must necessarily vary with the progress of these constructions And it is obvious that the formal principles are in fact continuous with and grow into the material principles, the two kinds of axioms bearing at bottom the same relation to one another that exists between

the judgment that there is a system, and the judgment that the system is of such and such a nature The former, of course, implies the rudiments of the latter, as the latter includes the import of the former Accordingly the distinction between the two classes of principles will correspond to the distinction between abstract and concrete science, [1] between explanatory theory and classification, between the law of sufficient reason and the conception of a teleological whole.

I call these principles by the name of Postulates, because when presented to us as abstract reflective ideas they operate as guides to knowledge [2] which lead to their own subsequent substantiation in a concrete form As reflective conceptions, then, they are postulates, i.e. principles which we use because we need them. But they only come to be reflective ideas because on analysis of experience they are found to be active factors in it from the first, factors which acquire their content *pari passu* with experience itself, of which they merely express the animating principle of growth They cannot therefore be taken in a definite form as hypotheses or axioms antecedent to experience. Experience may be said to begin with the certainty that 'there is somewhat'; and the postulates of knowledge do but express in abstract form the progressive definition of this 'somewhat'.

Among formal postulates of knowledge it will be sufficient very briefly to examine the four most notable; the Law of Identity, the Law of Contradiction, the Law of Excluded Middle, and the Law of Sufficient Reason. As a sub-form of the latter the Law of Causation demands no separate treatment.

Each of these laws may be interpreted in more ways than one, according to the degree in which we may acquiesce in its mere abstract form, or attempt to penetrate its further meaning But at any rate with a view to anything like a separation between intelligence and experience, as such a separation is

[1] This is in strictness almost equivalent to the distinction between physical science and philosophy But of course evolutionary science with the conceptions of 'higher' and 'lower' does not fall within physical science as thus defined.

[2] See the account of reflective ideas as guides to knowledge in comparison, &c., II, chap I

purely fictitious, there is nothing to be gained by cutting down the content of these principles to a minimum, in the hope of restricting their reference to thought as opposed to things

The Law of Identity.

1. The Law of Identity must be taken to signify *at least* that it is possible to make judgments that have a meaning and are true.

Tautology.

a. In the bare form 'A is A', however, a form which is not drawn directly from Aristotle or from Plato, the law does not *prima facie* possess this significance, and therefore indeed not any. If it means that A is A and no more, or is *mere* A, then it is aggressively untrue, for it denies the synthesis of differences which alone can make a judgment. If, again, the law is taken as a mere symbol of the pervading unity of the logical subject, and not as intended to exclude all differences from entering into it, then it is an inadequate symbol, erring by omission though not by exclusion. In an absolute tautology which excludes or omits difference, identity itself disappears and the judgment vanishes with it [1]

Symbol of Concrete Identity.

Therefore, *β.* we can only assign a meaning to the law 'A is A' if we take the repeated A to be not a specification of the identical content, but an abstract symbol of its identity. The law will then mean that, in spite of or in virtue of the differences expressed in a judgment, the content of judgment is a real identity, that is to say, has a pervading unity. It says that there is such a thing as identity in difference, or in other words, there is such a thing as genuine affirmation—synthesis of differences referred to reality—which yet is true, that is to say, does not interfere with (but in fact is indispensable to) identity.

Unity of Reality

And, *γ.* we are only expanding what is implied in the allegation of real identity if we say that the law 'A is A' ultimately asserts the thorough-going unity of Reality. A significant judgment, symbolised by 'A is A', lays down for itself no reservation beyond that which its own content may dictate,

[1] It is desirable to remark upon this peculiarity of the formal 'Laws of Thought' as represented in symbolic letters, because the error to which it tends is characteristic of false doctrines of identity and difference Locke defines Identity by saying, 'When the ideas—vary not at all,' and the notion of negation in formal logic is also that of 'mere' or 'bare' negation Cf. Euler's circles

and claims therefore to be true without any reserve Its simple affirmation leaves no room for any discontinuity in the real world, such that on one side of it the judgment may be true, and on the other false If there were such a discontinuity, the judgment, such is the claim of the categorical affirmation—and all affirmation *qua* affirmation is at least categorical—would have taken note of it within its content, and would in that respect affirm under a reservation But once true, always true. All reservations necessary to truth are included in the content. Reality, therefore, is one throughout Relation to time, for example, is not involved in the fact of affirmation, but only, if at all, in conditions belonging to the content affirmed which depend upon facts in time.[1] Affirmation as such is unconditional, that is to say, is not limited by conditions outside its own content, and so if true, is true without reserve. There is not one Reality of which it is true and another of which it is false. Reality is what it is, and if it turns out not to be what we thought, then we thought amiss, i.e. judged falsely.

Law of Contradiction.

11. The Law of Contradiction is but the complement of the Law of Identity. It supplies something without which the Law of Identity is not logically complete nor *distinctly* intelligible. But yet, by the fact of conferring distinctness, it is an addition.

Truism or false.

This Law also, as in its barest statement 'A is not both A and not-A', if understood to deny that A can be B, is either unmeaning or aggressively false Considered as the principle of the negative infinite judgment A is not *mere* B, it *corresponds* as the form of bare negation, to A is mere A as the form of bare affirmation In this form it is simply inadequate or unmeaning, and *equivalent to* 'A is at least A' But taken as the exaggerated abstraction of negative judgment in the sense 'No A can be any not-A', i e 'No A can be B at all', it *corresponds to* the more open interpretation of the Law of Identity as 'A is at least A', and is *equivalent to* the more aggressive interpretation of that law as 'A is mere A'.[2] For

[1] See 1, chap v, on Time in the singular judgment

[2] The *corresponding* meanings of the Law of Identity and the Law of Contradiction, judged by the latitude employed in interpretation, are not their *equivalent* meanings. The more exaggerated denial is *equiva-*

it then denies that *any* B (not only that *mere* B) can be united in a judgment with A This is simply a reiteration in negative form that A is mere A and no more

A genuine law of thought

But if, β. we take the Law of Contradiction in the obvious sense that a statement and its denial cannot both be true, it bears witness to the fact that a judgment *may* be truly denied, i e. that a judgment may be false, and therefore that there may be truth in a negation [1] It has been observed above [2] that, apart from the traditional distinction of quantity, the difference between the Logical contrary and the Logical contradictory, i e. between the principle of contradiction and the principle of excluded middle, disappears. But because they retain a meaning for vital thought although not for formal logic, even in the absence of quantitative distinctions, we will distinguish the two aspects of negation treated of by these two principles, and will speak first only of falsehood established by truth, and not of truth established by falsehood Though really, if our instance of contrariety [3] is 'A is B' and 'A is not B', we have before us both falsehood established by truth, and truth established by falsehood.

A law of Reality

γ. If we do not press hard on the implications of the Law of Identity, it may be said that the significance of the Law of Contradiction carries us one step further. But it is doubtful if truth can exist apart from the conception of falsehood, and

lent to the more tautological assertion, and the more pregnant or significant assertion to the denial of mere identity Thus

'A is mere A' *corresponds to* 'A is not mere B'.

is equivalent to

is equivalent to

'A is at least A' *corresponds to* 'A is never any B'.

[1] According to the traditional rule, a statement may be so denied that both judgment and denial are false But obviously in such a case some denial is true, though the one made is not To say that a judgment is false is to say that it is truly denied

[2] See above, on negation, p 295 ff

[3] I have pointed out before the inconvenient accident that the Law of Contradiction applies to Contraries only, while Contradictories or Logical Contradiction come under the Law of Excluded Middle

therefore it is better to say that the Law of Contradiction simply confirms and reiterates that assumption of the unity of reality which the Law of Identity involved. Reality, the Law of Contradiction asserts, is a *consistent* unity, which is merely to say over again that it is a unity You cannot, that is to say, play fast and loose with reality What is true at all, as the Law of Identity said, is true throughout Reality, but more than that, every such truth is double-edged, and carries with it throughout Reality consequences by which it affects and limits matters that are *prima facie* outside itself. To infer from 'A is B' that 'A is not not-B' means at bottom that A is determined by B in respect of C or D

iii. The law of excluded middle, expressed by Aristotle as 'Between the assertions of a logical contradiction there is no middle', i.e no third alternative, applies of course to all strict denial, for all strict denial is a logical contradiction of the judgment denied Law of Excluded Middle.

α. In its symbolic form 'A is either B or not-B' it lays down the ultimate formal *schema* of negation as the absolute alternative. Literally interpreted according to this symbolic form it has corresponding defects to those of the previous laws when interpreted in the same way. That is to say, all that it absolutely lays down is the form of bare negation which is *per se* not enough to constitute a judgment, because it involves the truth of the infinite judgment, but which in relation to anything further, for example to the intelligible antithesis 'A is either B or C', is only the affirmation of a possibility, and the hypothetical definition of a relation To invest a positive contrary C with the logical character of a contradictory not-B, is the work of determinate knowledge A Truism.

β. Interpreted in the plain sense, e g as by Aristotle, the law of Excluded Middle means that the *significant* negation of any judgment is an absolute alternative to it, viz that not only the judgment and its negative cannot both be true, but one or other must be true, and if true, we may fairly add, must be significant This means that falsehood can establish truth, or that negation can involve affirmative consequences In this sense the postulate in question is the essential principle of disjunction, which is an absolute alternative *between two* A law of Thought.

or more positive and significant members Therefore genuine disjunction has not the form 'A is either B or not-B', but has the form 'A is either B or C', which invests the positive assertion, in virtue of which C is taken to deny B as its contrary, with the absolute exclusiveness that only belongs of right to the bare form of denial, which has for its essence to express the contradictory In other words, the old account of the *contrary*, that it denies, and also asserts something more beyond the denial, must also be true of any *significant* contradictory

A law of Reality.

γ. The principle of Excluded Middle, then, ultimately affirms that Reality is not merely one and self-consistent, but is a system of reciprocally determinate parts In affirming that a significant or genuine judgment is possible, such that within it a negation [1] shall carry a determinate and explicit positive consequence—not merely, as the law of contradiction affirms, that a truth may carry with it definite negative consequences—the law of Excluded Middle fixes upon that reality which is constructed and maintained by judgment the character of a self-determining whole For a nothing can only be invested with the character of a something by being a precisely limited nothing that implies a positive nature in the limiting and sustaining something, such that in affirming the nothing we are not affirming an absolute nothingness, but are covertly alleging a positive something which is or is involved in *the* nothingness of something in particular From the mere and entire non-existence of mechanical cohesion, i e of any such thing as mechanically coherent substances, nothing strictly speaking could be inferred The idea would be the content of a bare denial, and unintelligible But from the failure of mechanical cohesion in the axle of a locomotive running at sixty miles an hour under precisely known conditions, all other substances retaining their mechanical properties, the most precise and detailed results could be predicted and must

[1] According to the *bare* scheme of Excluded Middle, the significant negation must be a negation of the negation, for though the falsehood of the affirmation involves the truth of the negation, yet in the phase to which such a scheme belongs we are hardly warranted in affirming that a negation as such has positive significance This use of double negation is a factor in substituting contrary with contradictory opposition

follow. This is a simple instance of the difference between the negation which has meaning and consequences, and that which has none

Law of Sufficient Reason and of Causation.

IV The law of Sufficient Reason, with its sub-form the law of Causation, is a corollary from that aspect of reality which the negative laws of thought have brought to our notice. Reality being a system of reciprocally determining parts, every part or feature of reality may be regarded as a consequent to which some other part or parts, or ultimately the whole, stands as ground Every consequent, so this law tells us, has a ground from which it necessarily follows. Necessity indeed means nothing but the inevitableness of the consequent when the ground is given [1]

In plain English, the Law of Sufficient Reason represents the demand of intelligence for the explanation of everything by something else. And it is plain that in the case of anything but the absolute whole this demand must go on to infinity, for outside any given content there is always something which can be regarded relatively to that content as something else We have sufficiently criticised the operation of this law, the law of natural science as such, in the construction of the would-be totalities of abstract time and space, and it is not necessary to repeat the proof that this aspect of experience, taken *per se*, generates and must generate the infinite series. For it rests on the relations of parts in abstraction from the whole, or in other words, without the element of totality.

One point must be noticed here Schopenhauer rightly maintains that *absolute necessity* is a *contradictio in adjecto*, because all necessity is *ex hypothesi* conditional We have therefore not spoken of an absolute necessity but only of a real necessity, namely a necessity rooted in a ground which is a fact We ought not to feel as if in this substitution the world had lost something of its rational coherence. Absolute necessity was a false ideal, and produced a fallacious preference

[1] See the admirable section 49 in Schopenhauer's treatise on the 'Satz vom Grunde' His attempt, however, to show (sect 50) that the law of ground and consequent in cognition does not entail an infinite series, although in causality or in space this is entailed, cannot be called successful It depends on his distinction between Causality and Sufficient Reason

of necessity to reality. For a *part*, necessity is a higher point of view than mere *perceptive reality*, because necessity involves relation to the whole, whereas perceptive reality, being isolated, is only formal[1] or potential reality But for the whole, reality is a higher point of view than necessity, for reality is its self-dependence as a whole, while necessity would at once depress it into a part

The above are the principal 'Laws of Thought'. We class them *not* as principles of intelligence apart from experience, but as principles of science or of rational experience as such, discoverable by analysis in every minutest portion of its texture, and capable of being regarded by a very easy abstraction as essential to its *existence* as contrasted with its special *significance*. They may therefore be ranked together as the formal postulates of knowledge, or as the formal aspect of the principle of Uniformity, in contrast with those which are not *prima facie* necessary to the existence of experience, or involved, at all events equally, in all reality as such; and which may therefore be considered under the head of material postulates of knowledge The reciprocal implication of the formal and material postulates in one another, of a teleological significance in a self-consistent system, is plainly a matter of degree, and our task is to analyse the mode in which it does exist, not to predict how it must exist

The material postulates of Knowledge

2 To emphasise the transition from the formal to the material postulates of experience, I make use of the following sentence from a distinguished writer [2]—

'It is conceivable that man and his works and all the higher forms of animal life should be utterly destroyed, that mountain-regions should be converted into ocean depths; the floors

[1] i e it has the contact with feeling which is the form of all contact with reality, but it falls short in content and is a mere fragment which has *something*, we do not yet know *what in particular*, of reality in it

[2] I quote and comment upon this passage purely because it is a striking illustration of my point I have not the least intention of imputing to its eminent author (Professor Huxley in Contemp Review for February, 1887) that he in fact undervalues those activities, the annihilation of which, according to this passage, would make no breach in the order of science I cannot but think, on the other hand, that any logical theory with which such a statement were compatible would be gravely defective

of oceans raised into mountains; and the earth become a scene of horror which even the lurid fancy of the writer of the Apocalypse would fail to portray. And yet, to the eye of science, there would be no more disorder here than in the sabbatical peace of a summer sea.'

Translated into simpler language, this sentence means that if all these things happened, they would happen without a miracle; or in logical phrase, they would be capable of explanation according to the law of sufficient reason. And this is undoubtedly a truth that we must lay to heart Our choice, being what we are, lies between the experience intelligible according to the formal laws, or none at all A 'suspension of the laws of nature', a 'supernatural interposition' or 'interference', is perhaps the one and only matter that if alleged as a fact can be denied on the sole evidence of the abstract 'laws of thought'. Against any phenomenon, any occurrence, however extraordinary, these laws, apart from more concrete experience, have no foothold and no purchase But the allegation that something is known and yet not knowable, nay more known *as* not knowable and in respect of the peculiar essence which makes it not knowable—this, if we would retain our sanity, we must refuse to entertain as conceivable And if supernatural means anything but this, any causation handled by superior knowledge and power within the unity of Reality, then *for logic* it is *natural* and we must treat it as we treat all natural phenomena. We deny no occurrence on the strength of formal laws; we only deny a theory about the occurrence. Formal laws do not care how extraordinary a phenomenon may be, anything *may* have happened or may happen, the only question is whether it *did*, or *will*.

Much unclear thinking and much false sentiment might have been avoided if the mechanical aspect of nature had been recognised[1] long since as Professor Huxley states it. Nature, as a mechanical system, is not teleological. Disease and deformity are as natural, as orderly, as much according to law, as health and beauty It is idle verbiage to enlarge

[1] e g by Charles Kingsley and teachers of his school who preach concurrence with and conformity to the 'laws of nature'

upon a contrast between law and lawlessness in the natural world, considered as a natural or formally knowable system. The only lawlessness is in the supposed supernatural within the natural Nothing that happens can escape from the principle of sufficient reason, and therefore nothing that happens is without an aspect of law

But these considerations, though true, are not the whole truth We unquestionably expect something more of the world than a capability of being known according to the law of sufficient reason It is the nature and the warrant of these expectations that I now desire briefly to examine

The maintenance of Life

i I do not think that it can be doubted that we expect an indefinitely prolonged—not necessarily everlasting—continuance of such conditions of the earth's surface as are compatible with human life It would not be justifiable to derive this expectation from the formal postulate considered above on any such ground as the necessity of a human intelligence to the existence—as we understand existence—of the actual world This merely logical necessity might at worst be satisfied by an appeal to our ignorance , for how can we know that the human intelligence is the only intelligence, in the system of things ? But in any case we are now compelled to accept as fact a state of the globe prior to the existence of the human race, or even of organic life, and if we find no insuperable difficulty of form in this view of the past, why should there be any in a corresponding belief as regards the future ?

It may be said, again, that our whole state of knowledge, and the absence of urgent warning from our scientific look-out men, justifies a disbelief in any imminent disaster or transformation of the earth's surface Now it is my contention in the present chapter that the postulates of which we are speaking simply sum up the pith and essence of our knowledge, and I have no reason to doubt that the actual state of scientific prediction is a large element in the practical certainty with which we regard the future of our globe Unmotived possibilities rightly go for nothing, and it is the case, I suppose, that there are not at present above the scientific horizon any seriously motived possibilities of a speedy end to man's existence

But I cannot think that this exhausts the question It appears to me that the real root of our conviction is ethical, and ultimately depends upon our confidence in the relation of our purposes to the scheme of the universe Such an ethical conviction is not a *πού στῶ* outside our knowledge, but is the very core of almost all that knowledge on which our distinctively human life essentially depends The purposes of the civilised world form the real teleology [a] on which our organised knowledge of society and of all human achievement is based, and it is on the conviction, inwoven in this knowledge, of the reality of these purposes in their essential content, that our faith in the future seems to me to be founded, and under present conditions of knowledge to be rightly founded

It may be said, with an appeal to eschatology, that such a faith is not even a '*quod semper, quod ubique*', &c, and that a speedy end to man's existence on earth has in fact frequently been expected by large bodies of human beings On this suggestion two observations are to be made In the first place, it would be interesting, both logically and psychologically, to know the exact effect of such a belief on the practical postulates of civilised life in those who hold it—to know, in short, the degree of reality with which, as a working belief, it has ever been held To some extent the doctrine has been specially directed to meet the dangers which it tended to cause, by inculcation of the duty of diligence in business and of orderly conduct as the best preparation for the end And then in the second place, as this adaptation of the doctrine shows, the conception of an ethical continuity of purpose is satisfied by the idea in question, although not necessarily under the form of a continued terrestrial existence [1]

[a] See notes, pp 99, 199, above The argument would be sounder in form, but not very substantially different, if we substituted 'individuality' for 'real teleology', the general principle is that our criticised desires though not our given desires have evidential value, where 'criticised' means divested of self-contradiction But anything more than the briefest indication of this problem does not belong to Logic

[1] Under this head, of a satisfaction for our ethical demand otherwise than in the form of our continued existence, may be classed in great part the curious psychological fact of the slight practical effect produced by prospective death even on men whose lives are by any cause gravely imperilled I ascribe this, though in part only, to our prospective

If the question were pushed home, and we were asked to translate our ethical postulate into terms of time and degree, we could only, I think, fall back on conceptions akin to the βίος τελειὸς of Aristotle, i e on the conception of a duration and environment of life adequate to the accomplishment of some worthy purpose And what catastrophes befalling the human race are compatible with the purposes of the world we cannot presume to guess [1] It would be hard to believe, for example, in the likelihood of a catastrophe which should overwhelm a progressive civilisation like that of modern Europe and its colonies, so that the history of the world would have to be begun anew, without any influence at any time arising, by rediscovery of remains, from the prior civilisation

'But we may be mistaken in our postulate' Certainly we may be mistaken, as in our present knowledge, so in the sum and substance of our present knowledge But unmotived

satisfaction in the maintenance of our essential activities and purposes by others after our death, and I do not think that this satisfaction would exist in view of a prospective extinction of the race Again, the truth that a belief in *some* continuance is necessary to *any* action, and that *some* action is necessary to *any* continuance, is merely the minimum grade of the postulate we are discussing

[1] The writer is aware of a strong prejudice in his own mind that a disastrous earthquake in London is an exceedingly improbable occurrence Not, of course, that volcanic agencies can act otherwise than they must, but that such a degree of inconstancy as to tempt an enormous heavily built city to be erected, and then to turn and rend it, would seem malicious on the part of Nature The prejudice is only mentioned as a psychological curiosity, and is not defended for a moment The writer believes it to be a blundering application of an ultimately genuine principle [1]

[1] I am aware that this note has been adversely criticised, and I admit, as I admitted in it, that the actual belief expressed in it is probably indefensible Yet I do not cancel it, because it calls attention to a technical point which seems to me fundamental, and which I doubt if my critics have observed It is this, that as inference from particular to particular is impossible, and you must always make the circuit through the universal nature of the system on whose manifestations you are arguing (see above, pp 22, 139), it is necessary to accept a heavy responsibility when the whole nature of that system is concerned, and if you cherish a general view about it, and allow this view to affect your inference, it is illogical not to formulate and defend your view If you believe that the world-system is wholly indifferent to the interests of civilisation, you shoulder just as heavy a logical responsibility as if you believe the opposite And you are bound to exhibit your view

possibilities of error must go for nothing, and in departing from the positive import of the knowledge which at present we possess, we abandon concrete reality for more or less abstract imagination In this first material interpretation which we have been putting upon the law of the Uniformity of Nature, we have simply been analysing a condition which is essential to the maintenance of human life as reality now presents it to us, viz. the prospect of continuance. If the constancy of content which this prospect demands were ever to become doubtful with good reason, the doubt would *ex hypothesi* show itself in our knowledge on positive grounds; but till then we must accept this constancy not indeed as an ultimate certainty, but as a leading characteristic of our actual world

The reality of values

11 But the uniformity of nature as materially understood goes at least one step further than to postulate the maintenance of human life on the earth's surface It also postulates the reality of those purposes and achievements which make man what he is

It is possible to fancy not merely a state of the earth in which the life of the human animal should be physically impossible, but a state in which though life were possible and actual, yet the apparent caprices of nature, however *formally* rational, should prohibit all advance in knowledge and civilisation. What degree of ambiguity in the appearance of natural bodies, in spite of an actual constancy of their properties, might make knowledge impossible, is a question which there is no sense in asking, as we have no measure by which to estimate the answer. Many ambiguities have been resolved by knowledge, but the operations of the intellect unquestionably demand not only the theoretical constancy of properties, but *some* degree of limitation in their variety. Exhaustiveness is, in short, though not theoretically deducible from the law of Sufficient Reason, a largely and increasingly essential element of knowledge. What would it help us that the specific gravity of gold is constant, if elements undistinguishable from gold by other ordinary tests, but differing in specific gravity, were continually to present themselves in our operations upon Nature? And although infinitesimal

variation is a predominant law of the organic world, yet knowledge is at least greatly facilitated by the existence of marked points of transition between species and species,[1] and it is plain that a succession of animals differing by variations of minute and equal value would not be compatible with our present modes of natural knowledge, although in geometrical matter the intelligence has displayed the power of theoretically grasping an absolutely continuous evolution

It might be said indeed that we were alleging above that 'If there is to be human life there must be human life', and are alleging now 'If there is to be knowledge there must be knowledge', truisms which amount to nothing. So far as content goes, this may, by abstraction, be true. A mere analysis of content is *eo ipso* hypothetical But the content which we are analysing is, moreover, affirmed of our real world as an integral element of the significance of that world, which significance is primarily ethical

And this significance for which I am contending is not an *a priori* postulate or axiom, from which any specific knowledge could be derived apart from experience If I am asked 'What is the material uniformity? How do you limit it? What does it imply?' I can only answer by pointing to the progressive content of knowledge itself The postulate of Uniformity is ultimately that there is such uniformity as our knowledge in detail reveals to us. Do I believe that mass and energy are constant, that gravity operates in the region of the fixed stars, that any of the heavenly bodies have animated inhabitants, or that the elements are ultimately reducible to a single form of matter? None of these, I should have to reply, are questions of an ultimate logical postulate Our convictions upon all of them must be determined by the state of our knowledge and by our estimate of its tendencies From an ethical postulate we can deduce nothing but the empty form of a logical principle, the form that 'what is involved in ethical [a] reality is real', the material details must come from science only.

[1] Owing, no doubt, to the extinction of intermediate forms

[a] In any new work dealing with these points I should not make use of the term 'ethical', which now seems to me an individualistic term,

The three degrees then which may conveniently be distinguished in the interpretation of the Law of Uniformity of Nature or of the Unity of Reality, considered as the postulate of knowledge, may be assigned as follows.

Three interpretations of Uniformity of Nature

(1) Reality is a mechanical system through and through. This postulate is expressed in the so-called 'laws of thought' which find their most explicit form in the 'Law of Sufficient Reason', or principle of Relativity.

Reality is mechanical.

(2) Reality as a mechanical system is adapted to the evolution and maintenance of life, i. e. is at least quasi-teleological. This is a first approximation to what is practically assumed as the material Uniformity of Nature

Reality is quasi-teleological.

(3) Reality as a mechanical system is further adapted to, or includes as elements within its unity, the substantive purposes of human intelligence, i e. is really teleological.

Reality is really teleological.

It is possible, by intellectual abstraction, to dissociate the first of these aspects from the others, as the import of 'mechanical' can be dissociated[1] from the import of 'machine'. It is not possible to dissociate either quasi-teleology or real teleology from a mechanically-conditioned system. The nature of a system can only be real in as far as the parts or differences that enter into it have a real mode of activity. Miracle destroys teleology, for it destroys the relation of part to whole. And activity or variation of activity, that has no ground in the one Reality, is miracle

3. It seems desirable to conclude the present work by bringing to a point the views that have been implied throughout it upon the ultimate nature of intellectual necessity, and upon the sense in which such necessity can be predicated of any elements within knowledge.

The ultimate nature of Necessity.

1. It would be a tedious task to analyse at length the

applicable to an attitude right and characteristic for a finite being within a whole, but neither to the whole, nor to the position of the finite being completely considered I should substitute some such expression as 'the conception of absolute reality' 'Human' is an instance, not an ultimate But obviously the problem goes beyond Logic

[1] This concession must be read subject to the reservations of p 99 This dissociation is not possible in an ultimate sense. But of course dissociation from any particular teleological scheme is abundantly possible, and that is the point of material importance.

A priori necessity and mediation.

components of Mill's discussions[1] relating to the basis of necessary truth But nothing could, in my judgment, be more conducive to a thorough mastery of the question than a careful study of the chapters referred to in Mill's Logic in the light of some plain distinctions, which, in default of a better guide, I will here endeavour to lay down By Mill himself, together with the writers whom he quotes, nearly every position of importance in the controversy is assumed in its turn, and the argument is a strange mixture of penetrating sagacity with unphilosophical confusion.

Mediate nature of Necessity forgotten.

a. Necessity, as we have abundantly convinced ourselves,[2] involves mediation or inference No isolated judgment *qua* isolated can have necessity Every necessary truth must, in so far as it is necessary, present itself as the conclusion from an antecedent. In the idle controversy whether axioms are known *a priori* or 'from experience' this aspect of necessity is forgotten on both sides

'From something prior'

(1) If *a priori* necessity is taken as inherent within the four corners of the axiom itself, the very nature of necessity is contradicted, and the only meaning which I presume the phrase *a priori* can ever have had is stubbornly denied to it.[3] '*A priori*' (ἐκ προτέρων) says in so many words that the knowledge to which this term applies is '*from* something prior to it', i e. is derivative, inferred and mediate. The metaphor involved in 'prior' no doubt created for Aristotle a problem about the series of premises, which, it would seem, must come to an end somewhere in an ultimate premise, a problem which could only be solved, as Aristotle, I imagine, was really quite aware, by making the series ultimately return into itself, and lose its successive character by transformation into an organised system.[4] But this difficulty about the

[1] Mill's Logic, Bk II, chaps v and vi

[2] Cp especially above, i 134

[3] I am quite unable, for the reasons assigned in the text, to subscribe to the views as to *a priori* knowledge which are stated in sections 355–6 of Lotze's Logik In placing the test of 'self-evidence' in an immediate recognition without any process of proof he appears to me to surrender altogether the rational character of knowledge His subsequent explanation, sect 358, seems to me exactly parallel to Whewell's practical retractation respecting the law of atomic weights See below, p. 227

[4] See p 269 below

ultimate premise of a series, even if unsolved, does not justify the neglect of the plain logical differentia imposed by the term *a priori* upon all that claims to be known *a priori*,[1] viz that it shall be inferred from knowledge, whatever this may be, other than itself

(2) If, on the other hand, by those who object to the '*a priori*' origin of knowledge, supposed to be alleged as not experiential, an appeal is made to any form of indissoluble association, originating no doubt in constant experiential conjunction, but operating finally through a sheer psychological inability to disjoin the parts which insist on presenting themselves together in the mental picture, here if anywhere we have the *vicious* doctrine of *a priori* knowledge in its most outrageous form For, it must be remembered, the past is past, the psychological history of our conviction cannot come into court when we wish to demonstrate the conviction to be true or false It is of no use to say, 'I have seen it so often that I cannot help believing it true' One might almost as well say, 'I have said it so often that I cannot help believing it true' The question is not how often you have seen it, but what you now know that you saw, and under what precise conditions If nothing in the content of the experience, as it now is in the mind, goes to exclude error or to carry conviction,[2] then *we believe it simply because we find it in the mind*, which is just the description of vicious or intuitional *a priori* belief[3]

Indissoluble association

[1] Prantl, Geschichte der Logik, vol iv, p 78, quotes from Albertus de Saxonia, A D 1390, as the oldest authority for the dualistic use of '*a priori*' 'Demonstratio quaedam est procedens ex causis ad effectum, et vocatur demonstratio *a priori* et demonstratio propter quid et potissima, alia est demonstratio procedens ab effectibus ad causas, et talis vocatur demonstratio *a posteriori* et demonstratio quia (that) et demonstratio non potissima' Nothing could be more sharply opposed to 'immediate' knowledge

[2] The distinct relations to the percipient, which make us sure that our recollection is not a fancy, are what perform this office in an act of 'simple' memory In fact, no act of memory is absolutely simple, as indeed no intellectual act of any kind is The truth of our recollection is inferred from content, not accepted because of mere psychical indissolubility

[3] In the discussion alluded to in the text, Mill is on the whole the champion of organised knowledge and inferential necessity against unreasoned conviction and mere indissoluble association But he

Organised and unorganised experience

β The distinction on which the relation of necessity to Experience really turns is the distinction between organised and unorganised experience The former can give necessity, the latter cannot give knowledge To maintain with Whewell that there is a necessity which does not depend on experience is to concede Mill's contention that necessity is a psychological illusion If there is no organisation of experience into a system, the latter view is obviously the truth, but with the necessity which Mill rejects there must in that case also be thrown overboard the knowledge which he maintains If there is organisation of experience, then the necessity which attends complete conception, although nothing irrational supernatural or immutable, is more than a psychological illusion It simply means that given this and that, being the conditions imposed by our knowledge of the matter in hand, then the other must follow because of the relation between them

This distinction may be I think, pretty thoroughly elucidated with reference to Whewell's treatment[1] of the principle, in his time quite a recent discovery, that chemical combination takes place between elements in certain constant definite proportions only Whewell was able to persuade himself that this law, when once understood by a mind with adequate scientific resources, could not but be accepted as a law whose falsity was inconceivable Of course such an assertion, made by a writer suspected of a belief in intuitional

wavers in his position, (1) by refusing to maintain against Whewell that a justifiable *necessity can* be generated by experience, and confining himself to the contention that an illusory show of necessity can be so generated—this means that the experience of which he is thinking is the mere repetition of sense perception and not a determinate system of science, and (2) by the constant appeal to the profusion of experimental evidence in favour of geometrical axioms, and in particular to the mental picture of parallel straight lines as the instrument, by a constant repetition of experiment, of generating the certainty that they are incapable of meeting Here we lose sight of the principle which owes so much to Mill's advocacy, that one good experiment will establish a law Sir J Herschel as quoted by Mill wavers in precisely the same way, appealing now to iteration of experience, and now to systematic knowledge Spencer seems to hold the view characterised in the text, not interpreting his test by conception into logical proof and therefore leaving us to suppose that it consists in psychical conjunction

[1] Fully adduced and discussed in Mill's chapters above referred to

or something like innate and unreasoned convictions of necessity, about a principle 'the discoverer of which was still living', was open to the ridicule with which Mill assailed it. But the interest lies in the explanations which Whewell subsequently offered, and which make the course of his mind in the matter tolerably clear. In order to perceive necessity in such a case, you must, he says in effect, understand the terms, you must conceive all the elements of the problem *distinctly*, and you must be furnished with a degree of scientific knowledge which not every man of science possesses. The 'intuition' of the truth, he says, 'may be a rare and difficult attainment'

There can hardly be two opinions as to what all this means Conception thus understood is simply systematic knowledge, and the reason why you cannot conceive the law false is that you have attained a thoroughly *mediate* insight that the system of science requires it to be true; i. e. that if it were taken not to be true your system of reality would be shattered and overthrown This necessity is read into the terms of the principle in question, the interpretation of which has been insensibly enlarged, and without careful analysis there is great likelihood that the principle will seem to possess a necessity involving no relation to anything outside itself. In the particular case in question it may be—though the suggestion is hazardous—that a confusion was operative in Whewell's mind between a very abstract principle which is involved in the place held by quantity in the real world, and the peculiar law discovered by Dalton as the law of atomic weights All quantity is definite, and every combination is a combination of definite quantities Nor can there be any doubt that every mixture has different properties according to the relative quantities of the things mixed together Wine and water will mix in any quantities, but the mixtures will not all be the same I venture to write down these platitudes, as Mill, in maintaining that the occurrence which Whewell called inconceivable really represents the general rule, almost seems to forget that every mixture is a mixture of definite quantities, and that a change in the proportion will make a difference in the mixture No doubt this comparatively

formal principle is a long way from the law which Dalton discovered, viz that the peculiar combination known as chemical combination would not take place at all except between definite proportions of the elements But yet, assuming the *constancy* of the resultant combinations, e g that there is only one kind of water and not two or more kinds, and also the *limitation of their number*, i e that there is not in nature a series of compounds containing the same elements as water but in slightly different proportions—and I should have imagined that the truth or untruth of these two suggestions must have been notorious to chemists before Dalton's time—then presupposing all this it does seem to an outsider as if the law of combination in definite and constant proportions[1] was pretty much rendered necessary by the mere nature of quantity At all events, without being so rash as to infer from the operations of my own mind to those of Whewell's, I may suggest that some such process as the above, which is obviously a mediate inference from matters of fact combined with a formal principle about quantity, constantly follows upon the discovery of a law We are apt then tacitly to presuppose the matters of fact, and to identify the new law with the formal principle which it interprets This, I venture to think, is the key to the general character of the process which Whewell's mind must have passed through in the case before us, with the result of his mistaking mediate for immediate necessity[2] In any case, his reason for believing Dalton's law plainly was, as he says in so many words, that he thought he saw the whole order of nature to be involved in it. If the intuition of an *a priori* necessity excludes mediation or inference, then this logical process was not the intuition of an *a priori* necessity

[1] The theory of atoms goes further than this in form But I understand Mill and Whewell to be speaking of the law only in as far as it refers to definite proportions

[2] It is said that men always begin by denying a new truth, and then say that they knew it before This is simply that the material interpretation or development of an accepted abstract principle is at first strange to them and they resist it, but when they have understood it, they pass it over into the old formal principle, identify the two, and become unconscious that they have made any advance.

In as far then as Conception means this complete insight, its necessity is clearly the sole test of truth, being simply identical with the necessity of knowledge. Mill's polemic against the test by mere Conception is largely justified by the ignorant use that was made of this test, as if it were immediate and operated by mere inspection In this polemic Mill shows himself alive to the true source of experiential necessity, although he rejects the term necessity except in reference to mathematical reasoning Thus, strangely enough, Mill re-introduces into knowledge the distinction between necessary and not-necessary truth, which the experiential school might be expected to deny And his account of the distinction is on the whole sound, referring it simply to the difference between the complete knowledge of the conditions, which is possible in mathematics,[1] and the partial knowledge of the conditions which alone is possible in ordinary physical investigation It would be better, however, either to abolish the term necessary altogether, or to extend it to all scientific knowledge as such

Mere *imagination*, on the other hand, as Mill rightly contends, though inclined to extend the contention erroneously to *conception*, has nothing to do with truth or knowledge either way. Allegations are not more likely to be true because we can imagine their content, nor less likely because we cannot.

It follows from the above considerations—

(1) That every judgment is *necessary* [2] and *mediate in as far as it is known*, and that no judgment has necessity or precision (which depends on the explicitness of the mediating conditions) if taken apart from the totality of knowledge,

and (2) That the content of every judgment, as well as its truth or necessity, is correlative to the one ultimate judgment, i e to the whole system of knowledge, and that therefore while we do well to maintain that the body of knowledge has certain indispensable functions, we nevertheless commit an error of principle if we deny that the identity of these functions is like other identities compatible with variation

[1] The view of mathematical conceptions as hypothetical does not concern us here See Bk I, chap iv

[2] See also chap i of this book, on the specific necessity of judgment.

Thus for instance knowledge, or reality as known, must have such a function of relativity as that which we express by the law of causation But to suppose that the shape in which that function happens to be familiar to us, involving perhaps homogeneity of cause and effect, is necessarily an ultimate shape, is one of the most mischievous results of the fallacy of an isolated necessity I do not think that there can be any doubt that even the conceptions of the straight line or of three-dimensional space are modified in their content by the explicit distinctions needed to save them from being confounded with arcs of great circles on a sphere surface or with space of more or less than three dimensions Unquestionably the new conceptions, however unreal, make themselves felt as restricting the absoluteness of the old ones. Every judgment is relative to the whole of knowledge and no judgment entirely escapes modification as this whole is modified

Rehabilitation of formal distinctions

ii In order to illustrate the true import and value of such conceptions as that of *a priori* truth or of necessary knowledge, I will venture to give a brief sketch of the process, tending to repeat itself in history, by which such distinctions are most thoroughly apprehended, and which, if only in the individual mind, is perhaps necessary to their apprehension

When, in an epoch of genuine enquiry, a student first opens his eyes, so to speak, in the philosophical world, he finds himself confronted by a multitude of traditional distinctions, some of which claim to be fundamental lines of demarcation Impressed with the ruling idea of all great epochs or earnest intelligences, that of the unity of reality, he assumes a protestant attitude towards these distinctions, which appear to him incompatible with the demands of his genius or of his time His iconoclastic zeal is inflamed by the justification which it finds in the obviously meaningless and mechanical rigidity of the tradition which it attacks, a tradition that has come to be in many respects a real offence against the primary postulates of intelligence And turning from his contemporaries to the great masters of thought whom they profess to interpret, he finds in them also the phrases and ideas which he has learnt to regard as the symbols of an unmeaning super-

stition And therefore, finding no help in man, such a protestant reformer in philosophy will proceed to reconstruct his world on the basis of that aspect of it in which its unity has been revealed to him, that is to say, in the case of logic, probably on the basis of sensation, of observation, of particulars, of inductive experience

But when with labour and pains some progress has been made in this reconstruction, then for the reformer or for his successors there arises a further stage. The duty now falls upon them of maintaining the essential distinctions of thought, between perceptive comparison and geometrical demonstration, between empirical laws and laws of nature, between induction by simple enumeration and the constructive processes of methodic science When these antitheses are fully developed, then the time has come for a rediscovery of the meaning of Plato and Aristotle The language which science is compelled to hold reveals itself as coincident with that of the teacher who first explained in what science consists The distinction between the province of self-contradictory opinion [1] and the province of coherent knowledge recovers for science the meaning which it had all but lost for scholarship When it becomes unavoidable to erect, within the whole of 'experience', which has been passionately proclaimed to be coextensive with knowledge, the included wholes of 'empirical' observation and mere fact, as opposed to deductive certainty and mathematical necessity, then it is understood how such distinctions as these when originally made were distinctions within the knowable world, and were not incompatible with the unity of experience No geometrician, I imagine, would accept the statement that the ratio of the diameter to the circumference of a circle, so far as ascertained, is ascertained by observation, because this would mean that it was found by direct measurement But, in denying this 'empirical' origin of the cognition in question, he would not suppose that he was alleging its

[1] De Morgan's Budget of Paradoxes is little else than the self-defence of science against opinion The failure to distinguish relations, which in the world of opinion makes difference into contradiction, is well illustrated by one of De Morgan's cases, an argument against the rotation of the earth which asks 'How can a man go 200 yards to any place if the moving superficies of the earth do carry it from him?' p 78.

independence of our acquired knowledge concerning space and spatial relations He would explain, I suppose, that no doubt the calculation in question was based upon spatial relations that could only come into the human intelligence through its being aware of a spatial world (however this its spatial perception is attained), but that nevertheless the conclusion is reached by a process of reasoning or calculation, and is not an observation in the sense in which it is an observation that there are ten volumes in the shelf at my right hand And by extending the same reasonable interpretation to Plato and Aristotle which we extend to ourselves, remembering, that is, that all contentions are relative to certain purposes and proceed on certain assumptions, it becomes possible to recover something like their natural meaning

The development of Logic in England from Bacon to Mill and Jevons is a good illustration of the process which I have attempted to describe. And on a still larger scale, extending to every side of life, I make no doubt that the Renaissance itself, and also the new Renaissance of Winckelmann, Schiller and Goethe, were examples of a similar phenomenon Ancient systems of thought or of religion can in fact only be interpreted in as far as their interpreters feel the necessities which were pressing upon their authors And thus the individual mind, in as far as its ideas develope from a root of genuine interest in reality, tends to pursue an analogous course If a great master of thought could come on earth again after some centuries, he would seldom find his true followers among those who have never deviated from the straitest sect of his exponents.

Thus a cynic might say that the history of philosophy is a process in which the meaning of Plato and Aristotle is periodically forgotten by their disciples and rediscovered by their antagonists, who then, perhaps, become their disciples, and so the cycle recommences And the observation would be just except in so far as it implies that in each rediscovery no advance is made on the meaning as understood before. The cycles of philosophy repeat themselves, but not with identical content The Encyclopaedia Britannica is a very different thing from the Encyclopaedia of existing knowledge as sketched in Plato's Republic.

iii It has been suggested by a great writer[1]—and the suggestion falls in with many current ideas about philosophy—that the necessity or propriety on the strength of which synthetic connections are derived from or combined into an including unity, ultimately the unity of the world, may be rightly described as 'aesthetic'. This conception contains elements of very unequal value, and I suspect that the element for the sake of which it is recommended is one for the sake of which it ought to be rejected 'Aesthetic necessity'

It does not matter, or ought not to matter, whether we speak of self-evidence, propriety, or necessity They all attempt to express the same fact, that in knowledge, that is in judgment, we are not free, but are under a constraint exercised upon us by the content of knowledge itself, such that some judgments have to be accepted and others to be rejected But if we express this fact by the term necessity, then in virtue of the explanations which have been given above we exclude, and rightly exclude, an interpretation which the terms self-evidence and propriety admit if they do not compel, that is to say, an intuitional interpretation.

'Aesthetic necessity,' then, would either mean something which we might accept as a fact, though we should pronounce its appellation unduly limited, or else would be a contradiction in terms. I will consider the latter alternative first.

a. An aesthetic judgment, like a moral judgment, is in everyday life, at any rate, not explicitly mediated It is the peculiarity of the aesthetic product, or of the aesthetic aspect of any object, that although coherent and rational, having passed through the medium of mind, yet nevertheless, *qua* aesthetically operative, it is not discursively analysed. Although in aesthetic judgment discursive analysis must play its part, yet such analysis is not the essence of aesthetic appreciation, but is on the contrary that which aesthetic appreciation has in common with scientific understanding, and is the mere organon of careful perception, by which the aesthetic product is constructed and brought to notice in the mind A work of art, or any object regarded as beautiful, makes an appeal to feeling, which, as such an appeal, must In one sense, a contradiction.

[1] Lotze, Logik, sects 364–5

be immediate, although the feeling to which it appeals is moralised or spiritualised, and consequently there is on both sides, in the work of art and in the spectator, a rational content This, though it appeals to feeling in an immediate form, is of course capable of being analysed in mediate form But yet, as the work of art is the outcome of a spiritual mood of feeling in the artist, so it appeals to such a mood in the spectator. It was not constructed by combination of abstract relations, and though its fabric must be coherent and charged with intelligence, yet no mere intellectual reconstruction of such a fabric can reproduce the spiritual mood which is the essence of the work of art This, if expressed in an abstract or inferential form, may retain a value for philosophy, but loses the differentia of fine art Therefore, as necessity involves explicit mediation, and aesthetic judgment in the strictest sense excludes explicit mediation, to speak of aesthetic necessity is a *contradictio in adjecto* It is this immediate or intuitional self-evidence, this appreciation by *feeling*, which, as I suspect, the suggestion before us intended to identify with logical coherence or necessity If recommended in this sense, the suggestion must I think be absolutely rejected Necessity only attaches to a judgment in as far as that judgment involves the whole of knowledge Unreasoned necessity is irrational belief

In another sense, a type of logical necessity

β If, on the other hand, aesthetic necessity merely meant that synthetic coherence of parts which every aesthetic whole shares with all universals whatever, then though we should admit the description to be true, and in one respect striking, yet we should have to add that it really did no more than refer us to one instance, and that an imperfect one, of the general relation to be described An aesthetic whole is, so to speak, a universal made easy In it the individual unity which belongs to everything real is not left to be toilsomely unravelled by reflection, but is presented in a shape capable of at once appealing as a unity to sense-perception or to imagination Hence the discursive analysis which is instrumental in the apprehension of a work of art, however subtle in its ultimate refinements, is *ex hypothesi* in great part evident and unavoidable In this sense and to this extent

the rational coherence in which all knowledge consists is strikingly illustrated, not by the aesthetic judgment itself, but by the analysis which accompanies the apprehension of a work of art in so far as this apprehension is of the same nature with the apprehension of any perceived object or complex of relations whatsoever For this reason it is not uncommon to take a work of art as an example of the compulsion by which the nature of a whole controls its parts, simply because this control, which is the essence of individuality, lends itself readily to analysis in a work that is pervaded by an especially harmonious unity [1] But precisely the same is the case with geometrical conceptions, and for precisely the same reason geometrical necessity, which is not only rational but also essentially mediate, is often taken as the type of logical necessity

Of these two classes of examples the geometrical conception is the more perfect in one respect and the aesthetic in another The aesthetic object is an imperfect type of necessity because its nature is not exhaustible by reasoned judgment, but consists in being such as to produce a certain spiritual mood As this mood involves and is accompanied by some degree of reflective apprehension directed to the coherence of parts in the artistic whole, which coherence is necessary, there is apt to be a confusion between the feeling and its concomitant insight which leads to an erroneous notion of *immediate necessity* And it may be added that in trained artistic perception there is an immediate reaction of repugnance or acceptance, analogous to the every-day moral judgment, which is right and accountable in its place,

[1] The famous simile of the statue in the beginning of the fourth book of Plato's Republic will occur to every one This simile, occurring at a critical point in an important work, is perhaps responsible for a current idea that the Logic and Ethics of Plato and Aristotle were 'aesthetic' or 'artistic' But the fact is that Plato and Aristotle dealt almost exclusively with the general principles which underlie all individuality and function, and illustrated these from fine art, from industrial art, and from science, almost indifferently They possessed indeed no specific term for fine art, and though they gave a just weight to the idea of beauty, yet nothing in their theories was aesthetic if that means sentimental or unreasoned If anything, they were too systematic and intellectual

but is the worst possible elucidation of logical necessity, with which the form of feeling is wholly incompatible The geometrical object is not open to this censure Its nature is to be capable of systematic construction through and through And the pervading nature in virtue of which the universal determines its differences, the root of logical necessity, is nowhere more explicitly formulated and applied than in geometry.

As to individuality, however, the matter is reversed A work of art, though not an embodiment of real teleology,—for it has not a purpose conceived as a definite reflective idea,—has nevertheless the content or nature of real teleology, being thoroughly penetrated with reason [1] in the form of feeling It is therefore individual in a special sense, as an outward and visible form thoroughly identified with an idea that pervades it, so that the work of art is distinctly relative to human intelligence, though it has no separable purpose embodied in abstract human thought. Thus a work of art is an exceptionally effective instance of an individual whole In geometrical objects the pervading unity is of the most various kinds, and sometimes, taking the imperfect form of a progression to infinity, appears to be incapable of constituting a whole complete in itself. Even space seems powerless to limit itself, and therefore its parts seem rather to lie indifferently behind one another than to constitute a totality in which each has its peculiar place and function. In this sense no doubt the *peculiar* and *specific* necessity imposed upon parts by the whole which they constitute is better illustrated by the aesthetic than by the geometrical whole

Yet the wholes of real teleology, the moral order, for example, as exhibited in a moral person filling his place in a community, illustrate the nature of rational necessity better than either the aesthetic or the geometrical system The identification of necessity with the idea of an intuitional or isolated self-evidence is the rock to be avoided

[1] Mr. Matthew Arnold's phrase 'criticism of life', applied to poetry, explains what is meant by saying that art contains reason That the reason must be in the form of feeling this term 'criticism' appears to ignore

Necessity, then, is a character attaching to parts or differences interrelated within wholes, universals, or identities If there were any totality such that it could not be set over against something else as a part or difference within a further system, such a totality could not be known under an aspect of necessity The universe, however we may conceive of it as including subordinate systems, must ultimately be incapable, *ex hypothesi*, of entering as an element into a system including more than it Strictly speaking, therefore, its relation to knowledge must be one of reality, not of necessity. But also, strictly speaking, it is a reality which we have no power to question or to explain, because all our questioning or explanation falls within it. There can be no meaning in talking about what might be the case if the universe were other than it is, or about what has been the case to make the universe what it is.

But except in the case of this unique and imaginary reference of that which is assumed to be the absolute whole to something outside itself, every judgment is the synthesis of differences, in a whole or identity expressed or understood, and is therefore at the same time the analysis of that identity. It makes no difference to the ultimate or actual import of a judgment whether as a process in time it took its rise from the synthesis of two data, or from the analysis of one. In every judgment there are differences within an identity. In every judgment therefore there is affirmed a necessity based on a reality The necessity itself may have for its content a further reality, or may remain an abstraction which can only be set down as descriptive or illustrative of reality. The latter is the case with the more extreme forms of the hypothetical judgment.

The various forms of universal which are the source of necessity and constitute the content of judgments, the comparative value of these forms for knowledge, and the affinities between them, are the object-matter of Logical Science. And because our intelligence creates and sustains our real world by a continuous judgment which embraces these forms, in their concrete connection, within the unity of its system, it is further true that Logical Science is the analysis, not indeed of individual real objects, but of the intellectual struc-

ture of reality as a whole In speaking of the intellectual structure of reality, it seems to be suggested that reality is modified by knowledge and is dependent upon mind How far and in what sense this is the case I have attempted to explain in some additional chapters dealing with current contentions of to-day.

Genetic Theory and Necessity

4. In more than one passage of the first edition of this work I referred to considerations bearing on the limits of a Genetic theory of Logic The development of systematic thought as it reveals its inherent nature in response to stimuli largely conditioned by its own advance, and according to its own necessity, appeared to me to be the true type of genetic analysis [a] It seemed to me further that a sharp distinction must be drawn [b] between varied forms and degrees in which the conditions of knowledge might be fulfilled in various environments, and any attempt at evolutionary explanation of its ultimate principles, such as the law of non-contradiction On the other hand I was careful to point out [c] that even such leading principles—the necessary functions, as I proposed to call them, of the rational mind—were not to be considered as formal propositions, given and self-evident each within its own four corners, and irresponsive to alterations in the general body of knowledge I regarded them as roughly [d] comparable with the main functions of an animal organism, which may be fulfilled in all sorts of shapes and degrees, but must be fulfilled if life is to reside in it.

There must, I urged, for example, be a function in the body of knowledge corresponding to what we know as the law of Causation But in what precise shape it is to be asserted has been and is still a matter of controversy, which will no doubt continue to lean in different directions from time to time according to the requirements of the matter to be dealt with

I propose here to return to the above question, which has been the subject of much recent discussion, and has lately met with a substantive treatment in a treatise on Genetic Logic.

[a] Vol I, p 2 [b] Ibid pp 7–8 [c] Vol II, p 229

[d] Roughly, because for life the environment is the surface of the earth, For mind it is the universe.

Reason as an Adaptation

1 There is, however, one general difficulty which may be dealt with in advance 'What matter?' it may be replied to me 'According to current views, and your own in particular, all genesis is only revelation, and there is no true creation *de novo* "Tout est donné" Suppose that reason is only an evolutionary adaptation, selected by the environment from variations of thinking, for you at least that makes its nature none the less its own. However moulded, it had to be as it has turned out What difference can the particular process make? What logical interest can you have in combating a view which professes to trace the genesis of the logical reason as an adaptation due to natural selection?' Now in a sense I am prepared not to admit but to contend that everything is modelled by the environment—the whole—which means, in the end, by something like natural selection But within this wide principle there are distinctions which must be taken Above all, we must distinguish systematic contrivance—the conscious working of a principle of totality and non-contradiction—from trial and error The conscious endeavour towards non-contradiction and totality is no doubt a powerful instrument towards survival through natural selection, but it is also a great deal more It is within itself a power of construction and of judgment, by its own law and necessity, in its own right This power is what we call reason and intelligence, it is characteristically self-contained and self-complete; and has its own necessity, which is systematic, and not the mere success *de facto* that comes through trial and error, the pure and simple form of moulding by natural selection.

How reason is truly creative I have briefly discussed above, and hope to consider more at large in another work. Briefly, I should urge that reason alone means creativeness, the continuity of the *bona fide* old with the *bona fide* new, and that there is nothing in the universe that is strictly creative except reasonable[1] and logical process—such for instance as we observe in every original work of art. That every such element is new and unparalleled is a quality secured to it according

[1] See above, p 182

to the degree of its individuality, which is one thing with its logical perfection To repeat or be repeated means imperfect individuality—a failure, *pro tanto,* to hold one's own as an element in the whole But I must not here digress further in this direction

Reason, then, however it manifests itself in answer to demands of an environment, has its own necessity and does its own selection in virtue of its own constructive principle. It is the nature of this necessity, and the organisation which embodies it, that we investigate in Logic, and from the standpoint of which, so far as the system has been apprehended, we can judge and understand its partial incarnations in the course of evolution This distinction between rational necessity and *de facto* survival will be further insisted on in the sequel

Imitation plus Selection

11. I understand it to be the claim of the recent Genetic theory of Logic to explain the rise and growth of the characteristic organisation of thought which we portray in logical science, by a process of Imitation through which thought-variations are suggested, together with one of Selection, through which only certain of these are permitted to survive owing to their fitness for social and practical needs. Granted these factors, it is held, the nature of logical thought can be accounted for by them and out of them.

My contention is, on the other hand, that Imitation is merely a later and partial aspect within the character of relevant response which belongs to the principle of non-contradiction developing in its world of Identity and Diversity of experience, which is to it as the world of organic being is to the principle of Life.[1] And further, that Selection is something which Thought does for itself—as in the relation of any theory to the experience it unifies—and from its own standpoint. A Genetic theory, therefore, I contend, may exhibit the *de facto* evolution of the thought system and thought principle under historical influences, but it can never derive from other factors that systematic necessity of reason, rooted in the principle of non-contradiction, in virtue of which it pronounces some judgments to be true and others to be false

[1] But see p 238 above as to the different environments of the two

Meaning of Imitation

iii Imitation is a partial case of relevant response, which depends on a recognition of Identity in Difference *a* I can not agree with the view that this distinction is verbal, e g. in its application to biology It is easy to show that a working whole cannot be represented in terms of similarities between its members. And for the same reason the nature of the members themselves must in a great measure be omitted from such a representation The Linnean classification, or the current 'natural' classification in botany, may be taken as a representation according to resemblances, though I should not admit that any scientific classification is so intended. But a region of the world, as a whole of competing and co-operating members, according to the light thrown by the principle of evolution, can never be represented in such a form as this. It can never bring together the things which have most to do with each other competing species of plants, co-operative plants and animals, the soil, the climate, and their effect on the living things Of course all this can be added in footnotes, as it were, to classification by resemblances; but it cannot be represented in the structure of the classification itself It would be like trying to explain a locomotive by arranging its parts in classes according as they resemble each other. The reason of the impossibility is that the parts or members have their connection through their differences, and in a classification by resemblances, these, though they have their weight as differences, have no weight as instruments of identity This whole subject is treated by Green,[a] and I think is too little understood. I should strongly suspect that the reform of logic in this sense in the great Idealist days promoted, or at least was akin to, the transition from Linnæus to Darwin

Imitation and Response

Why held a vera causa?

β. Imitation (I summarise in my own language) is alleged to be a *vera causa*, it *shows*, is psychical, genetic, you can see it at work, the operation of a universal (that is, an identity in difference) is an assumption, shadowy, almost, I think, *a priori*, mystic, antiquated, invisible I assume, it is urged, publicity, the common awareness of a situation in which more than one person is concerned, which ought to be explained.

[a] See p 183 above

Now I cannot see any ground for all this in the facts Imitation no doubt is a fact, and plays an important part in furnishing the self with material I quite recognise the value of the work which has been done on this subject But surely response and reaction, indices of communication through a common nature, are much wider and more primary facts, extending over the whole world, physical and psychical. The adapted response is earlier—is it not ?—than consciousness, and the process of its passing under the control of intelligence and being emancipated from trial and error, is fairly well understood, though still doubtful in some details But the adapted response, as controlled by intelligence, just means a consciousness of the situation based on an inference which *pro tanto* dispenses with the test of material action; an inference based on perception is substituted for a certain number of errors, as when a man sees at a glance how to open a gate, which a dog will paw at till it comes open There seems to me no assumption in this, it is a plain statement of fact, and of fact more general and fundamental than imitation, and requiring no more assumption.

With responses adapted by intelligence on the part of two or more agents you have 'publicity' or 'the situation'. What you want, to account for this, is not imitation, but the power of consciousness to combine perceptions and see their results—in short, the unity of consciousness As I understand, it is urged that this must not be assumed but can be and ought to be genetically accounted for This I will speak of when I come to comment on the meaning of the term 'genetic'. My present point is merely that imitation is the secondary, less general, and less completely stated fact, and that the assumption of it, while involving, as much as a response does, the assumption of the unity of consciousness, is in no special way a help towards explaining the apprehension of a situation as a whole

False separation of Imitator and Inventor

γ. The treatment of facts introduced by this theory seems to me precarious all round. Particularly is this the case with the separation of the imitator and the inventor I am convinced that a really critical study of any branch of history would demonstrate the crudeness of this antithesis when

offered as a matter of principle The advance of the human mind, independently, so far as can be judged, of individual original genius, is one of the most striking phenomena of history, and one is inclined to add that the deepest transformations are those which have taken place in this way. It is an old and true saying that man must advance or recede, to stand still is impossible for him. That is to say, the application of tradition to life is in itself a generator of inventions; it is impossible even to borrow ideas without drawing conclusions which the lender never drew And it is well known how rarely, if ever at all, an invention can be assigned to a single mind The history of art is very instructive on this point, e g. the education of a Turner.[a]

Truth or Belief?

iv. The second question (p 240, on evolution of Thought), seems to amount to this Does a genetic account of thinking explain by what character judgments are true, or only under what influence we have come in fact to hold (often wrongly) certain judgments to be true? And what bearing has either alternative on the theory of selective thinking?

Limits of Genetic account

a. I will say at once that I see no meaning in a genetic account of knowledge, except as a history of opinion, but I admit that this involves a history of mental organisation A simple illustration will do as well as an ambitious one. We constantly make such judgments as this 'A. B is a moderate Evangelical, he was brought up as an extreme one, in a family and circle whose views were extreme, but his work and intercourse with varieties of people have made him much more temperate' Here we have the true place of a selective theory of thinking, so far as I understand it, in a nutshell A B inherited a platform, an organised mental constitution and logical or quasi-logical system; i. e he acquired it by adaptation to his parents' and teachers' views, or imitated them Starting from this, he developed his later position through varied forms of social selection acting on his ideas, involving accommodation to practical needs, and he now has a mental content and organisation at once fairly

[a] See, in Mr Finberg's book, the drawing copied by Turner in boyhood from an engraving, set side by side with the engraving from which it was copied

harmonious with the circle in which at present he moves, and determined as a whole by the platform which he inherited. I do not doubt for a moment that a history of all of us and of the human race could be written in terms analogous to these with a great deal of truth And it would not omit the facts of mental organisation The metaphysician, the psychologist, the biologist, mathematician, and also the Englishman, Frenchman and German, would all prove to possess, yes, and to have acquired and developed, certain favourite categories, certain forms of logical or quasi-logical bias, and predispositions to accept explanations of certain appropriate types

In such a historical enquiry some theory of selective thinking might have, so far as I see, very interesting applications It would show by what needs and under what direction of attention the minds of nations and individuals had grown into certain structures, and had acquired certain logical predispositions

But even here it would be necessary either to expand very largely the sense in which, or to limit very strictly the extent to which, we affirmed action to be the instrument of selection. If action meant all change of consciousness directed to an end, then, in referring the course of cognition and mental organisation to the needs of action, we should be making cognition itself the standard of cognition,[a] and saying that it learns to act as it does act primarily by seeking its own ends and secondarily by taking account of a certain contact with material action Then we might fearlessly say that 'action'

[a] Mr Stout in his Manual of Psychology seems to me to agree on the whole with me, never blinking the relative importance of the cognitive system as compared with external action, nor the liability of social endorsement to be erroneous But in one place he seems for a moment, as I venture to think, to slur the distinction on which I am here insisting On p 547 he insists that because belief is a condition of activity, therefore activity must be a condition of belief And this remark he extends to theoretical activity, though, indeed, as referring it to the provisional acceptance of working hypotheses, he gives it a very restricted and innocent application But the point I wish to urge is this In a 'practical' activity the end is assumed to be given, and it is not a cognitive end, therefore in this case there is some tendency to adopt beliefs which purely cognitive processes might not confirm, i e there is a possibility of a real non-cognitive influence on cognition But in a theoretical activity, unless a preconceived opinion is to be supported

is the sole test and instrument of selective thinking. How 'action' operates, would be the further question, to which Logic would be the answer

If, on the other hand, action were taken in the sense of the production of change in the external world, we should return quite a different answer We should say that the influence of practical needs was a diminishing factor as the content of systematic knowledge increased [a] We should point out that when thought has become complex, action on the external world is to it as sensation is to science, a condition which is little more than negative, something, disagreement with which demands more or less modification of the discrepant thought, but any given agreement with which carries us but a very little way towards truth We should further urge that the much talked of 'social endorsement', as applied to systematic ideas, has no existence This is a very important point in its practical bearing Social endorsement does apply roughly to habits of action But to cognitive ideas, to the actual content of inventions, and to theories, as such, it has no application, only touching them in one or two points out of thousands, and to suppose otherwise is a very mischievous superstition [b] It is a transference of the ideal postulate of reason, that all valid judgment is valid for all intelligence, to the *de facto* social consciousness, to which it applies only in grades so contingent and varying as to be of no selective value whatever The leading ideas of society, so far as they can be conjectured from their expression, are always in

(which is an *aberration from* the theoretical consciousness), the end to be obtained is not given, but is itself a conclusion to be constructed It therefore involves *ipso facto* a modification of the beliefs ancillary to it, and the dangerous primacy of action over reason is not confirmed by this instance

[a] Mr Stout in his Manual seems to me perfectly clear on this point, and to be wholly free from the ambiguity whether thought is made true by being socially and practically selected, which I find in others.

[b] I hope I shall not annoy a friend who conversed with me in the U S A, in 1892, if I make use of his observation to me 'Sir, the people of these States have endorsed the philosophy of Mr Herbert Spencer' The example seemed too apposite to be neglected, as showing the laxity with which a rough coincidence in one or two points is construed as an 'endorsement'

arrear of the truth known to experts, and more especially are discrepant with its own habits of action, which do represent in a rough and unorganised form the external needs of life [a]

The exclusive importance attached to action on the external world, and to social endorsement, even as influences on the history of opinion, is, I hold, a mere paradox, unsupported by facts. The subordination of the vast cognitive systems and interests of mankind (which have, it must be remembered, their own relations, dictated by cognitive needs, with the 'external world' or sense-perception) to the test of action in the narrower sense of material external change, I believe to be simply an elementary blunder. If, on the other hand, we are only asked to call these interests and systems 'practical', as Aristotle carefully pointed out that they are, in virtue of their inherent conativeness, we are asserting, I take it, the contradictory of Pragmatism,[b] but are returning to obvious truths [c]

The Making of Truth

β. And when we raise the whole question of Pragmatism, i.e. as I understand not '*How do we come* to think something?' but '*What tests or makes* its truth?' the idea of selection by social endorsement, or by success in producing change in the external world, loses all claim to consideration, except as involving agreement with sense perception, which is provided by cognitive activities in a much more adequate form. As we have seen, nearly the whole of cognition is simply untouched by action on the external world. In such action itself the outward change effected is but a minor part, from which, as we know, e.g. in all ethical considerations, it is impossible with certainty to understand a man's mind; and when

[a] e.g. T. H. Green usually agreed with J. S. Mill on questions of public policy, though on all theoretical matters their minds were diametrically opposed. This is possible, just because theoretical ideas, even of social matters, have so very little of their content in contact with practice.

[b] Because Pragmatism says, as I understand, that the only ends of action are those which consist in change wrought upon the external world, and that, to these, cognition is a means. For me, cognition, as a harmony in our experience, has the character of an end of action, though not the whole end. But external change is *never* an end.

[c] See Stewart, Notes on Aristotle's Ethics, 1098, a 3, and citation from the Politics.

we come to the great cognitive systems the prerogative of such action vanishes altogether Indeed, there is but one criterion of truth, and that is, a fuller systematic cognition of the content whose truth is in question No history of opinion, no formation of a platform, no idiosyncrasies of mental organisation, can come into court when the question of truth is raised. Then we have to do with nothing but the systematic necessity of knowledge and the fact that fuller cognition can compel every false judgment to expose itself as flat self-contradiction.

Now the advocates of Genetic Logic seem to me to mean that selection by social and practical needs not merely accounts for our holding opinions, but also constitutes their truth or falsehood If so, then, as the problem opens out, we have the whole of Pragmatism on our hands, and are, as I hold, beyond the limits of legitimate genetic explanation Grant, e g for the sake of argument, that the unity of consciousness first appeared in practical action in the narrower sense given above (as it must have done if there was a time when consciousness was entirely 'practical' in its aim), or that it is motor in its nature, or that it appears in some sort of general sensory process. All that is interesting in the history of opinion, but has no bearing on the logical value of such unity This is only to be discovered by an analysis of the part played by it in the organisation of experience so as to avoid self-annihilation by self-contradiction. It is an old story; granting (what is not true) that we need not play the game, yet if we sit down to it we must observe the rules. If we are asked, Why must we? there is no answer but to show by analysis in any given case that in trying to evade them we are disguisedly throwing up our hand I can imagine its being replied, 'But you say that A B 's rules and platform are got by his history and education, then surely his truth is so too.' The answer is that his rules and platform are an imperfect appreciation of *the* rules and platform, and cannot stand against another, in him or outside him, which more nearly approaches them, and therefore is able to exhibit his as self-contradictory. His knowledge, or rather opinion, *qua* his, may be compared to his body, a *de facto* structure, accounted for by accident

and selection as well as nutrition and correlation But his knowledge *qua* knowledge may be compared with the work his body is now capable of as a machine—a test to which his genesis has nothing whatever to say Truth is the most organised organisation of reality in the medium of judgment, our history may excuse our failures in it, but cannot make them successes

Views like this suggest to some thinkers the idea of 'the mind, for no reason, and by no regular processes, making its truth what it will'; or of 'the essential mysticism of *a priori* formalism which prevailed before the rise of the genetic point of view'

This again is an old story The very error with which I am charged appears to me to be merely in the mind of my antagonist. The whole antagonism of principle between classical and modern logic, the whole conception of a modern development of the genetic point of view, considered as anything which affects the nature and criterion of truth, the whole idea of 'thought in itself' as opposed to the nature of the real in cognition—all this appears to me to be the merest mare's nest The truth of anything is for me simply its fullest nature so far as expressible in judgment, organised, as the fullest nature must be, so as to avoid diminution by the contradiction of its parts What I deny is, not that thought is the expression of organised reality, but that the organisation of reality is confined to the production of material change in things The nature of things is both general and special, and besides its more general and formal characteristics, there are all sorts of grades and variations as we push deeper and deeper into the heart of complex individuality These, as found by analysis, form respectively the more abstract and more concrete elements of Logic. But obviously all of them contain and confirm the general nature of truth

Why should not the universal 'be a mental experience which has for its physical counterpart the synergy of adapted action'? To me the answer seems simple—because there is very little thought, proportionately speaking, to which there is any adapted action, in the sense of external material

change, to correspond. I have said that I think that unity very likely first showed itself in adapted action But no thought, probably, ever had its content exhausted in the adaptation of external action, no thought of a cultured mind can ever be so exhausted to-day, even in the most practical of activities, and a very great part of life, a part which even economically and industrially is an immense and commanding interest in the world, has no end in external adapted action at all, but on the contrary uses and transforms such action by making it its means A great scientific laboratory, for example, has not its unity in a material operation to be produced, its actions have their unity in a cognition to be attained. The same point is very strikingly shown in the enormous material activities of a Wagner or Handel festival, whose whole practical business has for its determining purpose the production of a harmony in minds, of the same *general* (not specific) nature as a cognitive state The harmony is the end, the 'action' is the means

The formation of new reality, as a *bona fide* addition to the universe of what was not in it before, seems to me a contradiction in terms But the discovery of reality new to us, and the adaptation of intelligence to it, is surely a fact which no one has ever denied And what we call the discovery of reality, the coming to take part in it, as for example by education a finite mind must at some moment begin to—accepting its burden as part of the experience we have to work out—all this is not simply a finding of something pre-existent It is an element, an appearance, of the tension by which something maintains itself in ultimate reality; an element or appearance without which the self-maintenance could not be complete There are not two worlds, an original which pre-exists, and a copy which we make according to our discoveries of the first When we discover, we neither add to the universe nor repeat it We simply play our part, which as we are finite, has a beginning and an ending, in its self-maintenance.

Now if this distinction would satisfy the genetic point of view, I think we might come to terms. But if that view means *a* that new reality is created in the sense of being

actually added to the universe by no continuous self maintenance or logical process, and not created in the sense of being discovered as above explained, by entering into its act of continuous self-creation, and β that action on the external world, and social selection, are the determinants and criteria of truth, then I am afraid there can be no reconciliation between us.[a]

[a] See Bradley Mind 79, 323 ff where he urges that 'My act never is creative (p. 329) I take it that he does not deny that the universe, as a life, maintains itself in some degree through my act He only denies, I gather, that my act brings something new into the nature of the universe as a whole

CHAPTER VIII

THE ABOVE THEORY OF JUDGMENT IN RELATION TO 'ABSOLUTISM'

1 A THEORY of Judgment must be criticised on its own merits Still, it would point to some defect in such a theory, if, without the strongest possible reason, it should tie us down *ab initio* to an ulterior metaphysical doctrine

Our Theory prejudiced?

Judgment, we may say, should be an instrument of asserting whatever we want to assert If it trammels us in our beliefs, we must surely be interpreting it wrong

Therefore I understand it to be an attack on the theory of Judgment which, having originated elsewhere, is in the main adopted in this work, when we are told by distinguished writers [a] that it by itself [b] makes necessary the doctrine of Absolutism The attack is primarily indeed addressed to reveal the slightness of the foundation of the latter doctrine; but I take it that its suggestion is double-edged, and might be briefly rendered thus 'Your logical theory is a caprice which is enough by itself to force you to Absolutism, and will not let you say what you mean Your Absolutism is a metaphysical doctrine founded on little or nothing beyond this logical caprice' My aim in this chapter, as befits a logical treatise, is to combat the first of these two suggestions But so far as the 'by itself' is concerned, this cannot be done without reference to the positive grounds which force the theory of Judgment to define itself in favour of Absolutism And therefore to indicate the reasons for the latter doctrine, though strictly belonging to metaphysics, also becomes relevant here

The criticism runs thus If every judgment in ultimate analysis qualifies an existing reality by an abstract universal,

[a] Mr Russell, Philosophy of Leibniz, p 15, and Principles of Mathematics, p 448, Professor Stout, Aristotelian Proceedings, 1902-3, p 7, Professor A E Taylor, ibid, 1908-9, pp 202-5

[b] These words, 'by itself,' indicate a principal feature of the criticism in the sense in which I desire to repel it

it is impossible to arrive at a plurality of individuals which can be ultimate subjects of predication, because no combination of abstract universals can confer the uniqueness [a] which alone distinguishes an individual There can therefore be but one ultimate Individual to which all predicates must belong, and this doctrine is Absolutism

On this basis there are two charges against our theory of Judgment First, that its consequent, the doctrine of Absolutism, is false, and therefore the antecedent, the theory of Judgment, must be false also That is, there are many ultimate subjects of predicates, and a theory which involves the denial of this must fail

Secondly, in any case the theory of Judgment binds us from the beginning to a certain metaphysical view, which is an unfair and improper use to make of a logical caprice, or, let us say, of a logical analysis of the mere form of assertion

No finite real self-existent

i The set of facts upon which the former charge is supported, begins, as I understand, with the necessity of recognising real pluralities of terms in logic and in mathematics From this fact, as I read the argument,[b] is inferred, what is not included in it as a fact, the existence of a plurality of substances of which, on the strength of an appeal to our private experience, the self is taken to be the principal example And the existence of such true substances is also independently affirmed By substance is meant something which can only be a subject and never a predicate, and which is a uniquely individual existent, so far independent that as a part or element it is no less individually real than the whole, while the whole is no less individually real than the parts [c]

Now the doctrine of a single Individual Reality rests on the demonstration that no finite individuals are self-complete and self-contained, and that therefore none such can be self-existing substances, or irreducible subjects of predication [d]

[a] See Professor Stout, loc cit

[b] Taylor, loc cit It is very noticeable that this fact is not taken as itself including the existence of a plurality of substances, so that *prima facie* it is a proof that you can quite successfully work with a plurality of terms which have no claim to be substances [c] Ibid, pp 208-9

[d] Cf Bradley in Mind, 74, p 160, which I saw first after this passage was written

I suppose that every discussion of substance to-day must start from the position which Lotze has made familiar, [a] that to call anything a substance must indicate its mode of behaviour, and not an occult somewhat presumed to be inherent in it From such a point of departure we come at once to the position that substantiality, like individuality, is a matter of degree No finite real is wholly independent and self-existent, none, that is to say, when taken as a subject, can be concerned in judgments which are completely true either regarding its predicates or regarding its relations with other subjects [b] If this is so—and even the argument which I am disputing admits and maintains it [c]—there is no finite real which is in the full sense a substance No judgment in which such a real stands as subject or term can ultimately be true, the connections alleged about it must either be too much or too little

I must explain that to deny the self-existence of any finite real, is not to assert that Reality could be complete without it Nothing is self-existent, but nothing is non-contributory

Degrees of Individuality.

II The above argument is merely carried into detail in the doctrine of degrees of individuality This, if we start from Lotze's position, is indisputable, and is over and over again asserted both by implication and in so many words in the argument under discussion [d] There are terms, we saw it admit, which are not substances, and when we come to substances, it is granted that the finite self has less individuality than the social whole, which nevertheless most people would not admit to be a spiritual substance at all.[e] And in fact, no finite individual is self-contained, self-consistent, or self-dependent; all finite individuals differ in their degrees of these characteristics This seems to me to be one of the truths which are accepted without being believed I do not think that it is

[a] Metaphysics, Eng Trans, p 76 [b] See i 206 ff above.

[c] Taylor, loc cit, and Stout, Ar Proc, p 21, and cp Bradley, Appearance and Reality, in the arguments against Pluralism, pp. 141–3 and 607

[d] e g p 211, we might draw a distinction here between individual substances of higher and lower orders'

[e] I cannot reconcile this with the requirement that the element is to be as real as the whole, and *vice versa*, ibid, p 208.

or can be seriously disputed on philosophical grounds All the distinctions which philosophy takes account of between higher and lower grades of life and experience come back to this. Any logical or metaphysical system is enough to show how the positive value of the law of non-contradiction embodies itself in such gradations, and the argument of Mr. Bradley's Appearance and Reality is, as I understand it, the same in principle throughout Yet if this principle were not only accepted but believed there would surely be at once an end of pluralism and of the multitude of self-existent substances If all finite subjects have in various degrees their reality outside them, it is idle to speak of any of them as substances in the sense before us As a striking example of gradation, we may note how the series of individuals passes downwards into cases where individuality, and with it reality, touch a minimum, and fade into the extremes of self-contradiction and self-alienation, as in various grades of the animal mind, or in the terms, such as point or atom, for which it is not claimed that they are substances Confront an upholder of selves as self-existent substances with a brood of new-hatched chickens, and ask him whether these are individual substances or not They are certainly unique by their relation to presentation, but this, as we shall see, is a feature actually opposed to true individuality If, further, they are in behaviour and capacities true individual substances, or if they are not, the answer is equally fatal to the substance-theory In the former case an enormous difference of degree is admitted within the series, so great as to destroy the value of the self-existence claimed for its members, in the latter case a distinction of kind is admitted between grades of spiritual beings which cannot be other than arbitrary

Experience of self fatal to doctrine of substances

111 The appeal to our experience of ourselves is of all things the most fatal to a doctrine of self-existent substances. The evidence is so abounding that it is hard to know where to begin Perhaps it is enough to say that no doctrine of a monadic self, in the sense of a self which is single and substantial, has ever been able to deal with the self as it actually exists;[a] Plato's least of all, as he constantly admits and

[a] Cf Appearance and Reality, p 86 ff

maintains What we miss in him, what would have taken him fairly beyond the doctrine of the monadic soul, which hampers him as he himself points out,[a] is the conclusion which lies so near him, that the terrestrial course of the soul is the soul's opportunity for self-perfection, and that when struggling with multiplicity it has not fallen, but is for the first time (as compared with its self-existent purity) in the way to rise. The logic of his system in fact demands this point of view, and it might be said to be implicit in his doctrine of the earthly life,[b] but I cannot say that he has anywhere to my knowledge expressed it Some kind of unification at some stage of being, no doubt we with Plato assume as inevitable But what kind of unification, and at what stage of being, whether or not within the limits of an individuality that can be called ours—this is altogether a different question Nothing in our experience seems to warrant or even to suggest it, while our moments of fullest life seem most distinctly to deny it. That our self, or will, or mind, at its best, is not the self or will or mind of a unique individual, bounded, so to speak, by our normal circumference, is the one definite point on which spiritual experience seems unambiguous[c] And in the relation of the individual to society, described as before referred to, we have this admitted, and, as its consequence, that no individual which has a foreign environment can act in a way purely self-expressive What kind of individuality is that, which cannot express itself in its acts? It is to me quite astonishing that an appeal in favour of a doctrine of independent substances should be made on the ground of our experience of ourselves What all great masters of life have felt this to reveal has been a seeking on the part of the self for its own reality, which carries it into something beyond And social experience, like that of art, is absolutely conclusive on this point

Difficulty in Subject being Predicate.

IV But a logical objection is raised against an apparent subject being in any sense a 'predicate'.[d]

[a] Rep, IX end and X 611 C

[b] As the need and value of the Fall is in Christianity

[c] See Bradley's Ethical Studies, p 288.

[d] It is noticeable that Mr Bradley, in the Principles of Logic, spoke of judgments about non-temporal subjects as a class of singular

Class Predication.

It is urged that to predicate an individual subject of the Absolute has the self-contradictory result that the Absolute is included, so to speak, in that individual as a class of one; in other words, is identified with that individual.

Now the connection of Subject and Predicate in judgment has essentially nothing to do with the inclusion of the subject S in a class P Judgments which are so construed [a] mean something else, and are in no case the equivalent of assertions of class inclusion. And the importance of this is, that though in some cases class inclusion is inferrible from (in no case equivalent to) them, yet, following the straight line of what judgment really means, not only do we never at all deal with class as such, but when we come to the point where Logic deals with the universal in its true form, we leave the preconception of class behind us for good and all. From the point where the development of the judgment branches into the assertion of differences within individuals (the Singular judgment with proper name for subject [b]) on the one hand, and the abstract nexus of different contents (the 'hypothetical' judgment) on the other, the predicate has nothing more to do with class significance The meaning which has really underlain all its forms, the holding differences together within a whole of identity, becomes sole and unmistakable

The real significance of the Individual Judgment is revealed when it has developed into the Disjunctive judgment, in which you have a significant individual subject-system set out in the subordinate actual forms which it takes under

judgments in which the true categorical, undiscoverable elsewhere, might conceivably be found He gave as an example among others, 'The soul is a substance' (Principles of Logic, c 2, s 7, 41, 81) I should suppose that the criticism of Appearance and Reality is hostile to the claims of these judgments to express metaphysical truth, although they might still be recognised as forming a separate class in Logic

[a] Bradley's Logic, pp 162–8.

[b] It may be said 'But you can treat this as a class judgment', as in the old type 'Socrates is mortal' The fact is, you can play almost any trick with the forms of proposition, because they have so much in common But what such a judgment form really challenges us to do, is to develope the content of Socrates as an individual whole containing connected differences And so the predicates should be regarded as elements entering into him, just as his acts are

different conditions. 'When the Absolute tumbles into the water it becomes a fish,' so in asserting itself under this or that condition of its own imposing it becomes Mr Smith or Mr Jones Why not? No true relation of membership within a concrete universal can be expressed in a class-predication, or in any judgment to which such a predication is equivalent And no judgment, strictly speaking, expresses by its form a class-predication at all

Imperfect Individual can be conditional Predicate.

v. It is said that a predicate is always universal (meaning abstract universal), and that therefore an individual can never be a predicate, even of the Absolute

First, it should be remembered that the individual is the highest and only true form of the universal, and if a member of a system, has in it the nature of the whole to which it belongs. It can, therefore, as has just been pointed out, be a predicate of that whole, if the conditions under which it becomes so are specified

Secondly, no doubt a wholly self-complete and self-contained individual could not be a predicate of any other individual. No conditions could be assigned under which it would be so They would be two perfect worlds, and there could not be two perfect worlds either side by side or in any dependence on each other [a]

But we are free from all difficulty on that head, for it is amply clear and confessed that our individuals are finite and imperfect. They are, as we saw,[b] members within a whole on which they are dependent for their very self-hood and self-identity Such individuals can certainly be predicates, in the sense in which members of a whole are conditionally predicable of the whole The nature of an organism, starting as an equipotential whole, may express itself, in elements conditioned by being at one end of the body, as a head, in those conditioned by being at the other end, as a tail, or according to other conditions as eye or gills Of course these organs are predicates of the organism as a whole, subject to the conditions which have differentiated them.

But there is something more. Individuals which like all finite individuals fall short of true Individuality are, as we

[a] Appearance and Reality, chap xv end [b] Above, 254-5.

have seen, not, as they stand, terms in true judgments [a] They have no power to resist reduction when considered in the light of the demands of reality, and are in ultimate analysis connections of content within the real individual to which they belong [b] And this becomes important in the present case, for though no one doubts that the self has a degree of individuality, yet its real character consists not in individuality but in a claim to it, which may or may not so far as a first appearance can show, be realisable consistently with the existing form and arrangement of the self That the latter is the case in a considerable degree, i e that the self, in order to become anything like a true individual, must be very greatly transformed from what we are aware of it as being, I hold to be an obvious truth, guaranteed by all experience and by every serious religious creed and philosophical belief [c]

I repeat and summarise, a member in a whole can be predicated, under a condition, of that whole which is his subject [d] And further, an imperfect individual is according to the degree of his imperfection a subject whose connection with his predicates can only be expressed in judgments which are untrue, and to bring his nature into harmony with the truth of the whole it must be transmuted and rearranged so that it can be expressed as a true connection within the content of the whole. It is plain from the argument before us that this is so The kind of individual with which we are dealing is unable to express his own nature in his own behaviour. He is, in Spinoza's language, very largely 'passive'; the acts ascribed to him are not his own And therefore judgments in which he appears as a subject are not true

Take as an extreme case the minds of animals, of which we spoke above Are they not best treated, most adequately

[a] See Mind, 74, loc cit, and above p 254

[b] See Principles of Logic, p 93

[c] It seems to me clear that Plato, even if he holds the soul to be eternal, does not think the same of the self

[d] Professor Stout, Ar Proc, loc cit, p 21, seems to me to take an example of part and part for a relation of whole and part The roof or pillar holds in each the nature of the cathedral, though not so directly that of each other Certainly each of these is predicable, conditionally, of the cathedral, as an eye is of the organism

treated, as streams or eddies of mind in which some temporary combination, some minor aspect or centre within the universe demands transitory expression, in cases where the needs and conditions do not exist which give rise to an apparently individual subject? No one, who has loved a dog, can doubt that its mind has a value of the same kind, if remotely the same, as his own [a] No one, on the other hand, can well suppose that it has the distinctness and organisation of content which we should expect of anything that is to have a permanent place of its own as a separate member of the system of reality. Surely the solution must be of the general type which conceives this partial mind as contributing a character, some intensification of loyalty and affection, to some greater existence,[b] but not claiming in itself to be a unique differentiation of the real.

Absolutism rests on its own substantial grounds

VI So far then it seems clear that there are plenty of good grounds, akin to but not dependent on the theory of judgment in question, for presuming that there is ultimately but one true individual Real, of which all contents that can be affirmed in judgments are ultimately predicates In fact it would almost appear that this is not denied [c] And if so we have only to establish, in answer to the second charge, that our theory of judgment lets one say, for ordinary purposes, what one likes about 'individuals' as subjects of judgments, and about other pluralities of terms

Freedom of Judgment on this theory

2 And passing here to the second charge (p 252), we recall that this is a point which has been greatly misapprehended

i For all current purposes of logical utterance—and we have seen that the criticism itself is at a level which concerns nothing more, nothing really ultimate—it is the doctrine of a sole self-existent subject which has first given complete freedom to the judgment. For, in view of it, the traditional theory of a

[a] Cp Bradley, Mind, 72, 508

[b] Compare the well-known and attractive doctrine of the final annihilation of the wicked only, which in its modern form consists in conceiving an individual participation in eternity as something which is not for every mind, but perhaps for some

[c] Professor Stout, loc cit, seems to me to affirm a plurality of ultimate subjects, but to admit that they are not ultimately self-existent

Grammatical S P rejected

proposition divided into S and P becomes a mere superstition.[a] It permits the judgment to be formulated as any complex of terms and relations, any arrangement of a plurality of apparent subjects, as well as of a substantive and adjective standing in the regular scheme as grammatical subject and predicate. Ultimately all this can make no difference, for the one part of the judgment is not judged of the other, the whole complex, grammatical S P and all, is predicated of the ultimate Reality. Thus one shape of judgment is as good as another, except that the most natural and convenient is the best. No one has resisted more strenuously the folly of forcing every assertion into the S P schema than believers in the sole Real. One may use the form of the common subject, as the present writer has urged,[b] to express the condition or limitation under which the ultimate Real accepts the predicated content. But this is not what the common S P schema takes itself to mean, and is not intended for a defence of it.

Thus I infer that no objection lies against the doctrine of a sole ultimate subject from any tendency to interfere with the freedom of the judgment-form, which this doctrine, more than any other, has tended to establish.

Individuality demands Designation—a mistake

11. But there is a last intensification of this objection to deal with. It is contended that whatever may be the fact as to what we commonly call subordinate individuals, whether in reality they are genuine subjects or not, our theory does not leave the question open, but cuts away *ab initio* all possibility of distinguishing them as individuals. For it restricts us to universal predicates, and universal predicates can never, by any complication of them, distinguish and define individual subjects. This can be done, and can only be done, by contact with immediate experience.[c] This is to say, that Individuality rests upon designation, to which a predicate can never be equivalent, for it never confines us to a '*this*'; and therefore an individual can never be defined by predicates. And I reply at once that Individuality cannot possibly rest on designation, and that what does so rest is not Individuality.

[a] See Principles of Logic, p. 23, and above, 1, p. 76.

[b] See above, 1. 3, 75-7.

[c] Stout, Ar. Proc., loc. cit., 19–24, cp. Bradley, Mind, 72. 500.

but Particularism,[a] the very sign of negation and imperfection, which, wherever it applies, is the proof *pro tanto* of the absence of Individuality But yet the point is in a sense perfectly sound. *Our* individuals, so far as imperfect, do depend on designation for the recognition of their uniqueness. And this is a conclusive proof that they are not and cannot be genuine individuals. I cannot understand why Professor Stout calls them ultimate when, as it seems to me, he explicitly admits that they are not[b] For designation just excludes self-containedness and self-completion, and that uniqueness which comes of filling a definite place in an ordered whole[c] It is tending, I note, to become a commonplace that the individual can only be designated and cannot be defined But in truth, ultimately, the distinction should be reversed A true individual cannot be designated, but it alone, and nothing else, can be defined. Designation means pointing as with the finger; mute identification *ab extra*, but the true

[a] Bradley, Principles of Logic, 76-7, Mind, 72, loc cit, and 74 167-8.

[b] Ar Proc, loc. cit, p 21. I cannot think that the argument of p 23 successfully impeaches Mr Bradley's criticism of the analytic judgment of sense

[c] No doubt (Mind, loc cit) it is impossible to recover, on the level of ideas, the uniqueness belonging to mere designation, the mute and negative 'this not that'. Objections may always be raised such as Mr Bradley raises in the passage referred to You cannot specify a partial and imperfect individual by conditions which ensure that he is unique in the universe But this is not because he is individual, but because he is imperfectly individual And *so far* as an experience possesses individuality—*so far* as it has a complete and self-contained nature, positive and real,—*so far* it has uniqueness because it includes the conditions which protect it against repetition by assigning its place in the universe That these conditions cannot be complete when the individual itself is partial and imperfect is only natural But for all that, in principle, uniqueness depends on completeness of explicit conditions and not on designation, and thus we are intensifying and not enfeebling it as we tend to complete the organisation of experience through ideas If a perfect individuality is not to be experienced in the form of discursive thought, that is nothing surprising, and in no way suggests that it may not be approachable through that form As we have seen throughout, individuality varies *pari passu* with degrees of being and reality It is altogether perverse to find in it the character of the datum of mere contact, as of the undefinable See Green, Prol, sect 194 and author's notice of G E Moore's Principia Ethica, Mind, XII 259

individual, and any real so far as individual, identifies itself not mutely but explicitly, and not *ab extra*, or negatively,[a] but intrinsically, by what it has in itself If about any element you are able to urge that so far as we know there might be another just the same, that is a sufficient proof that the first is not genuinely individual If it were, it would contain, in its completeness, the reason why there could be no other beside it. Uniqueness by designation, by mute contact with our experience, is, we might say perhaps, formal [b] Uniqueness by adequacy of content and absence of need or room for repetition we might call material In dealing with the imperfect elements which meet us in our experience we have need of both, but nevertheless the predominance of the one or of the other practically measures the whole distance, within our world, between the poles of the unreal which is merely given, and the real which is coherent and self-complete

Doctrine of ultimate 'Subject' leaves philosophical theory free.

III We have sufficiently seen then I hope, that (*a*) the theory of the sole ultimate subject does not stand by itself, as a logical caprice, in necessitating the theory of the Absolute, but is the consequence of a comprehensive and well-supported philosophical attitude, and (*b*) that it does not, as a matter of expression, forbid our common so-called individuals any completeness that could be ascribed to them, but that they fall short only because they are incapable of receiving more, and, in the higher forms of judgment are denied only a means of distinction (the 'this'), which could not possibly be serviceable in establishing their individuality.

[a] i e by a mute discrimination against others

[b] See 1, 106 and 207 above on the relation of the Demonstrative to the Universal Judgment.

CHAPTER IX

Truth and Coherence

1 It seems worth while, for the reasons assigned in the Preface, to restate at this point the general attitude of the present work to the theory of truth. Disclaimer of 'Correspondence' Theory.

The main current doctrines on this matter have been conveniently designated in recent discussion as the theory of Coherence and the theory of Correspondence respectively. I should hardly have thought it necessary to explain that I cannot for my own part conceive how the doctrine of Correspondence can be adopted as a serious theory, were it not that in an elaborate criticism [a] of the first edition of this work it has been urged that I have myself adopted it.[b]

The genetic theorists have discovered the failure of the correspondence theory, and believing some of us to be old-fashioned, they attribute it to us and then attack it. But we think, or at least, I think, that no logician really of the first rank ever held it, and that our critics are belated in awaking to its impossibility However, whether the misconception is my fault or my critic's, it will be well to make a short restatement of my view towards the close of so voluminous a work. The details of the criticism will all, I think, settle themselves if the radical misconception is explained But I

[a] Cf. 'Bosanquet's Theory of Judgment' Miss Thompson, Chicago Decennial Publications, 1903. I may note that for myself I entirely disclaim the epithet 'epistemological' which Professor Dewey frequently employs in his introductory essay For I understand it to imply a theory of cognition in which truth and reality are treated as external to one another, in fact, some form of the correspondence theory He is indeed discussing Lotze and not any writer with whose views I am in agreement. But I am not sure how far his criticism is meant to stretch, and it is better to guard oneself

[b] Under all the circumstances, this supposition recalls to me a vulgar story current in my youth, of a doctor who, finding himself unequal to treating the patient's actual ailment, proposed to inoculate him with a quite different malady, 'for,' he said, 'I'm death on that.'

shall further take occasion to express my own interpretation of the present position in the philosophical world, with reference to logical theory, which offers certain highly suggestive antitheses.

An Expression which might be misleading

i. I will say at once that there is just one expression in my book which in my opinion may have given some sort of colour to what I must call my critic's fundamental error It is the phrase in which I stated the relation of Reality, as the subject to be defined, to thought as the process of defining it I said [a] 'It is an essential of the act of judgment that it always refers to a Reality which goes beyond and is independent of the act itself' Such an expression, taken by itself, or in a treatise framed on wholly different lines from the present one, would not have been inconsistent with a conception of reality as an existent world external to our thinking, in resemblance or dissimilarity to which lay the truth or falsehood of our ideas But in criticising a work which takes for its watchword the saying that 'The truth is the whole', offering this as the solution of the difficulty that a world outside thought cannot be laid hold of by thought, it does seem to me a gratuitous misconception And further, I think it is one which not only every paragraph of general theory, but still more the whole progress and structure of the book disowns The mere structure of the treatise is enough to explain the expression

Reality is independent of the Judgment in two senses. There is, of course, an ultimate Reality, a higher experience than ours; we must postulate that if we do not mean to accept e g all individuals' worlds of experience as separate and unconnected When I speak of this Reality as independent of our act of judgment, as it is in an enormous proportion, I do not mean to exclude the truth that our judgment, in an infinitesimal degree, contributes to sustain it, and forms an element in its life. In this limited sense the two forms of Reality are interdependent. But their interdependence is not correspondence, and their independence is not that of original and copy

This Reality then shows itself in our world of experience

[a] 1, 97

in a way which is independent of our act of judgment in a second sense For our immediate experience, our feeling, our possession of a contact with a world, has individuality in a mode which as a mode of experience [a] our judgment cannot confer or originate, but can only attempt to restore by a secondary process, when its unity is transcended The contact in feeling has existence and quality together, and primarily is satisfactory and self-contained, though carrying a sense of diversity which challenges analysis in judgment, but is as such independent of interpretation through judgment.

We construct our world as an interpretation which attempts to restore the unity which the real has lost by our making its diversity explicit. This construction is our intellectual world. It is a form of reality, possessing some of its characters, and there are other forms, higher and lower. But none of them can be a world external to our thought and yet acting as its standard. The thing is a contradiction in terms, not because of the metaphor of externality, but because of the vital autonomy of the thought system.

If we ask, how we know our interpretation to be true or false, to possess or not to possess the character of reality, so far as its discursive form allows, the answer comes from the principle of non-contradiction, which is only another form of words for the principle that the Truth is the whole. This could easily be shown at length [b] The important point is that the principle of non-contradiction is positive and constructive; its force cannot be evaded by a logical quietism, by saying nothing. For you cannot get away from the world; if you try to say nothing you are in contradiction with a mass of experience, not with a presumed external world, but with what enters into your own being; and you leave it in contradiction with itself

11. Our doctrine of truth is therefore wholly immanent. There is no external standard, and, of course, no possibility of applying it if there were one The criterion [c] identifies itself

The Criterion immanent

[a] For its *content* is modifiable See below, p 297

[b] I hope to go over this ground in much greater detail in a forthcoming work of a more metaphysical character

[c] We have been warned that a criterion is properly a label, extraneous to the character which it indicates But any such criterion in highly

absolutely with that imposed by the doctrine of coherence.[a] And the structure and nisus of the treatise is a simple embodiment of this principle It is a progressive interpretation of the 'this', the contact with reality in which we possess both existence and quality It is an advance from one form of individuality to another, from individuality which has never gone beyond itself to individuality which has experienced contradiction and is being approximately restored as an explicit system of non-contradictory content It is a product of the interest and purpose to explain all that you can, to push the explanation further and further in response to the demand for removal of contradiction in the relative whole of experience at every stage This interest and purpose is the clue pursued by the effort of judgment from beginning to end It is the special and distinctive cognitive interest. And a treatise like the present endeavours to trace in its genesis the system developed by the action of this interest which is of course inclusive of all more special stimuli and occasions The whole interpretation, as referred to the individuality that appears solid, but therefore only implicit, in the 'this', possesses the character of reality, viz individuality, not perfectly, but in the degree in which the form of finite thought can achieve it[b] We know this by the fact that this character, the character of a systematic whole, is the condition of our possessing a world of experience at all. If we let a contradiction stand, we possess so much the less of reality Something cancels something, and we are the poorer and dissatisfied

Truth its own test

111 Immanence is the absolute condition of a theory of truth. It is this that makes the fundamental contrast between the coherence and the correspondence theory As I said at

organised matters is a bad, i e. highly fallible criterion And it is well to insist that in such matters the only sound criterion is the character itself or some important element of it

[a] See the author's Knowledge and Reality, p 331 (publ 1885), for a criticism of the simile of the foundations of knowledge This criticism is always decisive of a writer's attitude to the correspondence theory. Cf Bradley, Mind, 71 335

[b] Need I say that errors in the personal thought-process are expected and admitted ?

starting, truth is individual [a] This is only another form of words for the principle of non-contradiction, the principle that the truth is the whole, and the doctrine that coherence is the test of truth and reality Truth is then its own criterion That is to say, it can only be tested by the more of itself. Your completest system at the moment cannot be further tested You can only test it further when you are in a position to make it more complete Then what interferes with its greater completeness must go

Is it necessary to say a word about comprehensiveness? Sometimes we are told that our criterion is mere formal consistency This can mean nothing but that the critic has not thought the matter out to the bitter end By coherence or consistency we mean the consistency, so far as attainable, of the whole body of experience with itself Nothing less would satisfy the law of individuality or the necessity of non-contradiction But in this interpretation of consistency comprehensiveness is obviously included

Not all correspondence means copying.

IV One word more about correspondence If an identical principle operates in different worlds, e g in the experiences of different spiritual beings, the products are likely to correspond And I notice a tendency [b] to aid the process of inoculating us with the malady which is not ours, by insisting on this obvious truth. If the fundamental principle of reality is operative in the sphere of finite thought, of course this sphere will show a character that possesses certain common features with those of other spheres or of the ultimate real But if correspondence,[c] i e identity in certain characters of two or more systems, must result, that is no argument that correspondence is the criterion for either system If two men add up a sum right and therefore the same, that does not mean that the sums are right because they are the same, or that one man has copied from the other Of course, there is a reality which is more than an individual's thought. There is, at least, the thought of other individuals And undoubtedly these will correspond, i e. will show a structure identical in

[a] 1, p 3 above

[b] See even Joachim in The Nature of Truth, p 174

[c] On the nature of correspondence, see Essentials, p. 18.

principle but different in details But that is nothing against the character of both being immanently determined

I shall return in a later section to the question in what sense the coherence theory fails

The Logical World to-day.

2 I will now venture to state what I believe to be the cause and tendency of the peculiar logical movement of to-day

'Life,' 'Practice,' 'Feeling'

1. It is plain that the last half century has brought to philosophy in general a great revival of interest This revival has coincided with a marked increase of the tendency, traceable in European thought ever since Rousseau, to emphasise the philosophical value of feeling, of practice and action in the plainest meaning of the words, and of what has come to be called, in an almost technical sense, 'life' and 'living'. The movement has conceived itself as a sort of democratic revolution in the things of the mind,[a] and is obviously connected with the change of affairs in society and politics. A supposed aristocracy of intellectualist principles is to be dethroned. Truth is to become more vital, more accessible, its touchstone more obvious and more easily applied. Life, one may say, is to be substituted for thought as the central object and impulse of philosophy

All this has had and is having the usual effect of revolutionary demands in philosophy [b] The new theorists are insisting on something which was really vital in the older tradition, and the result of their movement will probably be a certain alteration of balance and emphasis in the formulation of that tradition One can hardly suppose that a movement so widespread and so popular will bring with it no elements of gain at all If it brought nothing but its adherents' interest in philosophy it would already have brought a good deal.[c]

The misconception which governs the movement

11 But the movement itself, I am sure, is conducted under a misapprehension It has hold of something very partial

[a] Cp the author's Philosophical Theory of the State, ed. 2, Introd. Nietzsche represents perhaps the 'Saviour of Society' who attends upon some democratic movements

[b] Cf the analysis p 231 ff above, of the relation of epochs of empiricism to the traditional distinctions of Logic

[c] As will appear, I believe this to be far the greater part of the gain it will bring

and consequently sees, and, as I have pointed out, further produces by its assumptions, a fundamental opposition where there is really nothing but a part unduly contrasted with its whole

Let us particularise Genetic Logic, the treatment of thought as a system or at least an aggregate of adaptations evolved in response to the needs of practice, has in principle adopted and popularised the coherence theory of truth This doctrine, being as we have seen a doctrine of immanence, is essential to all vital philosophy and logic, and, to the best of my belief, no other has in fact been held by any leader of European thought from Plato downwards [a] But by restricting the coherence which is to be the standard to the coherence of adaptation with external action, at first (as no one can doubt) in the purely normal and everyday usage of the latter term, it has on the one hand voiced a popular demand, but on the other has precluded a real understanding by itself of its own philosophical position And so it strongly tends, as we saw, to assume that in the older philosophy, which it feels to be in some way its antithesis, the view opposed in principle to its own, that of correspondence to an external standard, must be the prevailing one And it conducts its controversy on this basis, reinforcing its attitude by utilising another popular demand, that for actual individual endeavour and modification of things, which it is unable to unite (the great and ultimate test of a philosophy) with the belief in a perfect and timeless real. And the completer form of its own logical view, the coherence theory of truth, it is apt to stigmatise as a mere formal consistency [b]

[a] See, for example, my remark on Aristotle, p 224 above. I know that this has been adversely criticised, but I believe that when we consider the full meaning of apprehension by *νοῦς* as the sort of insight which comes, for instance, by induction, my view will be seen to hold good Cp for example, Burnet's Ethics of Aristotle, pp xxxvii and xlii. I may say in general that I should have guarded myself much more emphatically against the correspondence theory if I had ever imagined that it could by any mischance be imputed to me

[b] As I suggested above, the controversy is thrown completely askew if you take Lotze as typical of philosophical Logic The whole statement of the issue, as based upon the contrast of thought in general with reality in general (Dewey in Introductory Essays to Chicago

Fallacies of Genetic Logic

iii. Thus in a very able statement [a] of the contrasted positions of genetic and the older philosophical Logic, I seem to myself to find three connected misconceptions at the very basis of the whole representation

First, there is Dualism Thought is from the beginning conceived in contrast to its occasions It is taken as reflective, as what arises now and again when we set ourselves consciously to 'think' That is to say, this is the limitation of the thought with which the writer deals Something called Constitutive thought is mentioned in contrast with it, but whether this is simply the working thought by which we carry on unreflective life, or some theoretical construction of a creative force in the universe it seems impossible to tell What is clear is this much, that not merely the limitation of thought as a distinctive form of reality which operates through ideas, but the special limitation of 'pale reflective thought' as against 'active endeavour', or of 'abstract description' as against 'living appreciation' are accepted as formulations for the object of the new conception of Logic [b] 'Thought arises in response to its own occasion' Then, by removing only the definiteness of the occasion, which ought to be retained, and retaining the dualism of nature between constructive and discursive thought, which ought to be removed, an antithesis is created against philosophical logic which assigns to it as its characteristic problem the relation of thought in general to reality in general, as the epistemological issue out of which its whole treatment springs [c] And an apparent corroboration of this attitude is found by giving a predominant place to an analysis of Lotze's position

University Decennial Publications, 1903) appears to me thus utterly falsified If we want to deal with a master of philosophical Logic why not select Hegel or Plato or even Green? That is, if one was not going to take the obvious course of considering Mr Bradley's whole position with regard to Thought and Reality [a] Dewey, l c

[b] This takes us back to the conception of thought as decaying sense, which, whether right or wrong, is sharply opposed to the conception of it in the masters of Idealism I should explain that Idealism, in the sense in which I use it for the philosophy, say, of Hegel, is the antithesis of what is commonly called Rationalism But I know of no other name that would carry the reference

[c] Dewey, p 6, and cp Green, Prolegomena to Ethics, p. 27.

This idea of the situation—I say it mainly to make my own conviction clear—seems to me wholly and utterly false. The relation—the nature of the antithesis—is in my view altogether different from this

In Logic as I understand it, attempting to follow out at a long interval the practice of the masters, there is no epistemology in the sense supposed,[a] no treatment of thought in itself as opposed to reality in general, no question of a bridge from the one to other In analysing the thought-world it holds itself to be analysing the structure of reality, the detailed and articulated responses by which the living body of experience exhibits its endeavour to approximate as a system of ideas to a non-contradictory whole Of course all these phases could be construed as responses to the environment. But the environment for thought is not the sphere of external action but the universe of experience The occasions which evoke responses of thought within specific limitations are merely a fragment of this total environment The genetic theory, so it seems to me, has merely insisted on an arbitrarily limited fragment of the genuine logical theory.

From this, therefore, it is separated in degree rather than in kind, by a further error involved in its naive Dualism, an error for which I can find no better name than Occasionalism. Thought, we are told, is always within the limits of a specific occasion, a specific purpose It is charged against what is treated as general logical theory [b] that it disregards these limits, or only regards them as throwing light on the terms on which thought transacts its business with reality 'But in the end all this is incidental In the end the one problem holds. How do the specifications of thought as such hold good of reality as such? In fine, logic is supposed to grow out of the epistemological problem, and to lead up to its solution' [c]

[a] The explanation i, p 3 above, was intended to guard me against the appearance of dealing with 'epistemology' or 'a theory of cognition', by which I mean an examination of the nature of knowledge as something apart from the reality which is then taken as its external standard

[b] It should be remembered that this is not accepted as a just title for philosophical logic

[c] Dewey, p 6

All this, as I see the situation, is the same old half-truth turned into a complete delusion There is no discussion of a relation of thought in itself to reality at large No question arising out of it determines the course of logical investigation But it is perfectly true that thought (in a way, as we shall see, comparable to life, about which the same error is made) has in all its specific responses and adaptations the universe implicitly before it Its adaptations, like those of an organ in an organism, are controlled throughout by a system of functions which is a response to something continuous in the nature of the environment—as in life, to the conditions of organic existence on our earth's surface, so in knowledge, to the condition of belonging to a universe Occasionalism, the insistence on response to specific occasions as the condition of thought, thus misses its underlying and continuous character, as the active form of totality, the nature by which all experience strives of itself towards the whole Thought is essentially the nisus of experience as a world to completion of its world The intervals of conscious reflection are merely one of its forms of advance, and are not, in their paleness and meagreness, characteristic of thought, which is essentially organic concrete and constructive In its Occasionalism again the genetic theory is saying something so far true, but fragmentary, and is again taking it as the basis of an antithesis which has no existence, except as a relation of a partial to a more comprehensive view

And lastly, Dualism and Occasionalism take shape in Adaptationism This is more than a recognition—which would be justified—that all thought may be regarded as a response or adaptation to surroundings It consists (*a*) in neglect of the character of thought as a system of functions adapted to the removal of contradiction throughout experience and having always this complete systematic function operative in controlling specific responses or adaptations, And (*b*) in the suggestion that, considering the complete explanation of evolutionary growths to be only possible through regarding them as adaptations to their environments, each to each, the antithesis of origin and value ought to be treated as superseded, and psychology, for instance, should become in

its aspect of a historical science a serviceable instrument in logical valuation

a As to the former of these points, it is now I think recognised that to consider a living organism as a mere box of patent[a] contrivances, a collection of adaptations to particular situations of environment, is to consider it inadequately. Every adaptation is built on a system, and the system is determined by essential functions, which may be regarded if we like as a great general adaptation. But these functions, as a system, it must be borne in mind, constitute a large proportion of the environment for every specific adaptation. In every adaptation life is there as a whole, and has the whole nature of the environment in view, not as a general abstraction, but as a concrete whole that enters into every specific situation. So with thought It is, if we like, all developed as responses, but it is inadequately considered if it is considered as a box of tricks. Thought never really forgets the universe There is always more in it than its occasion brings, or rather, it makes its occasion more than it is. Thought not a collection of devices.

b. And the idea that evolutionary explanation has disposed of the antithesis between genesis and value seems to me more particularly to invert the real relation. It is true of course that natural history is much interested in natural selection; but the decisive point for logical theory is that natural selection is not in the smallest degree interested in natural history. One may fancy oneself pleading before the court of natural selection. 'Only give me time, and I can explain everything! The fact is, I was not adapted to to-day's environment, but only to yesterday's That is why I am not equal to the situation' But the court, I take it, replies, 'My dear sir, in the court of history that would be interesting, but in this court it is wholly irrelevant. We must ask you to deal with the situation of to-day, or—' It is being equal to the whole situation that is the criterion for Logic as for Morals Past adaptations can justify no theory of to-day. Have we or have we not a system Value not dependent on history

[a] The Mendelian theory is not quite this But even its way of regarding an organism, as, if I grasp the idea rightly, a group of more or less independent factors, seems difficult to accept without further explanation.

which gives the possible maximum of non-contradiction, in the construction which it puts upon the fullest conceivable experience? This is our standard for the present, and in it, for the past And Professor Dewey says what seems to me equivalent to accepting this standard. 'The historical point of view explains the sequence, the normative follows the sequence to its conclusion, and then turns back and judges each historical step by viewing it in reference to its own outcome.'[a] Yes, but the sanction lies surely not with the history of adaptation, which shows a certain stage to be *de facto* the outcome; but with the court of natural selection, which applies the test of adequate or inadequate adaptation, that is, of power or impotence to deal with contradictions, taking the whole body of experience together as constituting the concrete situation This is the test, the test of coherence and non-contradiction, which philosophical Logic accepts; the immanent test of the presence of the character of Reality within the thought-form as one of the many branches or appearances of the real

Psychology passing into Logic

iv. Psychology, from anthropology upwards, beginning with a natural history conditioned by quite other environments, leads gradually up to a situation in which, as the proper character of mind emerges, the logical test by present adequacy of working supersedes the historical explanation by past adequacy of work in a less complete environment. 'Working'; that is the apparent watchword, the name accepted on both hands for the test which might bring the two theories together. But to cover the problem of philosophical Logic it must take the environment as the widest conceivable experience, and must recognise the fact and right of cognitive interest[b]

When once the ultimate criterion is accepted, with the extension of the supposed new view to its natural boundaries, that is from practical working to dealing adequately with experience, I cannot understand how the relation of Psychology to Logic should present a difficulty. I have observed above that the epistemological attitude which the

[a] Chicago Publications, p 16

[b] The true type of the relation of Psychology to Logic is in the relation of associated contents—impure universals—to pure logical connections.

new theory is attempting to force upon the old is nowhere, so far as I know, accepted by it [a] And in fact the historical method, the explanation of past phases in the light of their environment, was not derived by philosophy from the historical or evolutionary sciences, but rather by them from it.[b] It is a notable characteristic of Plato, and could hardly have been more prominent than it is in Hegel's Phenomenology and in his Philosophy of Mind It is unfortunate that there is really no word free from irrelevant suggestions for what we mean by Idealism when we apply it to the philosophy of Plato or of Hegel But taking 'Objective Idealism' as a more or less accepted equivalent, we may say that the history and estimate of thought-adaptation in relation to the environment has always been the peculiar pride and province of objective idealism Only, the actual test of truth, of the character of reality in the thought-form, was by it always kept separate from the historical estimate of imperfect forms, the justification of which had shown itself, as we may say, doubly relative [c]

v This then, is one part of the logical situation as I feel obliged to conceive it It is well to vindicate for Logic the sphere of Life and practice as against an imaginary heaven of ideas—to which however no master of thought has relegated it It is well to bring the development of thought together with the conception of adapted response, and to apply to it the general idea of natural selection It is well to vindicate for the individual mind a living share in the self-maintenance of Reality as against the idea which Plato repudiated of a statue-like immoveable system All these are attitudes of special emphasis due to the philosophical and semi-philosophical movement of the last fifty years But if the reforming theorist limits practice to the sphere of external action, adaptation to the history of *de facto* success apart from the principle of its determination, and our living concern with Reality to effecting in it

Summary —Defects of the new attitude

[a] I have explained why I think it misleading to take Lotze as a specimen for criticism

[b] A remark of W Wallace I have not the reference

[c] 'Relative' as falling short by the standard of our best experience; doubly relative, because that standard is itself not absolute.

ultimate change, in a time which is ultimately real, then his view remains fragmentary, and he has failed to grasp the inheritance which is coming within his reach

Realism as Immediacy

3. Complementary to the view of truth which I have just attempted to explain—the view for which thought is an adaptation, and truth along with reality is *bona fide* in process of being made—is the reassertion of Realism in the modern world Realism, indeed, however opposed to the conception of a universe in actual genesis, belongs at bottom to the same impulse of modernism The very same flowing tide which carries with it the demand that truth shall be a mere adaptation to vital needs, brings also the antagonistic requirement that truth shall lie in a relation to simple given fact On both sides we have the demand for immediacy, here the immediacy of satisfaction, there the immediacy of apprehension. And the second, as we admitted of the first,[a] has doubtless, even from our point of view, contributions to offer. The first, we hoped, would bring about a correction of the confusion of Idealism with rationalism, and destroy the conception of a pale and meagre thought, identified with decaying sense. The latter, we hope, will undo the unhappy connection with mere psychicalism or mentality[b] and bring into prominence the more robust conceptions of a philosophy which admits true differences of kind within the whole.

I propose to devote the following chapter to explaining the attitude involved, in the theory of truth which has been followed through the present work, to mental states and the claims of naive realism.

But here some remarks will be in place concerning a doctrine of truth which, as far as I grasp it, shares on one side only the position of naive realism and simple apprehension, while on another side committing itself to a special theory of existence with which naive realism has directly nothing to do[c] The doctrine of simple apprehension, and the true meaning of the

[a] p 268 above

[b] See e g Mr Moore's Refutation of Idealism, cited and commented on in Joachim's Nature of Truth.

[c] I think that even in their theory of existence the two have an impulse in common, that of hardening into isolated existence purely relative objects

principle that knowledge makes no difference in what is known, will be spoken of in the following chapter

The world of simple things

a 'The world is a world of many things, with relations which are not to be deduced from a supposed nature or scholastic essence of the related things In this world, whatever is complex is composed of related simple things There is no identity in difference, there is identity and there is difference, and complexes may have some elements identical and some different, but we are no longer obliged to say of any pair of objects that may be mentioned that they are both identical and different.' [a]

The core of the view, as is well known, is the rejection of what have been called 'internal relations', i. e. relations grounded in the nature of the related terms; and the assertion of mere external relations, i e as I understand that there is no reason why relations should be so grounded [b] The phrase 'internal relations' seems to me not quite satisfactory, as suggesting relations between parts within a given term At least the view which to me appears reasonable would be better expressed by some such term as 'relevant relations', i. e. relations which are connected with the properties of their terms, so that any alteration of relations involves an alteration of properties, and vice versa

The following reasons for accepting a doctrine of relevant relations appear to me to be unimpeached

Relations relevant to kinds.

(1) In a large proportion of cases the relevancy of the relations to the properties of the related terms involves a community of kind You cannot have a spatial relation between terms which are not in space You cannot have a moral relation between terms which are not members of a moral

[a] Russell, Philosophical Essays, p 169 I do not think it is maintained on our part that relations can be deduced from the properties of single terms which are in relation I understand the point of interest to be that you cannot explain one term of a complex without explaining the rest Every complex, it must be remembered, has a special quality of its own, and every member of it has a quality relative to this; see 1, pp 139–40

[b] Op cit, p 161 It would be important to know if it is maintained that relations cannot be so grounded, because then we could ask for the author's explanation of the more obvious cases in which they appear to be so

world. Why is it absurd to ask for the distance from London Bridge to one o'clock? Surely because the one term is in space and the other in time. This is not a general argument that if the relation were other the terms would be other, from which any possible conclusion might follow.[a] It is an analytic determination of a common positive element on which both property and relation depend.

The meaning of terms relative to their grouping.

(2) There is further no case in which on philosophical scrutiny[b] the relevancy of relations to properties is not perceptible. I do not say that the relation can be reduced to a fact about the one object only together with a fact about the other object only.[c] The point of the relevancy of relations, as I understand it, is that each of two or more terms can only be understood if all are understood. 'Father' and 'Son' is a vulgar traditional instance. But I do not see that it is not a sound one. And in every case, I think, the basis of such a necessity can be shown. This or that observer may not possess the knowledge or the acuteness required to formulate the element which changes with the relation in precise detail. But it can always be shown what sort of thing must be relevant to the relation. So much so, that I cannot think this to be really and totally denied of so-called external relations. And I will pass on to a point of view which raises this question.

Relations express behaviour of terms in groups

(3) Relations are true of their terms. They express their positions in complexes, which positions elicit their behaviour, their self-maintenance in the world of things. This is really the all-important argument. And I cannot believe that if the doctrine of mere external relations were completely stated, we should not find the same thing admitted by it, in one way or another.[d] If the relations make no difference to the terms,

[a] Russell, Philosophical Essays, p. 166.

[b] I have in mind Mr. Bradley's argument in Appearance and Reality, ed. 2, p. 572 ff. [c] Russell, p. 161.

[d] As I understand the appearance of this is avoided by connecting the mind with the relation straight, so to speak, and not through the terms. But this seems to me simply a bold omission of a fact in the complex. Does not the conception of a 'sense' in a relation like love necessarily admit this? The term A is different according to the 'sense' of the relation of love between A and B. Or take spatial

it follows that things do not react or behave with reference to the complexes to which they belong Yet if Charles I had died in his bed, he would have died in a different bodily attitude from that in which he died on the scaffold.

b. I do not understand relations to be adjectives of their terms They are not adjectives because they involve other terms which are as substantive as any of which we might be inclined to pronounce them adjectives Relations cannot be reduced to qualities, nor qualities to relations Relations are just the way in which discursive thought represents the unity of terms which it cannot make adjectives of one other. As Mr. Bradley has said that they are a *modus vivendi* between predicates of the same subject whose unity we cannot really construe to ourselves, so it might be said they are a *modus vivendi* between terms in the same universe, of whose unity in the imperfection of our experience, the same is true

Relations not asserted to be adjectives

None of the objections which have been put forward appear to me to touch these points [a]

I quite understand that on the doctrine offered to us Identity in Difference must go. And I quite see for myself that it must go 'in the end', that is to say, in any experience for which objects are self-contained, and cease to transcend themselves What our pluralist realists [b] are grasping at is therefore justly anticipated Undoubtedly the Real is self-complete and self-contained But I insist on the words 'in the end' because it is their repudiation of them [c] that I take to be the root of their failure. They are the extreme Absolutists. They are not content to have the Absolute 'in the end', as we more modestly claim it, not meaning after a lapse of time, but in so far as what are fragments for us point out to us a completion beyond them And there is surely a difference of completeness in different experiences. But they

relations in the visual field When a new object is inserted in the field, every object in it becomes a member of a new pattern, and so necessarily exhibits a new quality

[a] Of course I am following Mr Bradley, Appearance and Reality, loc cit, though he is not responsible for what I say.

[b] I do not wish to use a name that will be disliked I merely invented an appellation that seemed to be fair, for shortness' sake

[c] Russell, pp 159, 163

will have the Absolute here and now ; and to make it handy and adaptable for everyday use they split it into little bits. A universe of tiny Absolutes , that is really what they offer us [a] But if any of these Absolutes imply any term beyond themselves their absolutism breaks down And we have tried to show that in all relations this is the case

Truth and Error not absolute.

c As to error we have only to bear in mind that degree of partiality of the truth asserted must combine with a belief that it is the truth, the whole truth, and nothing but the truth, in constituting the degree of erroneousness. And also, for practical purposes and within certain limits we let imperfect truth pass as absolute. A repudiation of the phrase 'in the end' denies these distinctions But surely in denying them it denies nearly all the facts of life Presupposing these reservations, what has been said in satire [b] is surely a plain truth, which only needs complete application to make it obvious. A man who accepts the view that all his judgments have only partial truth is certainly *pro tanto* less wrong in each of them than if he believed he had got in each the truth, the whole truth, and nothing but the truth But obviously, in this form the principle is only a general warning, and cannot directly amend the actual partiality of what a man judges as truth For this follows from determinate reasons, and in each case he must judge or not judge A purely general warning cannot guide his judgment. But it can stimulate him to caution and criticism, and this is an obvious excellence in his whole cognitive system, which is excluded by the belief that partial truth can be absolute A man who has grasped the warning that you must only believe about one-half of written history is certainly *pro tanto*, i e if both have the same positive knowledge, nearer historical truth than one who thinks he may with safety swallow it all And though this caution alone will of course not tell a man *which* half to believe, yet it will place his cognitive system in a much truer relation to the facts, than that of a man, who, making the same judgments as the other, believes them to represent absolute truth It is the

[a] I suppose this is a familiar idea in the case of the Atomists and the Eleatics See Burnet, Early Greek Philosophy, ed 2, p 387

[b] Russell, p 155

case no doubt, that you cannot, out of caution, half make a judgment, you must make it or not. But it is further true that to make an additional judgment, 'there is a good deal more to learn about' this or that character or incident, puts your positive judgments in a proportion to the facts which is likely to be much more in harmony with them, than if you entertained no such critical principle

d I believe the fact to be that the doctrine of which we are speaking gains its vraisemblance and its apparent clearness, from clinging to just the region of so-called plain and simple fact, the illusory hardness and isolation of which—really a defect of low-grade knowledge [a]—it takes for absoluteness. And in this region it does seem *prima facie* absurd to take error as partial truth You must be, it appears, either right or wrong The fact is fixed, and you are in relation with it or are not There are no degrees of truth, and nothing which is truth in the beginning and not in the end, or in the end and not in the beginning I will try to show the nature of this delusion, as it seems to me, by a few words on truth of fact and truth of system

The illusion of simple fact

'It is plain—that the truth or falsehood of a given judgment depends in no way upon the person judging [it is common ground that there must be a mind to judge], but solely upon the facts about which he judges' 'Thus the judgment that two terms have a certain relation R is a relation of the mind to the two terms and the relation R with the approximate "sense" [= direction of the relation from A to B or from B to A], the "corresponding" complex consists of the two terms related by the relation R with the same "sense". The judgment is true when there is such a complex, and false when there is not The same account, *mutatis mutandis*, will apply to any other judgment This gives the definition of truth and falsehood' The complex, it is to be remembered, is composed of simple related things [b]

[a] Of course no one uses the whole of his own experience in his theories. He uses what for some reason has struck him and seemed typical to him I do not think it is without precedent that men of very high attainments should rely theoretically on very naive types of experience I think analogies for this are rather common

[b] Russell, pp 169, 173, 184, cp Stout, Ar Proc. 1911

I wish to explain, by a comparison of judgments differently related to 'the facts', why it appears to me that, in the first place, truth and falsehood depend on the judging mind in another and more vital sense than is here admitted, and not on the mere presence or absence of a complex of entities corresponding to the judgment, and, in the second place, that the facts themselves, though they are real, are not real in the way here asserted, as bits of reality, immediately accessible to apprehension, and corresponding each to each with the terms of our commonplace judgments

Stating a fact may be telling a lie

(1) Let us begin with Charles Reade's mediaeval physician, who, having a grudge against a reluctant patient, tried to have him arrested, laying an information that he intended to fly the country But 'his sincere desire and honest endeavour to perjure himself were baffled by a circumstance he had never foreseen nor indeed thought possible. He had spoken the truth AND IN AN AFFIDAVIT' For the patient *had* fled

Here the doctor told a lie, but in telling it, he spoke the truth; if, that is, we judge by correspondence with the facts. One might urge that his assertion, being contrary to his belief, was not a judgment at all, but a form of words intended to produce action in another's This I think is true [a] But it does not seem to me to destroy the point of the instance Why could a form of words, corresponding with the facts, be in his mouth nothing but a lie? Because it was contrary to his belief? But what does that mean? Belief is not a chance thing, sprung from nowhere It means that it was contrary to the system of his knowledge as determined by his whole experience at the time.

Stating a fact may be an error.

(2) Take another case—the so-called true conclusion from one or more false premisses. Here again we have truth, if judged by mere correspondence with the hard fact. For example, on a local railway I know, the signals are down all Sunday A stranger unaware of this practice might infer that a train is due And it might well happen, three or four times in the day, that at the moment of speaking a train was in fact due Judging by hard fact this judgment would be

[a] See above, 1, p 34

true But would the man be right in his judgment? It is a point on which probably his companions might wrangle with him *ad nauseam* He seems to have got a truth which he had no business to have got If he had known a little more—possessed a little more truth—he would not have got it. And the truth, as he possesses it, is felt to be unsatisfactory, and half or more a falsehood, because its dependence is wrong. that is, it is judged, as a truth, in part at least, by the system of judgment with which it is connected And more than this, it is infected, in its own nature, by the faults of this system Its logical stability is highly incomplete, it would be upset by a second trial ten minutes later, or by a most trifling bit of additional knowledge But logical stability—incapability of being confronted with a contradictory experience—is, we shall see, the very core of truth.

In the closer tissue of a science, this defect amounts more obviously to actual falsehood The 'true' conclusion participates so definitely in the character of the system from which it issues Those theorists who held that agriculture is especially and peculiarly a desirable industry held, in this view, I suppose, what corresponded to an indubitable fact. But when they deduced it from the view that wealth is not genuinely produced in any other occupation, they connected it with grounds which destroyed its value, and made it a dangerous falsehood, by including in it an unjustified presumption against other forms of industry

Strictly speaking, there is no reason for dropping the premisses in stating a conclusion And if they, being false, are retained, the falsehood of the conclusion, though apart from this corresponding with facts, is exhibited on the face of it Here again, it is obvious that the truth or falsehood of a judgment depends not merely on correspondence to a complex, but on the completeness and comprehensiveness of the system with which it is connected in the mind [a] Its truth is threatened, we have seen, both if it is at variance with the

[a] You may say there is nothing in this but that one judgment about one complex is true, while another about a fuller complex including the first is false But what is shown is that correspondence to *this* complex is not enough to make the first judgment true

system, and if this system fails to give true connections, *prima facie* outside the judgment directly in question

How we come by 'simple' facts

(3) Now let us take the strongest instance in favour of non-dependence on the judging mind This, it appears to me, is to be found in the current knowledge of facts currently admitted, forming the stock-in-trade of daily life and conversation, and considered out of the context of science or of any critical analysis[a] 'Charles I died on the scaffold.' This judgment most people would describe pretty much in the language cited at the beginning of this section It is true, they would say, because it corresponds to a complex of terms and their relation, which are or were facts or things There is or was such a complex of things and such a relation between them, and therefore the judgment which expresses the mind's relation to it is a true judgment. What the facts are or were is taken as a matter of general agreement; it would be held pedantic to ask where we get at them, how we apprehend them, what precisely they are or were, what meaning the judgment actually carries with it Our intellectual outfit for everyday use consists of 'facts' postulated in this way—the normal furniture of our mind; what Plato called the world of opinion We take the material hurriedly from authority and tradition; or from negligent perception interpreted by authority and tradition[b] We do not pursue their context. We do not fix their limits or analyse their detail Thus we let them shrink and harden into isolated counters dealt with by our thought, worn and defaced by rapid and careless exchange. And it is of these current counters that our world of fact is constituted, which we take to be self-existent, independent of our minds, each fact independent of the others, related to them but unaffected by their relation, complexes which are the standard of truth to our judgment If our judgment corresponds to facts as presented to us in these current counters in which we commonly believe, that is all we ask

[a] It is such facts, I suppose, which another school would consider to have received Social endorsement, and to be made true by answering their purpose

[b] Could even a scholar, for instance, as a rule, exhibit a convincing argument that the works ascribed to the ancient authors were really written at such times and by such persons as is commonly supposed?

(4) Now I am not suggesting that these facts are not actual, and that the judgments which correspond to them are not true, in a sense sufficient for their purpose My contention does not tend to making less of the facts, e g to reducing them to mere ideas, but to making more of them, i e to showing that as realities they cannot stop at the arbitrary point we have adopted And, no doubt, it follows that the mind has had much to do with them already and must have much more to do with them as they proceed As they stand, they are a selection out of reality for everyday use, carelessly handed down or observed, clipped, worn, their interconnection neglected But they do well enough as a standard for everyday truth, and our judgments, which we take to 'correspond' with them, do well enough as everyday truth.

The facts are more not less than 'simple'

But, even within this world of what we conceive as correspondence to hard fact, we do acknowledge differences of truth, or, if this language is preferred, degrees of correspondence to fact, according to the furnishing of the mind 'Charles I died on the scaffold,' we commonly assume is not so true in the mouth of a child who has just learned it by heart as in the mouth of a schoolboy who knows something of the history and significance of the seventeenth century And in neither's mouth is it so true as in that of a historical student to whom the seventeenth century is a familiar world and a living interest It is not a thing which is true or false by touching or not touching [a] From the first, it is an appreciation of elements in a system, and of their determination by the system, and is a matter of degree Our ordinary estimate of truth fully admits this to be the case

We have been urging so far that the system of the judging mind is an element in truth, and also, in the last paragraph, we come in sight of an inference affecting the actual things or facts which are taken as the standard

(5) I will pass to an instance which clinches both these points.

In the full facts the 'simple' facts are lost.

When we come to consider the knowledge of any leading historical authority on the period of Charles I, we find two

[a] See Aristotle, Metaph , 1072 b 21 *θιγγάνων καὶ νοῶν.*

remarkable things First, quite undoubtedly, and in all common usage, such a man's judgment 'Charles I died on the scaffold' is far truer than that of the child or the schoolboy or the ordinary conversationalist This shows how much depends on the mental system of the judging mind

Secondly, when at this level we begin to look for the single standard of fact which we are accustomed to rely on, it is not to be found [a] At first sight, it is absorbed into the great historian's knowledge For us, something picked out of that knowledge is the standard. Our 'facts' as we used to call them, now show as little bits or threads of reality, which we or others for us have selected out of the huge web of the world as known by such an authority as this Of course the facts have not turned into any one's mere mental system But they seem essentially continuous with mental systems We do not mean to deny that they—as much of them as is warranted on good authority—are real facts. What we are saying can only mean that he helps us to get at them That is all very true, but then, when we get at them through his knowledge they are much developed from what they seemed when we were readily passing them from hand to hand among each other Now we see that even in their discovery they are not simple or independent They depend for being discovered and warranted on an enormous constructive work of criticism, starting from present experience, and continued through heaps and heaps of testimony and evidence all of which is instrumental to that view of facts which will give the highest degree of coherence to the system so constructed [b] Yes, but 'the facts',

[a] Compare with this the difficulty which the layman often has in asking a question such as a scientific man can answer To the layman a point appears simple and single which to the expert is full of distinctions and reservations The writer once procured a meteorological record of temperature with a view to its bearing on a stoppage in the building trade But he found, of course, differing readings of several instruments under different conditions, and could not tell, without further enquiry, which of the temperatures was important for his purpose. The simple 'fact' vanishes as you come nearer, as a headland breaks up into an intricate outline of planes and edges as you approach it, or if you try to read a book with a microscope

[b] Cp The Presuppositions of Critical History F H Bradley Parker, 1874

it will be urged. All this is getting at 'the facts'; but the facts were there all the same, however hard to get at; and when got at, by whatever means, can be and are the standard of truth.' Well; but we must consider the point that the facts are not to be found simple of themselves, as we incline to imagine in our everyday exchange of them. They are not and cannot possibly be the working standard of first-grade thought You may copy them in your judgment, when the historian has found them out for you. But the working standard, which determines them, is not themselves, but his immense critical construction Accounts of eye-witnesses, e g are nothing but material; and, as a rule, very contradictory material.

The facts, then, though bits of reality, are mediated to us by an immense mental construction, and are not really separable from this They are not and cannot be, as simple and isolated, the first-hand standard of truth We may select certain results and make them up into a standard for a certain level of truth, e g one good enough for examination purposes, and that different for different examinations But that is simply an artificial extract

That is one point. The facts, in history at any rate, are not simply there, so that they can act as a given standard, correspondence to which is truth The primary working standard is critical system, or, what is the same thing, scientific investigation.[a]

(6) But then there is another thing. When we get our facts, our results, what we take to be real, it is something much beyond what we were wont to take as facts. It is a commonplace that in the higher knowledge we are beyond what is commonly called fact [b] We may say that our current counters were fact, but they were neither the whole fact, nor nothing but the fact What is the full significance and implication of the death of Charles I? And could we seriously say that a judgment about it is true in which its full significance and implication is ignored, more especially as on the other hand the picturesque and immediate aspect of the event is

The full facts are comprehensive systems.

[a] See, for an example of what is involved in a simple measurement, if it is to be precise, Knowledge and Reality, pp 330–1.

[b] See e.g. Bradley's Logic, pp 92–3

certainly not affirmed? The facts are not 'in the end' isolated and independent There is a stage when they seem so, but you cannot arrest them at that stage. As coherence with a system is the standard by which we establish facts, so the part they play in a system of reality, their influence and importance, which imply a further transformation, is the standard by which we judge their degree of reality, and therefore the degree of truthfulness of the judgments that affirm them Ultimately, these two systems are one, the system of experience, a critical system which is always transforming the facts, as we know them and rank them, towards a higher logical stability

Of course these remarks contain nothing that is new [a] But I hope they clearly explain my view about the relative places of correspondence and coherence in the meaning of truth, and about the alleged independence, both as regards mind, and as regards each other, of the things or facts of the real world.

Coherence does not imply correspondence

4 The standard of system or coherence is a standard applicable to discursive thought It is the standard of truth, which itself does not pretend to be the perfect or all-inclusive experience [b]

A judgment is true, as I understand the term, when or in as far as its self-maintenance as a judgment is perfect That is, in other words, when the whole system of the judgments, which experience forces upon the mind which makes it, contains less contradiction in case of its affirmation than in case of its denial Such a judgment is 'true' because on the whole it cannot be denied—not, that is, till there is a change, other than its denial, in the body of experience

[a] The line of the discussion is closely akin to that of Plato's discussion of trueness and reality, which agree in the character of logical stability. See Companion to Plato's Republic on 479 ff, 509 ff

[b] It is perhaps hardly necessary at this time of day to say that I have now in principle adopted Mr Bradley's view of the relation of thought to reality, with which the ideas of my early work, Knowledge and Reality, were more or less in conflict I shall refer below to a reservation on this view which I still entertain, and which I think is consistent with the attitude of this work The point is merely that there is more analogy between the work of thought and solid and complete reality, than Mr Bradley, treating thought as solely discursive, seems to allow.

1. Stated in this way, which appears to me to be the right way, the doctrine that truth consists in the self-maintenance of judgments, which again consists in their systematic coherence, does not seem to me to fail quite in the way which has recently been imputed to it.[a] Judgment professes to express the nature of the real so far as it can be uttered in a system of predicates and relations. It does not propose or suggest, so far as I can see, that the real is another system of predicates and relations, which that constituted by judgment pretends to reproduce or to resemble Therefore its failure is one and decisive, simply consisting in the fact that it is not, like the higher experience which we suppose to be the sum and substance of all Reality, solid and immediate as well as perfectly individual and non-contradictory It does profess to qualify Reality, to tell us about the nature of Reality; and in as far as it arranges content in a non-contradictory system it does so tell us and qualify Reality It sets out the content of the real in a shape of special interconnection and emphasis, the definiteness and varied accentuation of which in the diverse worlds of knowledge constructed from different centres, obviously proffers a side of the whole without which the perfect experience would in certain respects fall short of perfection. In the dissociation of the perfect experience involved in *finiteness*, this side appears alone [b]

Coherence does not fall back on Correspondence

But, so far as thought is discursive, it does not profess to furnish any appearance of Reality but its own, and if it is said to be 'about' the 'other' of thought,[c] that involves no claim to represent the fuller experience in its own character. Reality is operative *in* truth. The nature of the latter's self-maintenance as tested by the principle of coherence, non-contradiction, or individuality, (all of them expressions for the same character) leaves no doubt of that. But the claim to have Reality at work in it, subject to special conditions, involves no appeal to correspondence, though correspondence in a sense must result [d] And in my view the fallacy above

[a] Joachim, Nature of Truth

[b] That is, markedly distinct in character No side of experience is ever really alone

[c] Joachim, pp 170–2

[d] See above, p. 267

signalised—a sort of *post hoc ergo propter hoc*—is involved in the assertion that 'current Logic, consciously or unconsciously employs the nature of truth as correspondence, and if that notion is challenged throws the burden of justification on metaphysics'.[a]

The failure or limitation of the coherence theory of truth lies then, I urge, simply in the fact that judgment, to which it belongs, is an appearance of reality in relational form, doing its best to attain individuality in that form, which up to a certain point it achieves,[b] but which, because it is relational and points endlessly beyond itself for completion, it can never thoroughly attain But it possesses, as we have suggested, merits of its own, clearness, special interconnection, emphasis, apart from which it is easy to divine that the ultimate Reality would lack an element

No 'approximation' to an original

11 Thus I suggest that the enquiry I am referring to leaves its own true track in emphasising the impossible demands of perfect coherence,[c] as an attribute or essential of perfect truth, instead of adhering throughout to the position that the perfection of truth is not within its own character, but must he in a reality different in kind The importance of this point is that in this way an imaginary perfect type of truth and coherence is set up, by their 'approximation' to which actual truth and coherence are to be judged The term

[a] Joachim, pp 119–20 This suggestion seems to me quite fatal to a working logic

[b] I shall return to this question, in speaking of the reservation above alluded to

[c] Ibid, pp 170–2 'A theory of truth as coherence, if it is to be adequate, must be an intelligible account of the ultimate coherence in which the one significant whole is self-revealed, and just before "any partial experience", e g human knowledge, is "true" more or less, according as it exhibits a character more or less *approximating to* the complete coherence' (my italics) I suggest that the 'ultimate' or 'complete coherence' is not an intelligible expression Coherence is the substitute, possible only in a system of predicates and relations, for the immediate unity, transcending mediateness, which we are compelled to ascribe to a perfect Reality I repeat that the affinity of two exhibitions of a principle, or of two kindred principles, has nothing to do with correspondence in this discussion, which means correspondence of a copy with the original by which it is to be judged The application of it in other senses in this context involves the fallacy of *post hoc ergo propter hoc*

approximation, I take it, involves the correspondence theory to which accordingly at this point the enquiry harks back Thus we lose the immanent standard, and with it the whole merit of the coherence theory. But reality in all its forms and phases can defend and maintain itself according to the principle of non-contradiction. It never depends for its relative logical stability upon approximation or correspondence to anything else.

The ultra-truth is not truth.

111 And further I suggest that it is a confusion to use the conception that even truth is not quite true to suggest a recurrence to a correspondence theory[a] The meaning of this conception is very simple when we once have grasped the point that no experience short of perfect reality is altogether itself It is in that sense, that even the truest truth, such as the coherence theory of truth, is not quite true; that is to say its fullest completeness lies in something, a more perfect form of experience, which is beyond itself, and we may call this, to emphasise the relation of transcendence, a truer truth

But it is not truth in the form of truth, and there can be no question of truth in its own form possessing correspondence or approximation to its character Truth stands on its own ground, as a fulfilment under its own conditions of the nature of reality, and it can be tested as truth under these conditions and under no others, and therefore, as we have seen, by itself only and by nothing else in the universe. There is no meaning in the suggestion that 'the coherence-notion of truth *on its own admission* can never rise above the level of knowledge which at the best attains to the truth of correspondence'.

The coherence-doctrine is a theory, and so far is only truth But coherence does not further and doubly fall short not merely by being only truth, but by resting its claim to be truth on imperfect correspondence It rests its claim on the

[a] Joachim, p 174 'Since all human discursive knowledge remains thought "about" an Other, any and every theory of the nature of truth must itself be "about" truth as its Other, i e the coherence-notion of truth *on its own admission* can never rise above the level of knowledge which at the best attains to the "truth" of correspondence Assuming that the coherence-notion of truth is sound, no theory of truth as coherence can itself be completely true,' &c

working of reality within it, and not on any correspondence that may result from this; and to get away anything truer you would have to pass beyond truth into another form of reality. This may seem a needless subtlety, but it is important to avoid the implication that truth as such is something away and beyond, which the coherence-notion ought to correspond to, but does not quite succeed. If this is admitted we lose our immanent standard.

Our quasi-solid world

5. One reservation, it seems to me, must be made upon the doctrine that thought is essentially discursive and relational. It points only to an anticipation of the fuller experience, and as I am quite aware, not to an achievement of it But it appears to me suggestive, and more than that, I cannot see my way out of it.

It is nothing more than the recognition that the worlds we severally live in, with the spatial world of each of us, have been fundamentally transformed and reconstructed by thought working in and on perception and general experience They are now, as for example our spatial world with its full properties and qualities, worlds all different and peculiar, and yet solid and individual in an appreciable degree, possessing up to a point existence and quality in one The interest is, that if this is so—and I cannot open my eyes without finding it so—we have created for ourselves by thought originally discursive, a new immediacy, a new 'given', a new basis of feeling and object-matter of simple apprehension Nothing is more various, more relative, more progressive and personal, than the so-called simple apprehension of objects which we roughly postulate to be the same For if we are to admit such a thing as 'simple apprehension', we must take it as purely relative. Its object is a phase of our experience and not a stratum of it [a] Our worlds are all different, and yet all apparently solid, and clothed in inseparable contents, which nevertheless are of our own discrimination and attribution And these are not as a rule taken as predicates. They are taken as belongings of the quasi-subjects or rather quasi-substantive objects, although we can separate any of these contents and make them into predicates The objects of our

[a] See below, chap x, *passim*.

world, which are thus admitted as concrete subjects, are of course affirmed in the general judgment which sustains the everyday reality which we accept. But they are as I said just now not naturally subjects in the sense of dividing themselves according to an S P relation. The judgment which affirms them takes most naturally an impersonal or existential form

When treated as subjects, they are not naturally taken as subjects of their nearest habitual predicates. These have qualified and clothed them, and are presupposed, not explicitly affirmed, in judgment. It is only in textbooks of Logic that we say 'Man has two legs'. 'The grass is green' and the like. All this belongs in usage to the solid starting-point, not to an S P judgment proper. But these starting-points, though relatively given, are really artificial, and in some degree different for every mind

These relative data or quasi-individuals are indeed the so-called subjects which were to count as a plurality of things[a]. But the interesting point about them is their relativity. Thought has made them, and as may be seen in any criticism of their solidity, can unmake them. And to speak more obviously and without reference to abstruse speculation, we can see that it is always remaking them.

This is all I desired to point out, that a quasi-real world, apparently solid and individual, is always being deposited as part of the work of thought. I draw no general conclusion but this, that thought which can thus deposit an apparent solid individual, is not so far removed from the nature of the fuller experience as an exclusive study of the discursive S P judgment tends to make us suppose. This was the side of thought which e. g. to Green seemed characteristic and important[b] I do not in the least care to enter into a verbal controversy whether it is more properly called thinking or something else. But that our discursive judgment itself is always building up a world which its operation then presupposes—the world in which each of us lives, and takes it as actual—this I do think is an important part of its character and a striking analogy between it and ultimate reality.

[a] p 277 above

[b] e g Works, III 144-5.

Does Truth copy this world? No, it is plastic

6 One word more This quasi-real world of our own making is always passing at its edges into the discursive S P process of science and synthetic judgment. And on this ground it may be objected to our view of coherence and correspondence, 'But here you have a real and immediate world, actual in your experience, and your synthetic judgments are about it Does not this mean that your truth is correspondence—the right representation of your relatively real and solid world?' And I answer, 'emphatically, no' For our 'given' solid immediate and real world, in which all these characters are merely apparent, is absolutely plastic,[a] as in all immediate judgment and every object of simple apprehension. It is just as likely that it may have to yield to Science or Speculation as that they may have to yield to it. Nothing in the whole field is a fixture to which all other elements have to correspond Nothing is certain except the necessity that the whole should be coherent

[a] This is not plasticity of ultimate Reality, but may perhaps have been mistaken for it.

CHAPTER X

The Relation of Mental States to Judgment and to Reality

It seems desirable, for the reason stated in the Preface, to conclude with a brief discussion of the sense in which, if at all, mental states enter into judgment and into the real world. The Mental States in question.

By mental states or facts in this connection I mean such as are taken to be concerned with judgment, and not volitions or emotions, unless of course these happen to be the content of judgment.

1. The view of the relation of mental states to Judgment[a] which has been adhered to in the present work consists of two principal considerations, Doctrine of the present work.

First, α, that no mental states in a human consciousness are mere mental states, but all contain matter that has been and may be significant, but secondly, β, therefore, that the difference between mental states and ideas with a meaning lies in the 'use' of the former.

α. In the first place, then, all sensational or perceptual contents, at least in a human consciousness, bear the stamp of some symbolic relations and hold their place in the systematic judgment which affirms our world. There are no ideas which are not directly or indirectly affirmed of reality, and therefore *a fortiori* none which are not symbolic or significant. All sense-content significant, and any of it may become a 'state' of mind.

In taking this view from the beginning,[b] I was strongly influenced by a fact almost too elementary to mention, but one which I am glad, nevertheless, to see plainly referred to in

[a] The view was suggested to me, of course, by Mr Bradley's Principles of Logic But he is absolutely without responsibility for my account or defence of it. I ought perhaps to refer to his footnote in Mind, N. S. 60 445-6, which indicates that he has intentionally modified the expression of his views on ideas since his Logic was written.

[b] See vol i, p 69 ff, and Knowledge and Reality, p. 142 ff., on ideas in fiction

a very valuable recent argument [a] It is the fact that for a human consciousness at all events there is, in the main, no storehouse of mere unapplied psychical material, no sensations which are unattached subjective states, nothing psychical which is not stamped and figured within its own sensuous being by divisions, relations, intensities, all relative to its meaning as signifying some object of thought There is in the main nothing psychical analogous to a painter's colours on the palette, before he uses them to represent objects It is a question for psychology whether sensational states can ever escape the despotism of significance, and be a something in the mind which belongs to and suggests nothing more than itself [b] I believe that if they could be proved to do so, it would be little more than a curiosity of research These would be, so to speak, immediate, or mere mental states, because they had not attained objective determination; because they were, crudely speaking, *below* objective apprehension and nearer to what we might conjecture of some non-human consciousness, in which, however, lacking the contrast to the objective, they could not be distinctively 'subjective' But it follows from the point of view we are now drawing out that the existence of immediacy, of mental states or psychical facts, does not depend on the reality of such limiting cases [c] Immediacy is a character that may be assumed by any mental complex or object, however logically articulate or external and independent of mind it may appear under certain conditions And no complex or object is altogether beyond it. Every one has its immediate mental aspect Thus, on the one hand, there is no mental state to be applied in judg-

[a] Professor Stout in Mind, January 1911 The present chapter has been influenced by this very valuable article Only, if I understand it right, it says that the sensational nuances are part of an immediate stratum, beyond which is the meaning to which thought is directed through them Thus you get—so it seems to me—a dualism, which I wish to deny, between the shaped and nuanced sensation, and the object.

[b] As I have asserted to be exceptionally the case, 1, p 71. I suppose it may be possible to have a sensation without taking it as meaning anything but just itself I do not believe it is a common experience, even, say, with pain or emotion, see below, p 300, and cp Hoernle in Mind, 61, pp 75-6

[c] See the other alternative specified, 1 71, above

ment, which is not already organised as part of a significant structure, an object of thought, to the nature of which, as we saw, its actual sensuous detail and constitution is subdued. On the other hand, there is nothing in this to prevent it from becoming completely and emphatically, what it always and necessarily is in some degree, a part of our psychical being, a particular mental state or occurrence, one with us in feeling and active in the total life of our mind And as we are convinced that there is nothing or almost nothing absolutely and finally immediate or wholly below mediation and determination, and yet immediacy is certainly a large element in the mind, it follows that this, viz psychical immediacy as one side of all ideas whatever, not only may be but must be a fact. Immediacy is not a stratum of our consciousness, but a phase which all or any of its objects participate in and may totally pass into. No doubt, what is an object of thought cannot in so far be an immediate state of ourself, and *vice versa* But it is always in some degree both, and there is nothing in its being an object of thought ever so determinate and elaborate to prevent its falling back into an almost complete immediacy, in which case it carries within it the full detail of the content which it possessed as an object of thought When we feel ourselves most at one with art or nature we are also furthest from being deprived of the qualities and distinctions of the content, rather we then live in them with the most sensitive completeness It is the objective relation—the externality of the cognised object—which is then in abeyance so far as immediacy is complete The content has really become all but a subjective state.

In a word, immediacy, or psychical existence, or being as a mental state, is a condition into which the whole mental content may pass, and into which it is capable of passing as a whole, the marks of thought and the stamp of objective relations being in no way obliterated by the transition. This is the old contention that sensation is full of the 'work of thought', as e g notably in the perception of distance, which is demonstrably not given in the peculiar sensation of the optic nerve, and yet is seen in a way prima facie indistinguishable from any visual sense-perception We cannot separate

sensation from thought, and it is not sensation only that can become immediate Immediacy, as was said above, is a phase and not a stratum of our experience, and mental states, and existence as a mental occurrence, are the same thing as immediacy

The 'use' of contents as ideas.

β Secondly then, it follows from this consideration that for practical purposes the difference between mental states, or particular existent mental contents or occurrences, and ideas with a meaning or universal thoughts of real objects (universal, because all real objects are universal, as persistent elements of reality) lies in the 'use' of the former I will put what I mean as frankly as I know how, to assist, if possible, both readers and critics rather than to guard myself There is, we have agreed, no great storehouse or constant new production of psychical states or psychical material such as to be primarily subjective, non-significant, mental existences or occurrences, waiting for objectification through a sort of christening and name-imposing process [a] But, all the same, we proceed in judgment as if there were Our world of known objects, of apprehension, sensation, perception, contains within it a mass of psychical stuff; and the fact that, as we have said, all this is stamped and appropriated *ab initio* does not in the least interfere with our treating it as a storehouse of such stuffs and using or appropriating it over again, and modified, in every new psychical production I hear a noise in the room overhead It is perfectly significant and I know exactly what it is; it is the patent roller-broom. But the fact that it is thus appropriated in judgment does not in the least prevent me from using it to think of a cab coming to the door 'A cab sounds just like that,' and this change of application of its content necessarily implies the recognition in it of an immediate or existential side, a treatment of it as something which is an occurrence in my mind and can be used to qualify a subject quite other than that which it qualified before. In this use its previous structure and nuance are partly disregarded and partly modified

Is this term 'use' a mere word? Can we not get nearer the fact which it indicates?

[a] Cp explanation, Introd, p 17 note

Mr. Hoernlé [a] has warned psychologists against the idea that the complex of word-meaning is formed by mere association. I suppose we may generalise his remark, and say that mere association is not enough to develop meaning It is different to think first of one thing and then of another, and to treat the one as a qualification or character of the other. No doubt this is true But we have surely learned of late that every association is at bottom a pure connection of judgment, and where judgment affirms a character of a subject you certainly have meaning This seems to give us the distinction we want. A content is 'used' when, in judgment, it qualifies a real world, when its nature, carrying us beyond its mental existence, makes us attend not to the latter, but to a quasi-independent subject, ultimately a condition under which that nature is true of reality. It is the work of thought as opposed to feeling, of what we know to what we are, if only we remember that thought adds no element to feeling, but merely re-organises its matter

Thus we use our stores and our fresh production of psychical stuff, in their original and acquired content,[b] or any part of it, to qualify subjects independent of those whose stamp and figure it bears *ab initio* within itself. And this is possible because it is after all in one aspect and more or less completely a psychical existent, and as such is not tied down to any significant structure, although primarily appropriated by one or another

And it should be observed as a general principle of judgment [c] that subject qualifies predicate no less than predicate subject; for the judgment is a brief expression of the same unity which is more fully uttered in inference, where premisses qualify conclusion just as conclusion qualifies premisses. This explains how a psychical content may appear to become more in meaning than it was as a content, when it is synthesised with a relatively [d] self-existent subject.

[a] Mind, 61, p 76.

[b] See vol 1, p 69, note, with reference to Professor Stout's paper in Aristotelian Proceedings, 1903

[c] See Appendix to II, chap 1, p 41 note [c], and vol 1, p. 69 Cp. previous note

[d] No finite subject is more than relatively self-existent On the

The term 'use' in short, implies the distinction between the existence and the content of a mental state When we take a character out of one complex and apply it to qualify another we may be said to 'use' it. It is a case of the law of thought by which the nature of an existent carries us beyond it. In being 'used' it becomes part of the clothing, so to speak, of a determinate subject in the world which judgment sustains; and in becoming this it is penetrated by new relations, and takes on a new stamp and new articulations in response to the nature of the subject which stands ready to receive it. 'The coat is rough,' 'The road is rough,' 'The sea is rough,' 'The man is rough' In each of these predications the common content which has become one with the word—say 'unevenness'—has taken on from its subject a special nuance and articulation But what is one with the content is itself a variable amount, and is affected by usage and by the nuances and articulations which it is in the habit of taking on The psychical existent or immediate is not a part of our mental formation but a phase of it. What is acquired enters into it as much as what is given, if indeed there is any sense in speaking of the given when it is impossible, finally and in principle, to draw a line between the given and the acquired *Any* content of apprehension or comprehension may become a state of our mind

The result of our discussion amounts to this.

All mental states are phases into which our objective apprehension under certain conditions may fall All our objective apprehension is something which is capable of taking the shape of a mental state, i.e. of becoming immediate. I may add as to the former point that none of the cases commonly alleged as cases admittedly of mental states or states of the self, e. g the experience of pain and pleasure, are wholly free from objective reference. A great part of the horror of pain depends upon this; that something seems to be devouring you, or growing within you, or crushing you, or piercing you, or tearing you The mental state in virtue of its content passes

fresh determinations acquired by a content in use see Professor Stout, Ar Proc, 1903 And on the self-transcendence of the existent, cp Hoernlé, p 75, who finds a difficulty in it

into affirmation, as affirmation in virtue of its existence passes over into the mental state. You cannot find a kind of experience which is necessarily a mental state and no more; nor can you find one which necessarily involves an apprehended object and no less. It is a fundamental error of principle to look for either. Immediacy is a phase and not a stratum of experience.

The Real not arrived at by subtraction.

2 Having thus discussed the relation of states of the self to judgment, it remains to draw the consequences of this relation in its bearing on the real world—to take a test case, though not the most important case, on physical reality.

The full detail of such a discussion as Mr. Prichard has recently devoted to Kant's Theory of Knowledge, goes beyond the purpose of a treatise on systematic Logic. I am only anxious to explain the attitude to the ideal construction of reality which underlies the present work, and for that purpose I shall refer to some of Mr. Prichard's arguments

I place in the forefront of my observations a principle, which I take to be fundamental, and to which I have already more than once referred. The significance of judgment and knowledge as of experience in all its forms lies always on ahead, and not behind; that is to say, in attempting to discern the real reality which justifies any experience you must go forward from it to the more concrete and more complete, and not retire upon something from which an element has been withdrawn. This is the well-known principle that the truth is the whole; the same is true of the reality, and it is in approximation to the whole, and not by disruption of the organism of experience into two opposing sides, that truth and reality corroborate one another. So far as the Realist movement of to-day is a reaction towards naive realism, it is a contradiction of this fundamental principle—an outcome of theoretical timidity and pessimism, which prefers in a difficult situation to seek safety in retrogression rather than success in advance. For the moment, I believe that this character is uppermost in it. But it must be, of course, at this time of day, naive realism with a difference; and I hope to indicate in passing that of necessity it carries elements within it which point to a real philosophical reform.

I will briefly observe on three points in which current

theory seems to affect the notion of apprehension and comprehension by the mind in relation to reality.

I understand it to be maintained

α That external objects, spatial and physical objects, are not in any sense or degree states of the mind, for in the first place, there is no need that they should be so in order to be known, and in the second place, if they were so, they could *ex hypothesi* not be known as apart from the knowing self, and it is an axiom that they are so known And as a subcontention to remove the apparent contradiction of the main doctrine, it is urged that knowledge is an activity *sui generis* and cannot be explained.

β It would follow that the present writer is wrong in attaching considerable value to subjective idealism as a propaedeutic [a] and a partial truth

γ It is dangerous to say that judgment sustains the world,[b] and untrue to say that *we* relate Predicate to Subject, that inference is a process of ideal construction, or that we ideally construct reality [c]

In my view, all these opinions depend on the one central fallacy pointed out above, that to find the reality independent of experience you must have recourse to a reality apart from experience

Knowledge involves Mental States

α We have seen that all judgment and its elements, not to mention sensation and feeling, are in certain aspects always, and in certain phases completely, states of the self

Reality lies ahead, not behind

(1) But it is urged that the objects thus known or the experiences thus experienced are not dependent for their qualities, for elements of their real being, upon judgment, sensation, and feeling, and therefore, though all these involve mental states, yet the objects known or the phases experienced in them have none of their reality in these states

I note at starting the extraordinary contention which this view makes necessary, that colours, sounds, smells, tastes, and sensations of touch are not qualities of things; [d] and with this, the attempt to show that because we know space to have

[a] Essentials, p 20 [b] Caird, Proc Brit Academy, vol I, p 106 [c] Prichard, 242–5

[d] Prichard, 86–7 To be on all fours with the other cases it should be what you feel in sensations of touch

three dimensions, though we can never see it so, therefore we know space as it is independent of perception This latter argument precisely illustrates the tendency which I call going backward instead of forward to look for reality. To know what space is involves perceptions of more than one kind, not merely visual, and, on the top of that, inference, i e a mode of conception which resolves the contradictions of perception. And because the knowledge of the nature of space involves the work of a percipient *plus* that of an intelligence, i e because the apprehended nature of space is a reconciled perception, going beyond the possibilities of actual perception into the realm of thought, we are told that the properties of Space do not involve a percipient at all [a] The fact is that they imply an intelligent percipient (if this needs to be formulated as though a percipient could be not intelligent) ; and that, just as the contradictions of perception have forced us to go beyond perception to something not presented in any actual perception, so the further employment of intelligence, in removing the contradictions of its primary constructions, may force us to assume as reality something which excludes the ultimate reality of any space at all You can only correct a perception by going forward in the positive process of removing contradictions. You destroy all positive reality if you attempt to go back by simple subtraction to a point anterior to perception and say that the the real is, what it is when perception is withdrawn—a predicate in this case involving, though not wholly given in, perception We see here the abstractions to which we are driven if we refuse to look for the self-existent reality in the inclusive whole which the effort to think things completely forces us to assume If we turn back in search of independence gained by omission, we cannot avoid committing arbitrary acts of abstraction like the foregoing at every step

Truth in 'Mind can only apprehend itself'

(2) Now we can see the explanation of the doctrine which we find ascribed to a mere assumption [b] It is held, we are told, that the mind can only apprehend what belongs to its own being, and that this is a gratuitous assumption. It is, if taken in one sense, a false assumption, but it is, as against

[a] Prichard, p 91

[b] Prichard, p 118

the opposite error, not at all gratuitous It starts from the obvious fact, now coming to be admitted even by realists,[a] that the object apprehended, say, in sense-perception or in thought, forms an integral part of our mental life and emotions, contributes influence to it, and derives predicates from it. But so long as this obvious fact is denied by a realism which so far remains naive (owing as I must believe to a very simple and popular fallacy which I will point out directly), any philosophy which respects the facts while bluffed into accepting this denial, must necessarily say that the objects of apprehension are simply psychical states and nothing more Subjective Idealism is the nemesis of realism What is wanted is to go forward, amending and expanding the experience which progressively approximates to giving us things as they are, under the full conditions which enable them to be what they are The reality thus attained altogether transcends at once our particular mental states and the thin abstractions which form the realist's actual world We need no assumption that the mind can only apprehend what belongs to its own being. We need only to recognise the obvious fact that what it apprehends at least participates in its own nature, a fact which the realist is forced to admit, both by the degree in which be truncates reality when he withdraws from it what he believes to belong to the mind ; [b] and by the degree in which the remainder which he is forced to leave to it still distinctly exhibits a living logical nature far transcending what can be ascribed to a physical object as physical Thus, he tells us, 'gold' is by itself and apart from mind 'a connection of universals'—i e it obeys the law of the self-transcendence of finite experience and embodies in itself a complex of conations.[c] And if we refuse to recognise this fact

[a] Who, so far as they admit it, are no longer naive realists.

[b] Prichard, pp 116–8

[c] (Prichard, pp 242–4) We shall see, too, below that inferential connection is to be a character of external reality *per se* Unless it is so, it is nonsense to speak of merely discovering connections. You can only *discover* conjunctions of fact. Necessary connections must be inferred by intelligence, unless they infer themselves I am perfectly certain that the present so-called realism, which does not yet understand whether it is naive or not, will have to hark back to an

in our theory of the reality beyond our particular mind, it must compel us falsely to concentrate the real into mere states of the finite centres themselves

(3) I note the same retrogressive impulse in the idea that explaining a thing means explaining it away, i e. deriving it from what is not itself [a] To explain a thing, surely, is to think it in terms of the whole A good explanation makes more, not less, of what it explains It draws the outline of its full individuality—which only its relation to the whole can exhibit —and calls attention to the law of its being Even an aesthetic product can be enhanced in value by good explanation. But I fully agree that it is impossible to have a theory of cognition, if that means a theory of cognition apart from a theory of reality, because to omit either is to omit what is essential to the full individuality of the other

What explanation means, viz. complete thinking

(4) And now we come to the supposed axiom of independent reality, and what I take to be the fallacy on which it rests. 'Knowledge unconditionally presupposes that the reality known exists independently of the knowledge of it, and that we know it as it exists in this independence' [b] If we construe 'independence' as = 'being apart from' I am quite sure that this statement is false Knowledge has no such presupposition We have no such conviction. The presupposition of knowledge, and our conviction, may be stated [c] in the first place briefly. 'So far as we know things, we know them as they are,' and then more precisely, 'Knowledge presupposes that the system of judgments in which it consists can maintain itself against any contradiction, and that the reality known is unmodified by knowledge except in the direction of being revealed as more completely itself' Thus the axiom as first stated rests on a plain fallacy *a dicto secundum quid ad dictum simpliciter* The nature of reality is not differentially depen-

Fallacy that Independent of =apart from.

outrageous form of Hegel's extreme doctrine, that every 'thing' is a judgment and a syllogism

[a] Prichard, p 124

[b] Prichard, p 118 'Independent' seems to = 'apart from', p 119.

[c] A difference of opinion about the statement of essential functions of knowledge is not at all a surprising thing The functions are relative to the whole, and our view of them depends on our view of it Cp. vol II, p 229

dent on knowledge, but it is a fallacy to go from that to the statement, 'Reality is what it is apart from knowledge,' unless, as seems to be partly the case,[a] you are relying on the contrast between knowledge and other forms of experience. It is a plain fallacy to say that because the difference between *a*, *b*, and *c* is not due to *x*, therefore *a*, *b*, and *c* can be what they are if *x* is withdrawn

And it does not suffice to suggest that to the special nature of any reality may be annexed the additional characteristic of being known.[b] The point is that the nature common to every reality—say, for instance, the admitted fact that it is a connection of universals,[c] involves its sharing the life and characteristics of experience. If it is argued that you must go to the special nature of every reality to determine whether or no it is dependent on mind (or, to put it more truly, and in a way that avoids subjective idealism, 'whether it participates in the life of experience'), the answer is that that question has long been considered and the result is not doubtful. The admission that the secondary qualities have special natures dependent upon mind is enough by itself to break down the principle that qualities of things must be independent of perception.[d] And if independence breaks down here, it cannot be maintained with apprehension and with knowledge The nature of reals is fatal to the axiom that we know things as they are apart from cognition The essence of this axiom is to look for reality in abstraction and isolation; rightly to accept the thing-in-itself as being what we know it to be; but without a shadow of justification or probability to assume that the being of the thing in itself is compatible with isolation instead of demanding as a condition of its existence the full context and interconnection of experiential life.

(5) This inclination is natural, because it promises to

[a] Prichard, p 118. Of course we do not say that knowledge is the only form in which Reality can appear, nor the most adequate But we do say that reality is inseparable from experience, and that is enough to negative the contention that it is what it is apart from mind. The only plausible case of being apart from knowledge is that in which, like pain, it is plainly not apart from mind, but another kind of experience takes the place of knowledge

[b] Prichard, p 116

[c] See above, p 304

[d] Prichard, p 120 ff, cp 86 ff.

common-sense a simple ultimate real, ensuring facility of treatment and finality in apprehension. But the anticipation of facility and finality in contact with the real is, I am certain, fundamentally vicious, and any theory which is guided by it stands *ipso facto* self-condemned. We have noted the involuntary recognition of this truth by the modern realist.[a] But as recognised under his dualistic assumption it becomes false. It tends to ascribe a psychical character to things apart from the mind. But what we want, and what an undistorted view of the presupposition of knowledge affords us, is not a psychical character of things apart from the mind, but a logical character of reality as revealed through the mind —self-revelation being essential and inherent in the real, whose nature apart from it is self-contradictory

A simple real must duplicate mental being.

(6) And in the proposition that the real is for knowledge the same that it is apart from knowledge I note an old elementary difficulty which seems to me insuperable. How can you separate the cognitive apprehension of an object from the operation of the organ of sense upon it? Grant, for the sake of argument, that if, *per impossibile*, you could compare a mind's cognitive consciousness of a perceived object with the full report of the living and healthy organ of sense directed upon it but minus consciousness, you would find that the bare addition of cognitive consciousness had not falsified the report, and therefore that the cognition had in no way modified the apprehension of the full object, the full object being the object of perception plus the modification effected in its physical operation by the organ of sense Then (omitting *ex hypothesi* the fact that in the absence of consciousness apprehension would not have taken place at all) under the impossible condition which I have granted you would be entitled to say that the intervention of cognitive consciousness left the object—the full object—precisely as it would be apart from such intervention [b]

If Independent = apart from, different physical objects are the same for apprehension

[a] Professor Alexander's contention, e g Ar Proc, 1908-9, that what are commonly called psychical objects are physical, cannot be carried through without a *rapprochement* between what are commonly called physical objects and the psychical. The comment of the text seems to me to apply to his view

[b] I omit for the sake of argument such matters as the influence of

But this, I take it, is not by any means the case which is in question when we are told that the object of knowledge is for knowledge the same that it is apart from knowledge In this doctrine I presume we are comparing the object as perceived through the organ of sense plus cognitive consciousness with the same as it is when withdrawn, not merely from the operation of cognitive consciousness, but from that of the organ of sense as well But that the object should be the same under these two conditions, of being known after transmission through a sense-organ, and of neither being known nor transmitted through a sense-organ, is surely a physical impossibility. A physical object cannot be the same when a complex physical condition is superadded to it and when that condition is withdrawn And the superadding of this complex physical condition is in every case inseparable from the apprehension of the object, therefore it cannot conceivably be the same when apprehended and apart from apprehension.[a]

For us this has no difficulty We do not doubt that the thing really is as it is apprehended, because its reality for us is its full self-maintenance under the completed conditions of experience But it seems to me absolutely fatal to the axiom of knowledge which was criticised above You may throw overboard, as appeared to be practically done by the view in question, the secondary qualities,[b] but you cannot in the full sense of the axiom secure even the primaries And I must insist that with the abandonment of the secondary qualities the axiom is absolutely surrendered If the real world apart from knowledge has no secondary qualities, it has hardly anything of what we care for It is not recognisable as our world at all

It may be argued that the nervous disturbance, or whatever the result of the sense organ's operation may be, is not the

apperception, and allow the hypothesis that cognition should bring no modifying influences with it to the report of the organ of sense.

[a] Note that in one place the theory we are discussing makes the real qualities of a thing exist independently not merely of knowledge but of perception and sentience (p 86) Thus sentience cannot be appealed to as presenting the complete physical object as it is apart from knowledge The real must be apart from sentience no less than from knowledge But cp. 306 sup and reff.

[b] Prichard, p 86

object of perception, and we are never so much as aware of it. That is a very good argument against saying that what we perceive are neural tremors and nothing more. But it is wholly irrelevant to the point that a vibrating violin-string is not sonorous in the absence of a hearing ear, and a candle is not a source of light in the absence of a seeing eye. These are simple physical facts; and they suffice to overthrow the doctrine that objects are the same for cognitive apprehension that they are apart from it [a]

In conclusion, then, the true axiom of knowledge is that we can know things as they really are And that means, that we know them in their full self-maintenance. There is no axiom that warrants us in arguing directly from the reality of full experience to the reality of abstraction, and if we adopt this procedure we must do it on the special ground that what we exclude is *ad hoc* irrelevant, a contention which always leaves us exposed to some degree of risk

And as to mental states, our conclusion so far is certainly not that the reality which we know either must or does consist solely of the mental states of particular minds. But, we urge, this proposition is false only because it would identify the real with a particular phase or character of mental being, and with that, moreover, as it is in each mind taken apart Our conclusion is, therefore, that the stuff of our mental states does really and truly characterise and belong to external objects, although, as characterising such objects, it ceases to be a mere mental state. It follows that the nature of external objects is continuous with that of the stuff of mind, and is physical, i e has variations relative to those of other objects, as well as psychical Physical or spatial objects are just as we know them, and truly have the characters which our knowledge (so far as it is knowledge, i e so far as it is consistent

[a] Professor Stout's doctrine as stated in Mind, 77. 12–13, seems to me not to be open to this objection, because, while holding that in presentation we have a glimpse of matter as it really is, yet in answer to the further question, '*What* matter?' he replies, as I understand, 'the matter of our body, and of other things only as affecting that,' not, as I read him, the matter of other things in so far as they do not affect that of our body I should not have thought that in this latter point Mr Moore would agree with him, as implied on p 9

with itself and with experience) attributes to them. The stuff of mental states, therefore, enters into them, and though the stuff of each particular mind is only a very small contribution to the real world, yet it is a contribution, and is capable, in principle, of furnishing some element which no other particular mind supplies

Objects then are only not mental states of particular minds first because they are not states of mind, but interdependent with minds and characterised by the stuff which is also experienced in states of mind; and secondly because if they were states of mind it would not be the states of any particular mind but the states of all minds It follows that the destruction of a single mind with its states neither destroys the objects apprehended by it, nor again makes no difference to their existence It diminishes, in principle, their amount of reality, and presumably moreover, in doing so, deprives them of some character which no other mind sustains in them, and the loss of which therefore is a genuine loss of being to the object So far mental states of particular minds are the material of objective reality, not merely as themselves psychical facts, but as facts which are necessary to the nature of real objects.

The object of perception *is* conditionally 'for me'.

(7) The same tendency to seek truth by omission is exemplified in the desire to treat perception as essentially less than judgment, and to deny it the right to use the word 'is'; setting down the expression 'is for me' as a contradiction in terms [a]

'The assertion that something *is* so and so implies that it is so and so in itself whether it be perceived or not, and therefore the assertion that something is so and so to us as perceiving though not in itself, is a contradiction in terms' The truth about this seems to me quite simple. Everything is what it is under conditions It is itself under full, normal, or typical conditions, the distinction of which from any partial or transient conditions is a matter of degree—of convenience for science or for common sense We cannot possibly attain a right conception of 'itself' by the withdrawal of all conditions That is the fallacy signalised above.

[a] Prichard, pp. 72 ff, 99 ff.

In perception, as in any other judgment, you may fail to be aware of the conditions to which your judgment is subject. In that case, judging unconditionally of what is conditional, you fall into error. But if you name the conditions, or indicate by a phrase of relation like 'for me' that you are aware of conditions though you cannot or will not mention them, the predication 'is' becomes appropriate at once Everything is what it is under conditions, and what it is 'in itself' is only distinguished by being under conditions which you accept as normal The thing which looks pink to a colour-blind man is pink to him [a] The condition of its being pink is present; that of its being blue is absent, and if it were not pink under the existing condition it would have to be other in itself than it is. The unconditional judgment of knowledge, interpreted as about the thing apart from knowledge, is false precisely in the same way as the judgment of perception is apt to be. An unconditional judgment must be false unless you interpret it to mean 'presupposing the normal or typical conditions'.

Subjective Idealism insists on vital continuity of universe.

β. I am therefore constant in the opinion [b] that Subjective Idealism is a valuable propaedeutic to philosophy and especially to Logic. I understand it to mean that we know nothing but states of our own minds It is not actually true, but it is truth of a higher order than that of naive realism. Naive realism I understand to deny that qualities which depend on minds can be qualities of objects, for it is essential to it that objects should be what they are apart from minds. This doctrine conditions reality by the withdrawal from experience, and so cuts the universe in two. Subjective Idealism is its nemesis. The latter urges that the qualities of reality are continuous with the psychical, and then, if and because you say that reality can have no psychical qualities, it repudiates the dead reality so offered, and limits the real world to what is admittedly self-maintaining in the context of a vital whole.

Thus it insists on the fundamental truths of the continuity of nature throughout the real, and the dependence of its being in some degree on every particular mind It is this continuity and this dependence—the life and wholeness of things—which appear more especially distasteful to realism, and which it

[a] Ibid, p 72 note. [b] Essentials, 19 ff.

avoids by the *ignava ratio* of withdrawing reality from any essential participation in the whole of sentience and intelligence Against this logical quietism—profoundly connected, I am convinced, with the timidity and pessimism of the day—the subjective Idealist raises an audaciously exaggerated protest. Whatever the object of knowledge may ultimately be, he contends, it can only attain or sustain what is worth calling reality in and through a share in the full and continuous life of feeling, sensation, and intelligence His error is to confine the life in question to the particular mind, and to reduce the real world to a mere combination of its states But this error means not that he is too idealistic, but that he is not idealistic enough He accepts the false doctrine which tells us that a real world independent of knowledge must be complete apart from knowledge, and rightly rejects a real world so defined. He cannot therefore suggest the only solution which can satisfy the logical need, a real world furnishing out of its self-dependent being a content to knowledge and experience, but of a nature continuous with theirs, and one in which self-revelation is inherent Nothing but this can satisfy the logical and metaphysical postulate that truth and reality lie in approximation to the whole And this postulate, which is one with the law of contradiction, lies at the root of Logic. Anything ultimately atomic, or independent, or non-plastic—any truth or reality not relative in respect of its qualities to its degree of participation in the whole, is irreconcilable with this postulate, and with a system of Logic resting on the conception of logical stability [a]

It is this fundamental principle that Subjective Idealism maintains, when it treats the real world, however erroneously, as a mere outgrowth or complex of mental states, and as dependent for its being on particular minds It thus emphasises what I may call the arduousness of reality, both of its theoretical conception and of its detailed apprehension Its conception demands to be maintained always on the level of the life of mind, that is, when we think of what we mean by reality we have also to think of feeling, sentience and intelligence in their various grades of perfection as actually involved and

[a] Cf. p 288 above

operative in sustaining the real in corresponding degrees. And its detailed apprehension in the same way depends for its completeness, not on mere adjustment to a hard datum, but on the degree in which the whole is brought to bear upon the given Therefore there can be no simple apprehension (except in the arbitrary sense that we may accept a certain level of insight as satisfactory *ad hoc*), and no such thing as a receptive attitude to data. The most obvious of perceptions involves the beginning of a critical process, progressive, arduous, and never finally accomplished All this is the logical lesson of subjective idealism, and therefore in spite of its failure to apply its doctrine to the self-dependent real object, I welcome it as a logical propaedeutic

Sustaining and constructing world

γ. 'When an idealist speaks of the judgment by which we sustain the world,[a] however adequate may be his explanation of such language, it is apt to excite a suspicion that his theories, if they were completely worked out, would lead to the individual being regarded as his own universe and his own God'[b]

'When it is said that our world, or the world *for us*, is due to our activity of thinking, and so in some sense *made* by us, all that should be meant is that our *apprehending* the world as whatever we apprehend it to be presupposes activity on our part'—'Nothing is implied to be made If anything is said to be made, it must be not our world, but our activity of apprehending the world.' 'Again, in judgment we cannot be said to relate predicate to subject . We must say that in judgment we recognise real elements as related or combined' 'Again, when we infer, we do not construct, ideally or otherwise.' 'Ideal construction is a contradiction in terms, unless it refers solely to mental imagining, in which case it is not inference' '—Inference would cease to be inference, if by it we made and did not apprehend a necessity of connection.'[c] 'The very nature of knowing presupposes that the thing known is already made, or, to speak more accurately, already exists.'[d]

[a] Cp e g vol 1, p 84 above

[b] E Caird, 'Idealism and the Theory of Knowledge,' Proceedings of British Academy, vol 1

[c] Prichard, pp 242-5.

[d] Ibid, p. 235

The points raised in the statements here quoted are primarily metaphysical, and as such go beyond the scope of this work. But they are greatly affected by a precise consideration of what we actually do in judgment and inference, and so far it seems necessary to offer some explanation in regard to them And if this explanation involves a word or two of indication as to where we stand, in a philosophical logic, with reference to the subject-matter of metaphysic, perhaps at the close of a logical treatise such an indication is permissible. The underlying question seems to be whether in cognition we are co-operating in the self-maintenance of reality, as ourselves organs within it; or are apprehending *ab extra* something finished and complete apart from us. Of these types of view it is the former that has prevailed in my treatment I am aware that it needs careful statement, and will endeavour to state it with due precision in what follows

How much then is meant and implied by saying (1) that our judgment sustains the universe, and (2) that in judgment and inference we make or construct reality ? [a]

Judgment involves a *world* and *one* world

(1) The expression in question was primarily employed to emphasise the idea of a total affirmative attitude to our world as distinct from the isolated judgment or proposition as commonly regarded in logic, *qua* selected and detached in abstract thought or in language

There were two things, as it seemed, to be insisted on, in conceiving such an affirmation as the genuine type of judgment. One was the overwhelming impression, conveyed alike by thought and by perception, of an affirmative and not merely receptive attitude in presence of our whole connected world. Everything, it seemed, had on it the stamp of meaning; and everything that had on it the stamp of meaning was *ipso facto* affirmed as soon as our thought rested, or our eyes were opened, on a whole including it. The point was not in any new account of affirmation; but that whatever we meant by affirmation in the fullest sense as applied to a definite judgment is true of our attitude to the world as a whole with all its inter-connected detail The example which seemed

[a] Cp with citations above, 1 42.

peculiarly insistent was the field of vision. It contained, as it seemed, innumerable judgments of magnitude and spatial inter-relation and other properties and relations, all affirmed at a stroke and so to speak as a solid whole on the simple opening of one's eyes. If the experiment is made of denying any one of them, any property or relation appearing within the field of vision, we discover beyond a doubt that we have affirmed it, seeing that we must either defend it by argument or admit that we were in error

And the second matter was the underlying wholeness and unity of this enormous world of affirmation [a] Everything in it is affirmed, but affirmed subject to being a part in the whole, and the consciousness of this reservation is essential to the affirmation. Every detail is asserted as in its way true and real, but only in its way, being modified by a huge co-ordinate affirmation, shading off into an undefined underlying unity, merely implied, but none the less affirmed [b]

This world of our affirmation, with its double aspect of innumerable detail and implied unity, seemed to be the world we practically lived in, and undoubtedly to be dependent upon our energy and capacity not for the nature of its content, but for being, so far as we had to do with it, more or less of itself. This was the primary sense in which our judgment could be said to sustain our world Its point was that our world was all in judgment, none of it in mere reception, and the amount of it so to speak, depended on—was coincident with—the amount of our particular mind.

This is the meaning of 'sustaining our world' with which logic is concerned It is merely an expansion of the customary application of the term 'judgment', demanded by consistency. It has the strictly logical advantage of putting in a strong light the unimportance of grammatical subject and predicate, which can hardly be found in the judgment so described, the ultimate subject being as always Reality; and also of emphasising the abstract and conditional character of the detached judgments which we make explicit as fragments,

[a] See 1 84 above, and Essentials, p 33 ff, and cp Stout, Mind, 77, pp 5 and 6.

[b] Cp Essentials, loc cit

attempting to compensate for their abstractness by precision of connection.

But if this meaning is challenged on metaphysical grounds, and we are asked how far *we*, we as finite fragmentary minds, can be said really to sustain the universe, we have not to disown anything that we have said. We are only driven a little nearer to metaphysic; and the answer is that we sustain the universe not only for ourselves, in the sense that it is through our own experience, and under its limitations, that we have to play our part in it, but in the more metaphysical sense that supposing a given mind and its states not to be,[a] the universe would actually be the poorer, however inappreciably, by certain elements of its self-revelation peculiar to the experience of that finite mind [b]

The former of the above views is all that strictly belongs to Logic, if we are at all to distinguish Logic from Metaphysics What Logic aimed at [c] was to show by what characters, exhibited in the structure of cognition, the object of cognition is revealed as more or less of itself and as possessing more or less of reality or logical stability

Metaphysic would have a return game to play It would show that finite minds which for Logic sustain the universe, are ultimately organs moulded by it and through which it sustains itself Both points of view are true, and it is the test of a philosophy to succeed in combining them [d]

Both Realists

(2) To say that before we know or apprehend an object it must exist, or its making must be finished, complete and apart

[a] We must not say 'to perish', or 'cease to be'. That would raise the question whether such events were not mere appearance—whether anything could be lost to the universe

[b] Not merely by lack of the mind as a member of the whole, as by lack of an unconscious creature, but by lack of its view of the whole

[c] 1 3 above

[d] In this respect the relations of Logic and Metaphysic are not unlike the relations of morality and religion In morality, too, the 'individual' seems to sustain the weight of the world. All is on his shoulders, he acts out of himself, all turns upon what he does next So it does, and the point of view is a true one But if we go on to ask what the 'individual' is, and how far he is really and truly an isolated and independent creature, then we get into the provinces of society and religion, which modify without annihilating the purely moralist point of view

from the knowing or apprehending, is a very simple and natural version of the presupposition of knowledge which we referred to above.[a] It will serve, indeed, as a first approximation to the true relation between knowledge and 'making' which I hope to suggest. It assumes, I repeat, a reality given as self-complete, and that apart from the apprehensive organ or process We, the knowing mind, stand outside a reality which is complete without us, without us, at least, *qua* cognitive subjects And I believe that to be consistent it would be necessary to add, without us as sentient beings either.

and Pragmatists put us outside reality

It may help us to discern the true relation of the process and object in question, if we first turn to the extreme opposite of the above theory of apprehension, an opposite with which I, quite as much as the new realists, am at war. It might be expressed by saying that in knowledge we create, and create not only truth but reality We make it, that is, *out of* nothing, and by means of nothing We do not make it by the law of the universe or of our minds or of the two together or of anything at all, out of anything that was given Against any idea of this type the reproach would be flung—'Then *tout est donné*'

This view presupposes a modifiable ultimate reality, outside which, in some strange fashion, we stand, with a miraculous power of adding new determinations to it in virtue of no existing nature, either of it or of our own [b] It is strange and suggestive that both the one-sided attitudes necessarily place the mind outside the reality; in the former case that the mind may not dictate to the real, in the latter, that the real may not dictate to the mind

Now we can discern what our attitude must be. We have rejected all ideas which make truth depend on copying or correspondence But again, we hold continuity and intelligibility to be destroyed if pure creation is possible—if, that is, ultimate reality is modifiable, if, that is, once more, the real is modifiable otherwise than by its own law.

[a] p. 305.

[b] It does not matter, I think, whether these consequences are accepted by the theorists in question or not The test is the rejection of continuity

For us, the universe is undoubtedly the object of apprehension and knowledge, prior to them and determining them. But it is not an object given and complete as a whole apart from apprehension and knowledge, still less, of course, from experience.[a] And the relation of apprehension to it is neither copying on the one hand, nor creation on the other These are fit expressions for the working of a mind from the outside upon a hard and self-complete real, or upon one unfinished and incapable of self-determination respectively. But for us the mind is a constituent of a living and self-determining real. We might apply to its activity the term apprehension, with explanations, or perhaps even the term creation, again with explanations But it will be best to look first at the nature of our object, and to judge for ourselves how the relation of knowledge to it should really be described.

Nature of object of Cognition.

(3) The object of cognition, then, is not a simple object It is not all given, and it is not given as a whole. If we try to get at it on the assumption that it is given, we find it eluding us, and extending into an infinite succession or extension of particulars in space and time And with all our trying we find in the end that even these particulars are not directly given. Now strictly speaking nothing but a given fact can be 'apprehended'. Therefore it is clear that our universe, *qua* object of cognition, cannot be in the strict sense 'apprehended'. To know it as we presume it to be, that is, as a whole and a system, we must 'apprehend' what is not given; and this qualification of the object at once modifies the idea of apprehension It is just as when we read in Plato and Aristotle about 'imitation'. It seems simple enough at first, to imitate a given object directly But then we find that one can 'imitate' a moral character with paint-brush or chisel, or even, perhaps, one can imitate what is not but ought to be. Such qualification of the object must expand the meaning of the verb; and so in knowledge the nature of our object changes the meaning of apprehension and cognition.

[a] If this latter point is to be taken as admitted by the new realists, it appears to me that the whole question is decided. It is impossible seriously to admit into the reality of the existent universe some forms of experience, and exclude others, *prima facie* of a completer kind

If you are to 'apprehend' an object by far the greater part of which has to be got at by inference, and all of which is continuously transformed as apprehension proceeds, you can only interpret the term apprehend in a way which has something to do with making I suggest as the key to the required meaning some such word as 'elicit'. Your object of knowledge has in a great measure to be elicited from the given. For the object is a whole, but not a given whole, and is always beyond itself, and is offering more to your apprehension than is already there for you to apprehend.

If we are substantially agreed on this character of the object, it becomes a verbal question whether we say that we construct [a] reality, that we *relate* a predicate to a subject in judging, that we *make* an inference, or that we *apprehend* the given and fragmentary facts with such modifications as logical necessity requires, and so again *apprehend* the necessity with which a predicate, not given, belongs to its subject, or a conclusion, *ex hypothesi* not given, belongs to its premises, or by which one factor of reality, given *per se*, conditions and is conditioned by another, also given only *per se*. In the latter case it is fairly certain that some corners of the given will have to be cut off through the operation of the conditioning necessity.

Now strictly, I repeat, it is misleading to say 'I apprehend' anything but a fact or a given conjunction of facts If you say 'I apprehend' the product of two numbers of five places each, I think it would be held that you must be a calculating

[a] I have myself commented on the term 'construct' as properly expressing only the auxiliary employment of mental imagery in inference (II 33), and there is a verbal inconsistency between that passage and the earlier one which speaks of the construction of reality, I 42. What I meant to urge was that the metaphor from physical construction does not really explain the construction of reality It is the latter that is the true construction, and the former derives its significance from it. You only construct even physically if you follow a principle and make what you meant Mental construction is mere imagining only if you slide from the relevant to the irrelevant content But if, being in possession of a mass of conflicting evidence concerning, say, a historical event, you frame a conception which harmonises a great part and explains the contradictions of the rest—the typical process by which we learn what reality is—I hardly see what to call the process if not construction

boy. But if we like to say 'apprehend' in the sense that possessing the given and the law of its system, and operating on it accordingly, we are able to apprehend it with modifications which belong to it, of course we may say so But any distinctive implication as to the priority of the *special* object of apprehension appears in that case to have vanished.

Thus it may be urged that all critical systems which support and elucidate 'facts' by bringing them into a coherent whole —e.g the facts of history [a]—all 'laws of nature' and appreciations of necessary connection, all intelligibility by which the given itself is known and framed in inter-connections and inter-dependences which are not and cannot be given, are ultimately 'found' in the universe and not 'made', and so are 'apprehended' and not 'constructed' But surely this is to go back to the naive empiricism which so much pains and labour have been spent in transcending If anything at all has been made clear in the history of philosophy, it is surely that as we get to truth, especially of the higher orders, we get away from facts It is not a contention especially of philosophical idealism It is as true for Mill and Jevons and Mr Bertrand Russell [b] as for Lotze, Green, and Bradley, not to speak of earlier philosophers No working Logic can be put together upon any other basis

Now of course the predications in question are presupposed to be true of the universe It is their general presupposition and prior object But it, if you omit from it mind as a constituent, does not furnish a given object of apprehension which corresponds to them or which they can be said to copy You may say our higher knowledge is a shorthand for innumerable facts That is, of course, extreme empiricism, and leads to the denial that the universe is a whole But this position is so far sound, that *if* you contend that judgments to be true must reproduce facts that are simply found, then our higher knowledge must be mere shorthand, or false If these higher

[a] Cp Mr F H. Bradley's Presuppositions of Critical History

[b] That is to say, so far as I can follow, all Mr Russell's higher truths, viz. those of Logic and Mathematics, are truths of implication and in no way deal directly with particulars Mr Russell would not, I presume, assent to the consequences which I suggest

truths are 'found' in the universe, they are only found as the statue is in the marble, that is, in such a way that the process has more to do with the result than the matrix material In the controversy about the relations of knowledge and practice it has become clear beyond dispute not only that practice, besides making the product always finds the real, but that knowledge, besides finding the real, always modifies the given To know the truth of anything, you always go further afield Even if you ask, 'Is the sensation of touch A a sensation of touch?' you mean to ask about it something further, e g 'Is something more to be detected than I have apprehended, which will be decisive that what I feel is a sensation of touch?'[a] This is the beginning of construction—ideal construction—which rests on the fact that all finite reality goes beyond itself, and is completer and completer in more and always more of its connections By this ideal construction it has—to borrow a phrase from another context—its own nature progressively communicated to it

But do we 'make' the construction? Is it not there as a whole, though unknown to us at first, and we apprehend it? Does not making exclude knowledge? The answer has been offered in the whole preceding argument, and it is for the reader to judge What seems to us is, that we certainly do not create the construction, but no less certainly do not in any natural sense 'apprehend' it It is true—conditionally true—of our universe, and in that sense, is 'there' before we make it But it is not given before we make it, and this does not apply to individuals because of fortuitous ignorance, but to the whole nature of what can be given compared with what can only be inferred If we rule out of the universe the living power of mind which alone can complete the given into a coherent whole, a systematic construction, modifying the facts with a view to their inter-connection, could never come to be given at all It has not only less—less irrelevancy—but more—more relevancy—than what can be given or found, and can never be adequately described as found or given. A necessary connection even between two given terms is a reciprocal inferential relation,[b] implying a system beyond

[a] Cp Nettleship, Remains, 1 180 ff. [b] Cp e g vol II, p 8 above

either, and not present in the apprehension of each alone or of both together

It is only by a misapprehension, as I have tried to point out, that such an interpretation of knowledge seems to conflict with its presupposition Knowledge is an essential form of the self-revelation of the universe, experience as a whole is *the* essential form In knowledge, the universe reveals itself in a special shape which reposes on its nature as a whole, and is *pro tanto* proof against contradiction It is, like the whole wealth of life and of history, elicited from a relatively given by the forming and interpreting activity of mind, which in this activity is an organ of the universe itself. Plainly, if you hold that to be a part of the universe disqualifies knowledge from being true (because without it or prior to it the universe is not completely there) you must, as I said, to make knowledge capable of truth make it external and additional to the universe

But the detail which it presents in the form of cognition *is* true of the universe, although falling within it, because the universe, *qua* object of cognition, in its self-maintenance against self-contradiction in that form shows that it must take the detailed shape which it does take and no other, and to know is to endow it with that form, making the given more and more of itself

And in Logic we study the character of the universe or the reality with reference to the degrees of stability, which, if we postulate *bona fide* employment, attach to the various frame-works at different stages of the structure arising from its self-utterance in that form.

The ' driving force of Idealism ', as I understand it, is not furnished by the question how mind and reality can meet in knowledge, but by the theory of logical stability, which makes it plain that nothing can fulfil the conditions of self-existence except by possessing the unity which belongs only to mind. External objects, therefore, are fragmentary and dependent.

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